Abstract
Drilling hydraulic optimization for drilling requires knowledge of the pressure loss in the system as well as the pressure loss through the drill bit. Energy per unit volume defines pressure and stress (fundamental definition).

A new nozzle coefficient (1.03) was described by M. Ramsey, et.al., (1983) and, independently by T. Warren (1989). In an independent study, reported in this paper, the nozzle coefficient was examined while drilling with 8 ½" roller cone bits with 3000psi ambient pressure. The data validated the 1.03 nozzle coefficient; however, an understanding was developed about the other values of nozzle coefficients which have been used in the past. Data is presented which clarifies the different coefficients currently in use.

Turbulent flow pressure losses can be calculated from the concept of kinetic energy per unit volume. The nozzle pressure loss equation currently used is derived from the energy/volume concept. One of the most popular nozzle coefficients (0.95) indicates that the pressure loss through nozzles is larger than the kinetic energy available (110%). The 1.03 nozzle coefficient indicates that 94% of the kinetic energy creates the pressure loss. Data analysis indicates why both of these conditions are correct in various situations. A phenomenological evaluation of drill string pressure losses serves as a basis for understanding how to calculate the pressure drop through bit nozzles.

History
While developing a telemetry system designed to transmit data through a wire while drilling, pressure was measured inside and outside a drill collar just above the bit. The results were surprising. The pressure loss in the annulus was calculated to be about 30 to 40 psi with equations popular at that time. The measured pressure loss was on the order of 300 to 400 psi. However, the pressure loss through the nozzles was measured to be about 300 to 400 psi lower than calculated. This meant that the standpipe pressure agreed with calculations but individual components were inaccurate.

On one field test (1) with the wire-line telemetry, the pumps were properly calibrated, and the nozzle diameters accurately measured. The nozzle coefficient (Cd) was determined to be 1.03 instead of 0.96 used by most bit companies in the equation:

\[
P = \frac{(MW)(Q^2)}{12032(C_d)^2(A^2)}
\]

Where P is nozzle pressure loss (psi), MW is the mud weight (ppg), Q is the flow rate (gpm), and A is the area of the nozzles (in²)

Tommy Warren (2), at Amoco Research, independently reported the same value of nozzle coefficient in 1989.

The industry was slow to change their method of calculating nozzle pressure losses as evidenced by the new API RP13D publication (3). The API Recommended Practices on Hydraulics indicates that the nozzle coefficient may be larger than 0.96 but fails to endorse the 1.03 value.

The lack of acceptance of data measured field and confirmed in the laboratory raised some questions about the nature of the nozzle coefficient. Why was it invented and what is the genesis of this value? This paper discusses the theory behind flow through conduits, such as drill pipe and nozzles, why nozzle coefficient is used, presents additional data which shows values of the nozzle coefficient less than and greater than 1.0, and explains why. First, some basic theory is discussed in terms of calculating pressure in terms of a more fundamental definition than force per unit area.

Introduction
Pressure is energy per unit volume. For a standing liquid, the energy is calculated from potential energy equations. For flowing fluids, pressure is calculated from kinetic energy equations.
Potential Energy
Pressure in a liquid, or stress in a solid, is the energy per unit volume. For example, in a static column of fluid, the pressure at any depth is the potential energy per unit volume at that depth.

Pressure

\[
P = \frac{\text{Potential Energy}}{\text{Volume}}
\]

Potential energy may be calculated from the equation:

\[\text{Potential Energy} = mgh\]

Where \(m\) is the mass, \(g\) is the acceleration of gravity, and \(h\) is the depth of fluid, or the height above the point of interest.

Pressure would be:

\[P = \frac{mgh}{\text{volume}}\]

From Newton’s Second Law of Motion:

\[W = mg\]

Where \(W\) is weight, or the force applied to a body by the gravitational attraction.

