Prediction of Frictional Pressure Losses for Time Dependent Drilling Fluids in Pipe Flow

Olufolahanmi Olusola, Tetra Technologies Inc; Tan Nguyen, New Mexico Institute of Mining and Technology; Arild Saasen, Det norske oljeselskap and University of Stavanger; Eissa Al-Safran, Kuwait University

Abstract

The objective of this study is to develop a numerical simulator to predict the transient frictional pressure losses in pipe under laminar flow conditions. The transient behavior described here is due to the gelation and thixotropic properties of the fluids, which is modeled employing a recently proposed rheological equation. Combining this rheological model with the solution to the momentum equation for fully developed, isothermal, laminar flow, we obtain an expression which is solved numerically for the prediction of the thixotropy-induced frictional pressure losses. A number of rheological experimental tests were conducted on water based drilling fluids with different bentonite concentrations in order to determine the parameters of Tehrani’s equation, which are used as inputs in the simulator. In addition, flowing tests were also carried out in a Hydraulic Testing Facility (HTF) to verify the simulator results.

The agreement between the simulator predictions and the experimental flow test results is good with possible improvement by refining the testing facility and laboratory experiment procedures as recommended in this study. The findings from this work show a great potential for application in improving drilling hydraulics predictions to ensure safe drilling and pumping operations.

Introduction

The accurate estimation of frictional pressure gradients while drilling is vital to any drilling operation to obtain optimum drilling efficiency. The choice of drilling fluid has been identified as the single most important parameter for drilling optimization, (Lummus, 1974), and is directly affected by the fluid’s rheological properties under various well conditions of both static and dynamic. These properties determine the effectiveness of any drilling fluid to achieve its primary functions of hole cleaning, cuttings transport, drill bit cooling and lubrication, borehole stability, filter cake formation, kick prevention, and assistance in the collection of information from cuttings, cores and logs. In addition, the fluid’s rheological properties are important to ensure prevention of formation fracturing and lost circulation during drilling.

Conventional hydraulics models consider drilling fluids to be time-independent. In other words, these models assume that when a constant shear rate is applied to the drilling fluid, the shear stress exerted on the fluid remains constant over time, hence the frictional pressure loss is also considered to be constant with time. In fact, many Water Based Mud (WBM) drilling fluids, which contain bentonite as clay mineral, exhibit clear shear and time-dependent (or thixotropic) properties. For instance, during drilling operations whenever there is a reason to suspend fluid circulation for short or extended periods or when the operation is shut down due to adverse weather conditions, the fluid within the wellbore tends to gel up and its viscosity increases. To resume circulation, an extra energy is expended by the pump to break the built up gel structure and get the wellbore fluid flowing again. This extra energy is a manifestation of the change in the properties of the fluid while it was at rest due to its thixotropic nature.

Thixotropy can be described as the reversible, isothermal transformation of a fluid from a colloidal solution or liquid to a gel or solid form. Consequently, the fluid’s viscosity decreases over time, due to structure rearrangement, when the fluid is subjected to a constant rate of shear (Darley & Gray, 1988).

Gelation is an important property of drilling fluids, closely related to thixotropy, which affects the ability of the fluid to hold cuttings particles in suspension during pump off periods. Both processes are due to the build-up and breakdown of the clay particles structure over time as the fluid undergoes periods of rest and shear, respectively. In addition to the effect of increase in shear rates whenever the fluid is subjected to increased velocity, the thixotropic nature of the fluid and extent of gelation also has a substantial influence on the magnitude of the pressure peaks and frictional pressure losses observed during a drilling operation.

The circulation start up process after a hiatus in operations as previously described, and the proper pumping pressures are presently hard to determine as the fluid behaves differently at different stages of the drilling operation due to variations in its shear history and the duration of the non-flowing period. To break up the gel structure, a common practice is to circulate the fluid at a low flow rate, for an indeterminate period of time, usually based on the driller’s experience. This not only results in increased Non Productive Time, but also leads to inaccurate estimations of the fluid bottom hole pressures, and is often a cause of induced formation fractures.

Therefore, developing a model that would accurately predict the thixotropy-related pressure drop trends in time-
dependent drilling fluids is of vital for the drilling industry to help drillers address this issue, and facilitate safer pump start up procedures in such situations.

