Cemented Casing: The True Stress Picture
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Abstract
The industry acknowledges that there is a change in the burst and collapse resistance of cemented pipe versus uncemented pipe; but the effects of cement and formation mechanical properties, in situ stresses, and nonuniform stress loadings are not well understood. Consequently, casing is designed routinely as thought there is no interaction with the cement sheath and surrounding formation. This paper presents the results of a series of finite element studies of the cemented casing under a variety of stress conditions for both burst and collapse and will demonstrate the inadequacy of accepted design equations under many cemented conditions. The burst and collapse resistance of casing may increase or decrease by 60 to 70%, depending on the true stresses acting upon the casing. The paper will discuss the design implications of these results and explain previously unknown causes of casing failures seemingly well within the conventional design limits of the casing.

Introduction
In casing design for burst and collapse, historically, the standard practice has been to design the casing while ignoring the cement sheath effects and surrounding formation effects. A primary reason for this method of casing design is that traditional analytic design equations are unable to incorporate the complexity necessary to accurately calculate the resultant stresses. Conventional casing design equations assume a uniform, predictable, hydraulic loading for both collapse and burst conditions. If these conditions do not exist, the underlying equations are no longer valid.

Many conditions exist where casing is subjected to nonuniform loading. Cement channels and voids, formation voids due to sand production, or eccentrically centered casing may cause nonuniform loading. Conventional casing design is unable to account for the stresses present in nonuniform loaded casing, and is unable to predict the actual stresses the casing must be designed to withstand. In many cases the casing does not fail, though there may be incidences of deformed casing, or “tight” pipe, where swedges or milling is needed to access the lower portion of the wellbore. In many instances, nonuniform loading is present when casing collapse is noted.

There is no standard casing design criteria for burst resistance, companies vary safety factors, and loading conditions. The American Petroleum Institute (API) does publish a bulletin for the formulae and calculations for casing properties. This bulletin defines internal yield resistance as the lowest of the internal yield resistance of the pipe or the internal yield resistance of the coupling. API’s burst pressure rating for the casing body is based upon Barlow’s equation, relying on the minimum yield stress of steel, the physical dimensions of the pipe, and a minimum tolerance to calculate a burst pressure rating. This assumes a uniform pressure loading. The calculated burst resistance is the basis of most burst designs.

\[ p_b = 0.875 \frac{2Y_h}{d} \] ................................. (1)

The collapse mechanism is a more complex phenomenon in comparison with burst. Elasticity theory cannot be entirely applied to compute stresses in the pipe wall for different loading cases and the outer diameter and wall thickness ratios (D/t). An underlying assumption on for all collapse calculations relies on uniform, predictable, hydraulic loading. Four collapse equations are used in order to calculate the four collapse ranges:

- Yield-strength
- Plastic
- Transition
- Elastic

Figure 1: API collapse modes and regions of applicability
Proper equation selection depends upon the D/t ratio as well as the material yield strength. API Bulletin 5C3 presents tables where the D/t ratio is given for different API tubulars. Figure 1 shows the four regions where the collapse resistance has different modes.

The obvious weaknesses of both these approaches is that the respective design equations only include the material and geometric properties of the casing, and assume a uniform, hydraulic loading on the internal or external surfaces of the casing. The effects of the cement sheath and surrounding formation, including in situ stresses, mechanical properties and nonuniform loading are not accounted for and assumed to be negligible. The magnitude of the error of these assumptions is difficult to predict and must be ignored for the use of classical casing design equations.

**Finite Element Analysis**

A method which is gaining wider acceptance is the use of Finite Element Analysis (FEA) to design casing using the mechanical properties of the surrounding cement and formations. The finite element analysis method of analysis is a numerical technique to obtain approximate solutions to partial differential equations. In finite element analysis, a continuous physical system is discretized into a series of finite elements. These elements are composed of a series of nodes, at specified intervals. At the location of these nodes in structural mechanics, deflections and stresses are calculated, based on the relevant material mechanical properties and the applied boundary conditions. A series of equation matrices are solved, allowing each node to affect the deflection and stress at each other node. As the mesh becomes finer in this analysis, the increments between nodes become smaller; increasing the size and complexity of the system of matrices that must be solved. A larger number of nodes increases the number of calculations necessary to solve the system of equations. Computer hardware and software has rapidly advanced to the point that complex modeling can realistically be accomplished with a desktop computer in a reasonable length of time. This allows practical analysis of problems in multiple dimensions.

The finite element analysis method can model various shapes and sizes of objects, which can be mathematically described and the interactions between those objects can be solved. Irregular shapes can be approximated, allowing shapes with ill-defined boundaries to be analyzed, such as cement channels for formation voids. Several different materials with separate mechanical properties can be easily modeled as a composite system, with a variety of surface-to-surface interactions possible.

**Von Mises Failure Criteria**

Experimentally, the yield stress of a material is determined from a simple, uniaxial test. The yield stress is indicated by the stress at which the stress-strain relationship ceases to be linear. Theoretically, the yield stress is determined on the basis of yield criteria.