A ratio of weight to volume is called density, \(\rho\). The equation for pressure becomes:

\[P = \rho h\]

To convert the units to oil-field variables and calculate pressure in pounds per square inch, \(\rho\) should be expressed as pounds per gallon and \(h\) in feet.

\[P = MW \left(\frac{lb}{gal}\right) \left[\frac{gal}{231 \text{ in}^3}\right] \left[h, ft\right] \left[\frac{12 \text{ in}}{ft}\right]\]

This equation reduces to the familiar equation used extensively in well control:

\[P = 0.052 \ (MW, ppg)(depth, ft)\]

Kinetic Energy
Pressure in a flowing liquid can be described as kinetic energy (KE) per unit volume.

\[P = \frac{\text{KE}}{\text{volume}}\]

\[P = \frac{1}{2} m v^2\]

\[P = \frac{1}{2} \left(\frac{m}{\text{volume}}\right) v^2\]

Where \(P\) is the pressure, \(m\) is the mass, and \(v\) is the velocity of the fluid.

Weight per unit volume is density \(\rho\). Newton’s Second Law: Weight = mg.

Substituting this into the equation results in

\[P = \frac{1}{2} \left(\frac{\rho}{g}\right) v^2\]

This can be converted to oil-field units where the density is in pounds per gallon, and the velocity is expressed by a ratio of flow rate \((Q, \text{ in gpm})\) and area \((\text{in square inches})\).

\[P = \frac{(MW)(Q^2)}{12032 (A^2)}\]

where the value of \(g\) is selected as 32.17 ft/sec².

This pressure is a function of the density of the fluid, and the square of the velocity \((Q/A)\).

Nozzle Pressure Loss Analogy
Before discussing nozzle pressure losses, consider flow through a pipe connected to a tank of liquid with a constant head. Calculate the pressure in the pipe at Points A, B, C, D, and E using the kinetic energy equation used to calculate pressure losses through nozzles.

Figure 1: Constant head flow through horizontal pipe

The velocity along the pipe is constant because the flow rate is the same at all points. The mass moving through the pipe is constant. Consequently, the kinetic energy \([1/2mv^2]\) constant along the length of pipe.

Pressure is kinetic energy per unit volume. The pressure at point A, B C, C, and E will be the same. Obviously, a pressure loss occurs along the pipe and is dependent upon whether the flow is laminar or turbulent.

The nozzle pressure loss equation currently used is derived from the equation that Pressure equals Kinetic Energy per unit Volume. The density [mass/volume] is unchanged along the length of pipe. The pressure calculated from kinetic energy would be constant. With a constant pressure all along the length of pipe, the flow rate would be independent of pipe length.

However, the pressure inside of the pipe decreases along the pipe as the fluid flows from the high pressure end to the low pressure end. Obviously, another term is required to properly calculate the pressure inside of the pipe at all points.

Consider the pressure losses resulting from placing a turbine at the end of the pipe. The pressure at the bottom of
the standpipe can be calculated from Potential Energy. As the fluid moves into the pipe, the constriction of flow streams results in a pressure loss. This is typically called a “velocity head loss” and is commonly observed in centrifugal pump curves. As the fluid exits the large diameter cylinder, the pressure may be calculated from the Kinetic Energy equation as discussed above. All along the pipe, however, the pressure is decreasing. This pressure loss depends upon whether the fluid is in laminar or turbulent flow. The pressure loss across the turbine blades depends upon the construction and restrictions caused by the turbine.

Nozzle Pressure Analogy

This example could also be related to what happens as drilling fluid flows through drill bit nozzles. Inside of the bit, the flow is diverted from a large diameter area into very small diameter jets [like the bottom of the large diameter cylinder]. A ‘velocity head’ loss occurs. As the fluid flows through the nozzles, a significant reduction in pressure occurs [as shown by the flow through the pipe]. The fluid exits the nozzle, strikes the bottom of the hole and reverses direction to flow up the annulus [similar to the pressure loss caused by the turbine at the end of the pipe]. In a nozzle, three pressure losses comprise the total pressure loss through the nozzle: 1. Entrance loss; 2 Through nozzle loss; and 3 Exit loss.