This work is divided into three sections. The first stage is the development of the model and simulator using Tehrani’s rheological equation (Tehrani & Popplestone, 2009). The second section describes the rheological tests performed to obtain the parameters of Tehrani’s equation which serve as inputs to the simulator for predicting the thixotropy-related frictional pressure losses. The third and final section involves a validation of the simulator model via tests performed in a constructed hydraulics testing facility (HTF).

Modeling Development

Conventional Hydraulics Method – Time Independent Fluids

The general form of the momentum equation for laminar flow of a single phase fluid can be written as:

$$\rho \frac{D \vec{V}}{Dt} = -\nabla P + \rho g - \nabla \tau$$

(1)

For fully-developed, isothermal, steady-state (i.e. no acceleration), incompressible fluid, one-dimensional and horizontal pipe flow, it can be shown that the shear stress at the wall of the pipe, \( \tau_w \), relates to the frictional pressure gradient as:

$$\tau_w = D \left( \frac{dp}{dL} \right)$$

(2)

where \( D \) is the pipe diameter, and \( \frac{dp}{dL} \) is the frictional pressure gradient along the length of the pipe wall.

Equation (2) can be coupled with the constitutive equation, such as Herschel-Bulkley equation (Herschel & Bulkley, 1926), to predict the frictional pressure loss in pipe flow for time-independent fluids.

Prediction of Frictional Pressure Losses for Time-dependent Fluids

In the conventional hydraulics solution as described in the previous section, the frictional pressure losses are considered to be due to the viscous forces or shear force, \( \nabla \tau \), alone. This shear force term depends only on the rheological properties of the fluid. More precisely, this shear force is a function of shear rates and is independent of time. Therefore, as a drilling fluid flows in a pipe with a constant diameter, the conventional hydraulics models assume that the frictional pressure drop is a constant.

However, many drilling fluids exhibit time dependence behavior, in which the frictional pressure drop is not constant, but changes with time instead. Therefore, it is clear that the viscous shear force is not only due to the velocity field gradient, but also due to the nature of the fluid as well (i.e. its thixotropic properties).

Therefore, the wall shear stress and frictional pressure gradient in equation (2) may be re-written in time-dependent form (or due to the gel structure parameter) as:

$$\tau_w(t) = \frac{D}{4} \frac{dP}{dL}(t)$$

(3)

There are several rheological models available for modeling the gelling properties of water-based drilling fluids. In this paper, Tehrani and Popplestone (2009) rheological model is chosen for two reasons, namely its form is essentially analogous to the Herschel-Bulkley (Herschel & Bulkley, 1926) equation and its accuracy for predicting the rheology of water based fluids compared to the Power Law or Bingham Plastic models (Hemphill, Campos, & Pilehvari, 1993). In addition, the coefficients in the Tehrani and Popplestone model can easily be evaluated using simple curve fitting techniques and shear relaxation experiments. As described in the paper, this model is proposed for modeling the gelling properties of time-dependent drilling fluids (Tehrani & Popplestone, 2009).

Tehrani and Popplestone used the concept of structure theory and an equation of state suggested by Cheng and Evans (1965) to describe the thixotropic behavior of fluids above yield point. By incorporating shear thinning properties, Tehrani and Popplestone developed the following modified Cheng-Evans equation:

$$\tau(t) = \lambda(t) \tau_y + [\eta_x + c \lambda(t)] \dot{\gamma}^m$$

(4)

where \( \eta_x \) is the viscosity of the unstructured fluid (value of viscosity at equilibrium conditions), \( m < 1 \) is the flow behavior index for shear-thinning fluids, and \( c \) represents a constant parameter in the equation of state of the Moore thixotropic model (Moore, 1959). Substitute Eqs. (4) into (3) gives,

$$\frac{D}{4} \frac{dP}{dL}(t) = \lambda(t) \tau_y + [\eta_x + c \lambda(t)] \dot{\gamma}^m$$

(5)

Eq. (5) is a partial differential equation which can be used to predict the transient or time-dependent frictional pressure losses due to the gel structure within a thixotropic fluid under laminar flow conditions. The wall shear rate in pipe flow for non-Newtonian fluids, \( \dot{\gamma} \), is described by Rabinowitsch, (1929) as,

$$\dot{\gamma} = \frac{3N+1}{4N} \frac{U}{D}$$

(7)

where \( N \) is the generalized flow behavior index and defined as,

$$N = \frac{d(\ln \tau)}{d(\ln \dot{\gamma})}$$

(8)

For time-independent fluids, \( N \) is a constant and equals to the flow behavior index, \( m \), if the fluid has zero yield stress
For time-dependent fluids, the shear stress, \( \tau \), is a function of time at a constant shear rate and hence, \( N \) also changes with time. For the simplicity, in this work, we assume that \( N \) does not depend on time and is a constant at a constant shear rate. In addition, if the yield stress is smaller than 3 lbf/100ft\(^2\) (1.4 Pa), the simulator will treat the yield stress as zero. In this case, \( N \) will be the same as the flow behavior index \( m \). This assumption simplifies the simulator program write up and enhances the speed of the calculation.

A computer simulator to solve Eq. (5) was developed to predict the gelation-related transient frictional pressure losses in pipes. The simulator was written in visual basic for applications (VBA). The input parameters for the model include the pipe diameter, fluid density, various flow rates, and the Tehrani’s and Popplestone model parameters, which were obtained from the rheological tests. The simulator outputs are the generalized Reynold’s number and frictional pressure loss over time.

The generalized Reynold’s number for yield power law fluids flowing in a circular tube is given by:

\[
Re_{(gen)} = \frac{8\rho v^2}{\tau_w}
\]  

(6)

where \( \rho \) = fluid density, kg/m\(^3\), and \( v \) = fluid velocity, m/s.

**Rheological Tests**

The OFITE model 900 rheometer was used to study the rheology of bentonite fluid mixture at different weight concentrations. Bentonite, a clay mineral consisting primarily of montmorillonite, is used as the test fluid as it is known to exhibit good time-dependent and thixotropic properties. The focus of this stage of laboratory experiments is: (1) to perform tests to observe the rheological properties of the chosen fluids, (2) to determine which fluid concentrations have strong time-dependent properties, and can this be used for the hydraulics tests in the next step, (3) to determine the constants in Tehrani’s model through curve fitting methods. The tests performed in this section of the project were shear relaxation tests in which the sample was sheared at a constant rate for a period of time, until equilibrium stress was attained for each shear rate, and each different fluid concentration.

The fluid used for these tests is an unweighted water based mud containing 20 g, 30 g, and 40 g of bentonite per liter of water, i.e., at 2%, 3%, and 4% weight concentration of bentonite. The bentonite fluid is mixed in a Standard Hamilton Beach mixer, according to API standards for mixing bentonite, and then allowed rest for 24 hours to ensure fully built up gel structure and full sample hydration. Then, the bentonite sample is transferred to a sample cup of the rheometer and heated up to the required test temperature before starting the constant shear tests. The test temperature for the laboratory experiments is selected as 120 °F.

The shear relaxation tests are run for about 30 minutes at the following constant shear rates: 20 rpm, 100 rpm, 400 rpm and 600 rpm. These shear rates represented a good range of shear operations for determining the parameters of the equilibrium flow curve.

**Hydraulics Testing Facility**

A hydraulics testing facility was built in order to validate the results obtained from the output of the computer simulator model. The experimental facility consists of a horizontal 1-in. ID steel pipe flow loop in which the mixed drilling fluid is circulated. Several instruments were mounted on the loop to measure the pressure drop, mass flow rate, and fluid temperature. A centrifugal pump, with 250 gallon/minute capacity, rated 250 psi, powered by a 100 hp AC motor is used to pump the fluid through the loop. During the test, the fluid is circulated through the flow loop, where a differential pressure transmitter is installed across a 3 ft section to measure pressure drop. The pressure drop across this length of pipe is recorded using data acquisition system. The fluid then flows out into the discharge section, back into a 100 gallon cylindrical tank, located about 7 ft above ground level. A mass flow meter is mounted on the flow loop to measure the fluid circulation rate and fluid temperature. The fluid temperature is kept constant by circulating cold water through a cooling copper tube installed within the tank. This copper tube helped keeping the fluid temperature variation within a range of ±3 °F of set testing temperature.