A phenomenological analysis of laminar/turbulent flow

Fluid mechanics provides methods for calculating pressure losses through pipes. If honey is flowing extremely slowly in a smooth wall pipe, the pressure loss will be proportional to the flow rate according to Hagan-Poiseuille’s Law:

\[ \Delta P = \frac{8 \mu L Q}{\pi R^4} \]

Where \( \Delta P \) is pressure loss,
\( \mu \) is viscosity,
\( L \) is length,
\( Q \) is flow rate, and
\( R \) is pipe radius.

The flow is laminar if the pipe walls are smooth and without obstructions. Hagan-Poiseuille’s Law can be written in terms of velocity, \( v \), instead of flowrate:

\[ \Delta P = \frac{8 \mu L v}{(R^2)} \]

Laminar flow can be illustrated with the following series of pictures.

**Figure 2: Pipe before flow starts**

**Figure 3: Position of points after one second of flow**

In laminar flow, all points across the diameter of a pipe would move parallel to each other. If the fluid wets the surface of the pipe, the first layer of fluid is bound rather tightly to the surface. The outer sections of the flow stream move much more slowly than the center layer.

**Figure 4: Envelope of velocity profile is a parabola.**

In the first picture, certain places in the flow stream are identified at time zero. If the fluid is moving with laminar flow, the places will have moved down the pipe but not all are...
moving with the same velocity, as shown in the second picture. The fluid wets the wall of the pipe and the first layer of fluid moves very slowly. The fluid near the wall of the pipe moves much slower than the fluid in the center. In the third picture, the new positions of the original places form a parabola. This is the envelope of the velocity vectors of the fluid moving in laminar flow in the pipe. Although not all fluid is moving with the same velocity, all flow streams are parallel to each other. This illustrated in the next picture.

LAMINAR FLOW

All the fluid will move in a straight line

Figure 5: Parallel flow streams in laminar flow.

Laminar flow means that all movement will be parallel to the sides of the container and no flow stream will intersect another. Achieving this requires careful planning and, obviously, very smooth sides. In this case the pressure loss along the pipe will depend upon the velocity of the fluid, and the viscosity of the fluid, as indicated by Hagan-Poiseuille's Law. Most drilling fluids are Non-Newtonian. The viscosity depends upon the shear rate, as discussed in Appendix A.

Turbulent Flow

Turbulent flow is much more complicated than simple laminar flow. Although the fluid is moving down a pipe, some components of the fluid are also moving in many other directions. A fluid at rest will not move unless a pressure is applied. This means that in a turbulent flow situation, many small pressure differentials are developed within the fluid which allows the fluid to flow in directions other than the direction of the main flow pattern.

Reynolds Number

The most common consideration to determine pressure losses of a Newtonian fluid flowing in a pipe begins with the calculation of Reynolds number. The dimensionless Reynolds number is a ratio of the inertia forces per unit area divided by the viscous forces per unit area:

$$R_e = \frac{\rho V L}{\mu}$$

Where $$R_e$$ is Reynolds number.

As the viscous forces diminish compared to the inertial forces, Reynolds number increases. In circular pipes, turbulence is damped out if $$R_e$$ is less than 2000. For $$R_e$$ from 2000 to about 4000, the flow is called transitional. For $$R_e$$ above 4000 the flow is considered turbulent. Another way to look at these numbers is to say when the inertial forces per unit area are less than 2000 times the viscous forces, the flow will be considered laminar. When the inertial forces are greater than 4000 times the viscous forces, the flow is turbulent.

However, the Reynolds Number is not accurately calculated for Shear Thinning Fluids like drilling fluids. A more complicated Hedstrom number must be used to judge whether the fluid is flowing in turbulent or laminar flow.

In laminar flow, fundamental laws produce a result that can be confirmed by experiments. Turbulent flow, however, is much more complicated. For example, at high Reynolds numbers the disruption of the laminar film adjacent to the wall of a pipe renders viscous action negligible. The velocity distribution and friction factors depend upon the magnitude of roughness or discontinuites in flow patterns rather than only the Reynolds number as in smooth pipes.

Head loss ($$h$$) in a pipe may be calculated from Darcy's equation:

$$\frac{h}{l} = 6 f \frac{v^2}{g d}$$

Where $$l$$ is pipe length, ft; $$f$$ is the friction factor; $$v$$ is velocity in ft/sec; $$g$$ is the acceleration of gravity, ft /sec$$^2$$; and $$d$$ is the internal diameter of the pipe, in.