The tested mud is prepared on location, by filling the tank with water to the desired volume, gradually adding the required weight of bentonite, and mixing using an impeller in the tank. Simultaneously, the prepared mud is circulated through the flow loop at a high constant shear rate of 30 GPM through the selected pipe section for an hour. This step ensured a proper dispersion of the bentonite in the water and a homogenous mixture. The sample is then allowed to sit for 48 hours for full hydration before any tests are carried out. Finally, the experimental data of temperature, differential pressure, flow rates, and duration of testing is recorded by a data acquisition system connect to a high speed computer located at the facility.

**Results and Discussions**

**Rheological Test Results**

Figure 2 shows the results of the typical shear-relaxation tests for a 4% weight bentonite at different shear rates of 20, 100, 400 and 600 rpm. The tests were conducted by shearing the fluid sample at a constant shear rate. The shear stress was observed and recorded until the equilibrium stress was obtained. Detail of the test procedure is presented in the Rheological Test section. In general, the curves show a distinct shear stress peak \( (\tau_p) \) followed by a steady decline in the shear stress values with time. Finally, the equilibrium shear stress, \( \tau_e \), for each shear rate was attained. The stress peak value represents the stress required to breakdown the fluid gel structure which built up while the sample was at rest in the sample cup. In addition, there is a faster breakdown of
fluid gel structure at higher shear rates, and the rate of breakdown decreases as time increases. This indicates that the fluid structures are broken down faster at higher shears.

Similar shear relaxation experiments were run for 2% and 3% weight bentonite mixtures. Progressively lower peak stresses and slower decline to equilibrium are observed at correspondingly lower shear rates and lower weight concentrations of bentonite. To illustrate the magnitude of difference between the peak and equilibrium stresses occurring in the fluid due to its gelation properties, a plot of peak and equilibrium stresses at various constant shear rates is presented in Fig. 3. This plot represents the results obtained for a 4% weight concentration of bentonite. The peak stresses in each case are consistently greater than the equilibrium stresses by at least 17%. A maximum difference between the peak stress and equilibrium stress of 45% is observed at the shear rate of 170 1/s. In other words, if the thixotropic property of the fluid is not taken into account, the conventional hydraulics model will always under-predicts the peak and equilibrium stresses and slower decline to equilibrium are observed at the lower shear rates.

For all the laboratory tests with the viscometer, the temperature is kept constant at 120°F. The reason for conducting tests varied from very low temperatures of 40°F in the mornings to about 70°F in the afternoon. The heat ing element was not efficient enough to maintain the fluid temperature at 120°F, about 80°F in tests location. It is also observed that the magnitude of difference between the peak pressure at start up and the equilibrium pressures is greater for the higher weight concentration of bentonite, and gets lower as the weight concentration of bentonite reduces. Transition from peak to equilibrium pressures for all three weight concentrations of bentonite is noted to occur over a shorter period of time when the fluid is flowing at the higher flow rate (20 GPM) than when flowing at the low flow rate (5 GPM).

Following the procedure proposed by Tehrani & Popplestone (2009), all the Tehrani’s parameters which served as inputs for the simulator for the three different bentonite concentrations are obtained and presented in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta(\omega)$ (Pa.s&lt;sup&gt;0&lt;/sup&gt;)</td>
<td>0.003</td>
<td>0.007</td>
<td>0.008</td>
</tr>
<tr>
<td>$c$ (Pa.s&lt;sup&gt;m&lt;/sup&gt;)</td>
<td>0.03</td>
<td>0.02</td>
<td>0.26</td>
</tr>
<tr>
<td>$m$</td>
<td>0.63</td>
<td>0.81</td>
<td>0.61</td>
</tr>
<tr>
<td>$b/a$ (s)</td>
<td>$9 \times 10^{-6}$</td>
<td>$8 \times 10^{-5}$</td>
<td>$2 \times 10^{-4}$</td>
</tr>
<tr>
<td>$k_1$ (Pa.s)</td>
<td>0.017</td>
<td>0.04</td>
<td>0.2</td>
</tr>
<tr>
<td>$k_2$ (s)</td>
<td>0.009</td>
<td>0.012</td>
<td>0.045</td>
</tr>
<tr>
<td>$\tau_y$ (Pa)</td>
<td>0.0015</td>
<td>0.9</td>
<td>2.7</td>
</tr>
</tbody>
</table>