For turbulent flow the friction factor is a function of Reynolds number ($$R_e$$):

$$f = \frac{0.184}{(R_e)^{0.2}}$$

The exponent of 0.2 on Reynolds number means that the effect of density and viscosity on head loss is small.

Actually, both the viscous forces and the inertial forces continue to contribute to the pressure losses in a pipe. This reveals the real problem of trying to calculate pressure losses for Non-Newtonian flow inside of drill pipe. At each tool joint, there is a turbulent initiator. The pressure loss in the turbulent zone will be proportional to the velocity squared. In other regions where the flow is not turbulent, the fluid viscosity will dominate and the pressure loss depends more on
the fluid viscosity. In Non-Newtonian flow, the viscosity varies with shear rate and, of course, temperature. Predicting where these transitions will occur is almost impossible, consequently precise calculations would be difficult to achieve.

The pressure loss for laminar flow is proportional to velocity and for turbulent flow the pressure loss is proportional to the square of the velocity. Viscosity does not appreciably affect the pressure calculation in turbulent flow.

As fluid moves through a conduit, some disruption in the laminar flow streams is entirely possible depending upon the velocity of the flow and the nature of the walls of the conduit. In the regions where flow is disrupted, the viscous forces are not as important as the inertial forces. Consequently, the pressure loss in a pipe might be a function of the velocity or flow rate raised to an exponent between one and two.

Summary

With laminar flow, the pressure drop will be a function of the velocity and the viscosity of the fluid. With turbulent flow, the pressure loss will be independent of viscosity and depends on the density and the square of the velocity.

What happens when there is a blend of the two types of flow? Suppose a pipe has discontinuities along the walls as shown in the figure below:

![Discontinuities Cause Turbulence](Image)

**Figure 6: Initiating Turbulence**

How can the pressure loss through this pipe be calculated? Most of the pressure loss will be proportional to the velocity (or flow rate) and viscosity; however, some components of the flow are producing pressure losses proportional the square of the velocity. The degree of turbulence in the flow stream is dependent upon the magnitude of the disruptions as well as the damping effect that Non-Newtonian flow properties can exert. See Appendix B for further comments on babbling brooks and other related turbulent events.

![Turbulence created by tool joints](Image)

**Figure 7: Turbulence created by tool joints**

The problem arises when calculating the pressure loss through a drill pipe. At each tool joint, a discontinuity in the flow stream can create a region of turbulence. The pressure loss through the drill string will be some function \( f \) of a combination of laminar and turbulent flow pressure losses:

\[
\text{Pressure Loss/length} = f(x \ P_{\text{lam}}, y \ P_{\text{turb}})
\]

Where \( x \) is the fraction of pressure loss in laminar flow and \( y \) is the fraction of pressure loss in turbulent flow. The amount of turbulent flow will depend upon the shape of the flow path and the fluid characteristics. If the fluid has a very low viscosity at the shear rates imposed, the turbulent zone will propagate a long distance down the next section of drill pipe. If the fluid has a very high viscosity, the turbulent zone will be damped rather quickly. The viscosity of a non-Newtonian fluid varies considerably with temperature, shear rate, and the exact ingredients in the fluid. The amount of damping will be almost impossible to predict.

Because the major component of pressure loss through nozzles seems to be the turbulent component, the Kinetic Energy equations are usually modified for calculating pressure losses in the drill string. With fully turbulent flow, the pressure loss is proportional to the flow rate (or velocity) squared. Computer programs use a flow rate exponent of 1.86 to compensate for the fact that the flow is not fully turbulent and not completely laminar. This exponent can be measured at the rig and exponents have ranged from 1.4 to 1.9. This technique to determine this exponent was published in 1982, and modified for longer bit runs in 2001 (1).

The analysis described above can be extended to describe flow through bit nozzles. Nozzle pressure losses are normally measured from a point just above the drill bit inside of the drill string to a point in the annulus just above the drill bit. Most of the flow will be turbulent, but some components may also have characteristics of the laminar flow pressure losses.