**Table 1: Tehrani’s Parameters at different bentonite concentrations**

**Modeling Results**

Solving numerically Eq. (5) using inputs of flow rate, pipe diameter, fluid density, and Tehrani and Popplestone coefficients for each bentonite concentration, the computer simulator predicts pressure gradient and generalized Reynold’s number vs. time for different flow rates. It is observed that the trend from the simulator is similar to those observed from the shear relaxation tests performed in the laboratory, i.e. at higher shear rates, and higher concentrations of bentonite, there will be higher peak pressures, as well as a sharper exponential decline from peak stress to equilibrium stress, as compared to the lower shear rates and lower bentonite concentrations whose curves show a lower pressure gradient and a smoother decline to equilibrium.

Figure 4 shows the computer simulated pressure gradient against time for a 4% weight bentonite concentration, flowing under various constant shears (flow) rates through a 1” diameter pipe. From this plot, it is seen that the simulator predicts higher peak pressures at startup and then a pressure declines to equilibrium pressure. The decline rate to equilibrium pressure increases at higher shear rates. Lower peak pressures and decline rates are observed at the lower shear rates.

A comparison of computer simulated pressure gradients against time predictions of the different bentonite concentrations at 20 GPM for 4 wt%, 3 wt% and 2 wt% bentonite concentrations (i.e. Re= 3533, 9328, and 23797, respectively), and at 5 GPM for the same bentonite concentrations (Re = 377, 457, and 1750, respectively) is shown in Figs. 5 and 6. The results for both shear rates indicate that there is a drop in peak and equilibrium pressures corresponding to the reduced weight concentration of bentonite. It is also observed that the magnitude of difference between the peak pressure at start up and the equilibrium pressures is greater for the higher weight concentration of bentonite, and gets lower as the weight concentration of bentonite reduces. Transition from peak to equilibrium pressures for all three weight concentrations of bentonite is noted to occur over a shorter period of time when the fluid is flowing at the higher flow rate (20 GPM) than when flowing at the low flow rate (5 GPM).

The Generalized Reynold’s number against time plots shown in Fig. 7 indicates a steady increase from a lower Reynold’s number at initiation of flow to a higher, constant Reynold’s number over time for each bentonite weight concentration. This increase is expected as the structure within the fluid breaks down over time due to thixotropy, until the fluid structure achieves an equilibrium state and frictional pressure drop becomes constant. The Reynold’s numbers are higher for lower weight bentonite concentrations fluid, and reduce as the concentration of bentonite increases.

**Simulator Results Validation (Hydraulics Testing Facility)**

Validation tests were run on a 4 wt% bentonite concentration fluid, flowing through the 1 in. ID pipe at a constant temperature of 80°F. The reason for conducting tests at 80°F was that the ambient temperatures in tests location varied from very low temperatures of 40°F in the mornings to about 70°F in the afternoon. The heating element was not efficient enough to maintain the fluid temperature at 120°F, which is the temperature at which the rheological flow parameters were established. Two tests were successfully
conducted at the Hydraulics Testing Facility (HTF), namely 20 gpm (Re\_gen = 3,533) flow rate test, and a 25 gpm (Re\_gen = 5,000) flow rate test, which both are under laminar flow regime and shown in Figs. 8 and 9. Note that all the rheological tests were done at 120 °F and so the simulator predicts the frictional pressure loss of fluid at 120 °F flowing in the 1 in. pipe. Therefore, the comparison of results from the simulator and the HTF is not carried out at the same fluid temperature. The effect of this difference in testing temperatures on the frictional pressure losses predicted by the simulator is discussed further in the penultimate paragraph of this section.

It is observed that the trend of the facility recorded gelation induced frictional pressure losses changes over time in a manner similar to the computer simulator pressure loss predictions. Even though there is a discrepancy in the magnitudes of the peak pressures and rate of frictional pressure losses, the equilibrium pressures are observed at similar times (approximately after 1 hour) for each flow rate.

For the 25 gpm (Re\_gen = 5000) flow, the simulator predicted a frictional pressure loss gradient rate from peak to equilibrium pressures of about 10 Pa/m/minute. The hydraulics test facility resulted in a frictional pressure loss gradient rate of 4 Pa/m/minute. A total frictional pressure loss due to gelation of 230 Pa/m was realized in the testing facility, compared to the simulator predicted frictional pressure loss due to gelation of 600 Pa/m. Hence, the predicted frictional pressure loss gradient rate exceeds the measured by about 40% for the 25 gpm (Re\_gen = 5,000) flow rate test.

For the 20 gpm (Re\_gen = 3,533) flow condition, the program predicted a frictional pressure loss gradient rate from peak to equilibrium pressures of 5.3 Pa/m/minute, while the measured value is 3.33 Pa/m/minute. A total frictional pressure loss due to gelation of 200 Pa/m was observed in the testing facility, compared to the simulator predicted frictional pressure loss due to gelation of 630 Pa/m. This represents a 32% discrepancy between the program’s predictions and the measured values.

Therefore, the computer simulator model under-estimates the average (equilibrium) pressure gradients at the chosen flow rates of 20 gpm (Re\_gen = 3,533) and 25 gpm (Re\_gen = 5,000) as shown in Figs. 10 and 11. In each case, it is found that the pressure gradient predicted by the simulator is about half of the pressure gradient measured in the test facility, i.e. 100% under-prediction. The most likely reason for this is that, as stated previously, the input parameters to the simulator were obtained from laboratory tests conducted at a temperature of 120 °F, which is higher than the fluid temperature in the HTF tests of 80 °F. It is expected that this temperature difference resulted in variation in the fluid rheological properties, and thus the obtained frictional pressure gradient. If tests at the HTF can be conducted at 120 °F, then the frictional pressure loss, which is a function of time, will be much less and the values of pressure gradient variation with time should be more closely matched.

Therefore, for future studies it is recommended to improve the temperature controller in the HTF as well as to conduct test during the summer time so that the desired testing temperatures can be achieved.

The start-up procedure for the test is also very important because of the tendency of bentonite to exhibit both time and shear-dependent properties. It is therefore important to ensure that the initial conditions for testing in the facility match as closely as possible to the initial testing conditions in the laboratory. The fluid was carefully mixed to ensure homogenous sample, and then allowed to rest in the tank for 48 hours hydration period. The fluid is also pre-sheared through the 1 in. pipe flow loop for 30 minutes at 20 gpm, which results in a shear rate of approximately 600 rpm, which is recommended in the literature for pre-shearing in the laboratory (Maxey, 2007). Then the fluid is given a 3 hour period of rest, during which the gel structure is allowed to grow before testing starts. For the successfully completed tests, the 3 hour structure growth period allowed for observable transient pressure gradient decline through the 1 in. pipe.

Concluding Remarks

The objective of this research was to develop a model to predict the frictional pressure losses for incompressible time-dependent drilling fluids, under steady state and isothermal conditions. Tehrani’s and Popplestone (2009) model was selected and coupled with the momentum equation to obtain the objective. Experiments were conducted on 2 wt%, 3 wt% and 4 wt% sample concentrations of bentonite in a laboratory using a rheometer. Frictional pressure gradient measurement tests were also carried out at constant flow rates in the hydraulic testing facility. The following summary and concluding remarks can be drawn from this work:

- For the tested fluids, the difference in magnitude between the initial peak pressures due to gelation and final equilibrium pressures may range from as low as about 15% to as high as 45% depending on the weight concentration of bentonite in the fluid. This difference must be accounted for, and managed by drillers to ensure that neither the formation nor the wellbore is damaged whenever the pump is restarted.

- The simulator prediction shows that the transient frictional pressure losses due to thixotropy has higher decline rate at high laminar flow rates. In other words, equilibrium flow is achieved over a shorter period of time at higher constant shear rates. For instance, for the 2 wt% bentonite concentration, at a constant flow rate of 5 gpm (Re\_gen = 3,771), equilibrium pressures are predicted after approximately 2 hours; as opposed to 1 hour at flow rate of 25 gpm (Re\_gen = 23,797). A similar trend was observed for the 3 wt% and 4 wt% weight concentrations of bentonite. This information is useful in drilling operation to implement a faster and
more accurate pumping schedule during pump start up.