Usually, the pressure loss through nozzles is calculated from this equation with the addition of a nozzle coefficient, \( C_d \), in the denominator:

\[
P = \frac{(MW)(Q^2)}{12032 \ (C_d)^2 \ (A^2)}
\]
Where $P$ is the pressure loss through the nozzles, $MW$ is the mud weight, $Q$ is the flow rate, $C_d$ is the nozzle coefficient, and $A$ is the total flow area of the nozzles. This equation indicates that the pressure loss is proportional to the square of the flow rate.

The nozzle coefficient is used as a ‘finagle factor’ to correct the pressure loss calculation.

The “viscosity” term also causes much confusion. Viscosity is defined as the ratio of shear stress to shear rate. With a Newtonian fluid, the ratio is constant – meaning that no matter how fast the fluid is moving, the viscosity remains the same. Non-Newtonian fluids, however, are used for drilling and their viscosities vary with shear rate. Viscosity also can vary with temperature and, sometimes, pressure. Drilling fluid viscosities have different responses to temperature depending upon the ingredients in the fluid. Above about 200°F, the viscosity of a drilling fluid at a shear rate of 100 sec$^{-1}$ can continue to decrease, remain constant, or increase. This creates a massive problem for calculating pressure losses for laminar flow even though computers stand ready to crunch lots of data. The equations do not exist to predict the behavior.

**Nozzle Coefficient**

During tests to try to develop a bit bearing monitor, the opportunity appeared to experimentally determine the nozzle coefficient for a drill bit. A bit bearing monitor was installed in two bits by different manufacturers. A facility was rented which provided the opportunity to drill very hard rock (tacomaite and granite) until the bearings failed in the drill bit. An ambient pressure of 3000psi was maintained at the bottom of the borehole. To decrease the cost of the experiments, a seal was not installed in one of the cones in each drill bit. A 10.1ppg, water–based, gel/Lignosulfonate drilling fluid was used for these tests. To assist a more rapid failure, 3% volume sand [1.5% vol. 75 mesh and 1.5% vol. 120 mesh] was added to the drilling fluid. Surprisingly, over 8 hours of drilling was required before the bearings failed. During these tests, the pressure inside and outside of the bit and the flow rate through the bit were accurately measured. The nozzles were callipered to provide an accurate nozzle area calculation.

The pressure loss through nozzles is normally determined with the equation:

$$P = \frac{(MW)(Q^2)}{12032 \ (C_d)^2 \ (A^2)}$$

This equation was derived above in the discussion.

The nozzle areas, in Table 1, were calculated from the micrometer callipered diameters.

### Table 1

<table>
<thead>
<tr>
<th>Mfg.</th>
<th>Nozzle Diameter Inches</th>
<th>Nozzle Area $\text{In}^2$</th>
<th>TFA $\text{In}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reed</td>
<td>0.3950</td>
<td>0.12225</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.4000</td>
<td>0.12566</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.4035</td>
<td>0.12787</td>
<td>0.376</td>
</tr>
<tr>
<td>RBI</td>
<td>0.3390</td>
<td>0.09026</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.3370</td>
<td>0.08920</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.3400</td>
<td>0.11254</td>
<td>0.2920</td>
</tr>
</tbody>
</table>

The nozzles were nominal 13/32” (or 0.4063”) in the Reed bit and nominal 11/32” (or 0.3475”) in the RBI bit. The difference in the callipered diameter and the nominal diameter is within the tolerance for nozzles BUT makes a very large difference in the calculation since the diameter is raised to the fourth power.

As an illustration of the effect of this calculation importance, consider calculating pressure losses through drill bit nozzles. Bit nozzles are given in 32nds of an inch. The tolerances on these, however, are one half of the 32nd of an inch. The calculation of pressure losses depends upon the fourth power of the bit diameter. If a 12.0 ppg drilling fluid is pumped through three nozzles at a rate of 400gpm, the pressure loss through the nozzles is calculated for three undersize, three over size and three accurate nozzles. For three 12’s, the pressure loss could be 1600 psi or 1200 psi. The range of differences becomes smaller as the nozzle sizes increase. The error is still significant.