- The verification tests performed in the hydraulics testing facility indicated that bentonite drilling fluid exhibits clear time dependent properties. However, the predicted frictional pressure gradient exceeded the measured values from the facility tests by as much as 40% due to testing temperature differences.

Future Work
The following points represent some suggested areas of further research:

- Determination of a correlation between the weight concentration of bentonite as well as temperature and Tehrani’s and Popplestone constants for direct field application of the model.
- Prescription of a repeatable pump start up procedure for better comparison of frictional pressure drop gradients.
- Investigation of frictional pressure losses in time-dependent fluids under turbulent flow.
- Effect of shear history of fluid on frictional pressure losses.
- Prediction of pressure losses in annulus with and without pipe rotation.

Acknowledgements
The author would like to thank his primary advisor, Dr. Tan Nguyen, as well as all committee members, sponsors and advisors of the Production and Drilling Research Project (PDRP) group and the faculty of Petroleum Engineering at the New Mexico Institute of Mining and Technology. The author would also like to thank Halliburton for providing the bentonite material used for hydraulics facility testing.

Nomenclature
Greek Symbols
\( \tau = \) Shear stress, Pa
\( \tau_y = \) Yield stress, Pa
\( \dot{\gamma} = \) Shear rate, 1/s
\( \dot{\gamma}_w = \) Shear rate at pipe wall, 1/s
\( \tau_a = \) Shear stress at the pipe wall, Pa
\( \lambda = \) Gel structure parameter
\( \eta_u = \) Unstructured fluid viscosity
\( \rho = \) Density, lb/gal
\( \nabla \cdot \tau = \) Viscous force term

Alphabets and Abbreviations
\( dv/dr = \) Shear rate, 1/s
\( N = \) Generalized flow behavior index, (dimensionless)
\( m = \) flow behavior index, (dimensionless)

U = Fluid velocity, ft/s or m/s
D = Pipe diameter, in or m
L = Pipe length, in or m
a = Structure buildup parameter (dimensionless)
b = Structure breakdown parameter, s
t = Time, s
e = Equilibrium
p = Peak
c = Moore model equation of state constant, Pa.s^m
k_1 = Equilibrium flow curve parameter, Pa.s
k_2 = Equilibrium flow curve parameter, s
Q = Flow rate, gal/minute or m^3/s
wt. = weight
r = Pipe radius, in or m
Re_gen = Generalized Reynolds number (dimensionless)
dP/dL = Frictional Pressure Loss due to gelation, Pa/m or psi/ft
gpm = gallons per minute
\( \nabla \dot{V} = \) Velocity vector term of Momentum Equation
P = Pressure term of Momentum equation
ID = Pipe internal diameter, inches
g = Acceleration due to gravity, m/s^2
\( (\nabla \dot{V})/Dt = \) Material acceleration, m/s^2

References
Figure 1: Hydraulics Testing Facility schematic

Figure 2: Rheometer tests steady shear stress relaxation results (4 wt% Bentonite and T=120°F)

Figure 3: Rheometer tests peak stress and equilibrium stress comparisons at different shear rates (4 wt% Bentonite)

Figure 4: Pressure gradient vs. time simulator prediction (4 wt% Bentonite and different flow rates)

Figure 5: Simulator frictional pressure gradient predictions (Q=20 gpm, different Bentonite)

Figure 6: Simulator frictional pressure gradient predictions (Q=5 gpm and different Bentonite concentrations)
Figure 7: Simulator generalized Reynold’s numbers predictions (Q=20 gpm, and different Bentonite)

Figure 8: Measured frictional pressure gradient vs. time (4 wt% Bentonite, Q=20 gpm and Pipe ID=1 in.)

Figure 9: Measured frictional pressure gradient vs. time (4 wt% Bentonite, Q=25 gpm, Pipe ID=1 in.)

Figure 10: Frictional pressure gradient validation (4 wt% Bentonite, Q=20 gpm and Pipe ID=1 in.)

Figure 11: Frictional pressure gradient validation (4 wt% Bentonite, Q=25 gpm, and Pipe ID=1 in.)