![Figure 8: Inaccuracies in Pressure Losses through nominal bit nozzles](image-url)

As an alternate condition, consider matching pressure drop calculations with standpipe pressures to validate the calculations. The process was to calculate pressure losses in surface equipment (relatively small), pressure losses inside...
drill strings, through nozzles, and up the annulus. The sum of these numbers sometimes matched reported daily report pressures. Problem: annular pressure losses calculated to be 30 to 40psi were actually measured in the 300 to 400psi range. AND the pressure loss through the nozzles was around 300 to 400 psi less than calculated. Also, the accuracy of the pressure drops through the nozzles were suspect because the diameter was not actually measured or was the flow rate down hole determined by calibrating the mud pumps. Jumping to conclusions is a great exercise for too many engineers.

Data
The flow rate was changed in relatively small steps while measuring the pressure drop across the nozzles while drilling with the Reed bit. The nozzle coefficient “C_d” was not constant but increased significantly when the flow rate increased, Table 2. The flow rate was not varied over a large range of values while drilling with the RBI bit.

![Graph showing smooth increases in pressure losses through nozzles](image)

Figure 9: Smooth increases in pressure losses through nozzles

![Graph showing nozzle coefficients change with flow rate](image)

Figure 10  Nozzle Coefficients Change with Flow Rate

<table>
<thead>
<tr>
<th>Reed Bit</th>
<th>Flowrate</th>
<th>Nozzle Coefficient (calculated)</th>
<th>Percent Kinetic Energy Converted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Loss psi</td>
<td>gpm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>49</td>
<td>0.805</td>
<td>124%</td>
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<tr>
<td>69</td>
<td>99</td>
<td>0.918</td>
<td>109%</td>
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<tr>
<td>138</td>
<td>147</td>
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<td>104%</td>
</tr>
<tr>
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<td>196</td>
<td>0.971</td>
<td>103%</td>
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</tr>
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</tr>
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</tr>
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<td>340</td>
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<td>97%</td>
</tr>
<tr>
<td>644</td>
<td>342</td>
<td>1.038</td>
<td>97%</td>
</tr>
</tbody>
</table>

<table>
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<th>RBI Bit</th>
<th>Flowrate</th>
<th>Nozzle Coefficient (calculated)</th>
<th>Percent Kinetic Energy Converted:</th>
</tr>
</thead>
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<td>Pressure Loss psi</td>
<td>gpm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>808</td>
<td>295</td>
<td>1.030</td>
<td>97%</td>
</tr>
<tr>
<td>833</td>
<td>299</td>
<td>1.028</td>
<td>97%</td>
</tr>
<tr>
<td>838</td>
<td>295</td>
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<td>99%</td>
</tr>
<tr>
<td>1075</td>
<td>340</td>
<td>1.029</td>
<td>97%</td>
</tr>
</tbody>
</table>

Results
At normal flow rates used to drill with 8 ¾” or 8 ½” drill bits, the nozzle coefficient for these nozzles seems to be well approximated with 1.03. At lower flow rates, nozzle coefficients may be smaller.

Analysis of Values for Nozzle Coefficient
The equation for pressure loss in turbulent flow is the Kinetic Energy per unit volume. Using the equation normally used to calculate nozzle pressure losses:
\[ P = \frac{(MW)(Q^2)}{12032 \left(C_d\right)^2 \left(A^2\right)} \]

This equation could be rewritten:

\[ P = f \left(\frac{KE}{VOL}\right) \]

Where \(f\) is the fraction of kinetic energy converted to pressure. The value of \(f\) is also the inverse of the square of \(C_d\).

This leads to an interesting situation. When \(C_d\) is less than 1.0, more pressure is produced than is available from the kinetic energy of the situation. For example, at the very low flow rate through the Reed bit nozzles, \(C_d\) was only 0.85 which means that 138% of the energy was converted into pressure. This is not very likely. The problem is that some of the pressure drop was developed by turbulent flow and some by laminar flow. Using only one part of the pressure drop (i.e. turbulent flow) but omitting the other part of the pressure drop, can lead to an estimation of the pressure loss but may not be very accurate.

When the flow rate was increased to values normally used in the field, the \(f\) value was about 94% of the Kinetic Energy. In this case, probably most of the flow around the drill bit was turbulent.

This also explains why the value of \(C_d\) seems to be dependent on mud weight or PV, although no tests have been performed to validate this. If part of the flow around the bottom of the hole has some laminar components, the pressure loss will have some characteristics attributed to viscosity as well as the normal ‘turbulent variables’ of mud weight and velocity squared.

**Conclusion**

Use 1.03 for \(C_d\), the constant in the Nozzle Pressure Equation.

**References**


**APPENDIX A. VISCOSITY**

Viscosity is defined as the ratio of shear stress to shear rate. If the shear stress is measured in dynes/cm² and the shear rate in sec⁻¹, the viscosity will have the units of poise. Unfortunately, many confuse rheological models with this definition. A rheological model attempts to describe the entire shear stress vs shear rate curve. The ratio of any point on the curve can be used to calculate viscosity. Most frequently, a curve which represents the relationship between shear stress and shear rate for a drilling fluid is made confusing by a line from some point to the origin. A statement is then made that this would be the viscosity of a Newtonian fluid. While that is true – if there was a Newtonian fluid with those shear stress-shear rate values – it would be equally true that this value would be the viscosity of a Hershel-Buckley fluid, or a shear thickening fluid, if the curve passes through that point. Relating a point on a Shear Stress-Shear Rate curve to a viscosity of a particular rheological model becomes very confusing to students. In the Shear Stress/Shear Rate graph below, a Shear Thickening Fluid, a Newtonian Fluid, and a Shear Thinning Fluid all have the same viscosity (30cp.) at 300RPM.

One other confusing point for students interested in Fluid Mechanics is fact that the concentric cylinder viscometer dial reading at 300RPM is the viscosity of any fluid at that shear rate. It is frequently confused with “Newtonian” viscosity. To change the dial reading to the unit of dynes/cm², the dial reading is multiplied by 5.11. To change the RPM to reciprocal seconds, the RPM is multiplied by 1.70. This will convert any ratio to the unit ‘POISE’. The normal unit is centipoise, consequently the 5.11 is normally multiplied by 100 to convert the value to centipoise. The ratio of 511 divided by 1.70 gives a conversion factor of 300. In other words, when a ratio of readings from a properly calibrated oilfield concentric cylinder viscometer is multiplied by 300,
the ratio is the viscosity in centipoise. Obviously, the dial reading at 300 RPM will be the viscosity of ANY fluid at that shear rate. The dial reading at 600 RPM will be twice the viscosity (in centipoise) at that shear rate.

APPENDIX B. Comments
There is a tendency to think of fluids in terms of a static situation as described by Pascal’s Principle. Fluid in motion does not have the same pressure throughout the fluid at a specific horizontal datum plane. If it did, a babbling brook could no longer babble. Ripples on a mountain stream would not exist. There would be no rapids with great rough surfaces to thrill those in canoes or rafts in mountain areas. The surfers would disappear from Hawaii’s North Shore because there would be no wave action. On the other hand, few would notice because the low pressure zone caused by rapid flow of air across the wing surface would not exist, consequently, no planes would be able to transport surfers to Hawaii anyway. Closer to home, the lack of a change in pressure caused by rapidly flowing fluid would eliminate mud hoppers. [They rely on the Bernoulli principle.]

The extreme complexity of flow patterns in a turbulent fluid is one of the reasons that coefficients are used to approximate pressure losses in flowing fluids. Fluid must have a pressure differential to flow. Each of the curved stream lines in a fluid must be in response to a pressure differential causing the fluid to move in that pattern. Chaotic flow profiles must have a great variety of small pressure differences creating these eddies. The viscosity of the fluid in response to these small pressure differences determines the velocity of the fluid in each of the eddies. Drilling fluid viscosity depends upon the shear rate within the fluid. So a tremendously large matrix of viscosities and flow patterns would be required to accurately describe all of the pressure differentials in a turbulent drilling fluid.