Experiments have shown that for ductile materials, there is reasonable agreement between experimentally determined yield stress and a maximum octahedral shear stress condition, from which the energy-of-distortion shear stress condition, from which the energy-of-distortion yield condition failure criteria is based on the assumption that yielding first begins when the strain energy density (after discarding the portion of strain energy density associated with the shear stress) reaches a critical value.

\[
2\sigma_{\text{y}}^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \quad \text{(2)}
\]

Although the physical basis for the Hencky-von Mises theory is not well known, it leads to a workable theory of plasticity that has been found to fit experimental data for ductile materials with reasonable accuracy. Failure is assumed to occur if the combination of the three principal stresses exceeds the uni-axial yield strength of the material, and plastic deformation of the material is thought to occur. The authors use the von Mises failure criteria for determining the state of stress in the finite element analysis.

**Assumptions and Model Construction**

It is important to realize that every analysis problem requires some kind of simplification otherwise it cannot be solved in a reasonable way. For the purposes of our work, all FEA models are defined as two-dimensional models. Two-dimensional models deliver accurate results under the following assumptions:

- The body forces, if any, cannot vary in the direction of the body thickness.
- The applied boundary forces do not have axial components and the forces must be uniformly distributed across the thickness.
- Loads may not be applied across the parallel planes bounding the top and bottom surfaces.

ANSYS, a commercial finite element program, was used to simulate all models. The program operates in three modes. The preprocessor mode is used to design a model. During the solution mode, the loads and boundary conditions are applied to the matrix equations and which are solved. The postprocessor mode allows both a detailed graphical and numerical analysis of the results. It draws images of the ultimate deformation and stress conditions of the model and presents the numerical values of each node. For all models the following assumptions were made:
• All materials, i.e. casing, cement and formation are isotropic.
• The axial length of the model was long enough to allow the use of planar two-dimensional models.
• This assumption is valid if the axial length of the object exceeds eight times the lateral diameter.
• The formation and cement sheath are assumed not to fail under the applied loading conditions. That means that the material remains together, even if the calculated stresses are over the materials’ ultimate strength.
• The wall thickness of the casing is uniform.
• The cement sheath is of uniform consistency and perfectly joined to the casing and the formation.
• The effects of temperature on the yield strength of steel are neglected.

Burst and Collapse Design Considerations
Consider a casing string which is unsupported by cement and is not constrained in any significant manner by the surrounding borehole (see Figure 2). The resulting casing stresses are identical for both burst and collapse loading between the FEA predictions with the triaxial theoretical model proposed by von-Mises using the Lamé’s thick wall cylinder solution. In this case, conventional casing design methodologies accurately predict the resultant stresses\(^6\) (see Figure 3).

The model represented by Figure 4 was used to examine effects of burst and collapse pressure on unsupported casing surrounded by cement sheath. The outer, coarsely gridded region represents the cement sheath and the inner finely gridded region represents the casing.

The results for the burst resistance of the casing are little different than for casing unsupported by no cement sheath at all. Cement has little tensile strength of its own and fails in tension before lending significant support to the casing. The assumption of no contact between the cement sheath and borehole is unrealistic, but illustrates the dangers to cracking the cement sheath by generating a high internal pressure in casing, especially during casing pressure tests after cementing.

Casing collapse resistance is greatly affected by pres-
ence of a cement sheath, if the collapse stress is applied to the external surface of the cement sheath. This would be possible if the external stress is due to the formation insitu or grain-to-grain stresses are being applied to the cement sheath and are being transmitted to the casing through the cement sheath.

For both very low and very high values of Poisson’s ratio in the cement sheath, increasing the Young’s Modulus of the cement reduces the radial stress which is applied to the casing as illustrated by Figures 5 and 6. Figures 5 and 6 give a radial stress distribution resulting from applying 5,000 psi to surface of the cement sheath, 1.5” thick on the surface of the 5.5” casing. The amount of radial stress transmitted through the cement to the casing is dependent on the Young’s modulus of the cement. For both cases of high and low Poisson’s ratios, the higher the cement Young’s modulus, the lower the radial stress that is transmitted to the casing. Figures 7 and 8 graphically depict the Von Mises equivalent stresses through both the cement sheath and the casing. For realistic values of Young’s Modulus and Poisson’s ratio for cement, there is very little difference in the results; the critical item is to fill the annulus with cement. Combining the lower radial stress applied to the surface of the casing with the casing distortion constraints of the cement significantly lowers the risk of casing collapse.

Using the model in Figure 9, the combined effects of casing, cement and formation were studied. The casing, cement and formation regions were discretized and burst and collapse stresses were applied.

For casing collapse resistance, the formation properties can have a dominant effect on how much stress is transmitted to the casing. Figure 10 clearly illustrates the dependence in calculated von Mises stresses at casing ID
to the formation mechanical properties. The stresses in this section range from 51,969 psi for brittle cement and 49,770 psi for ductile cement, to 19,772 psi for brittle cement and 19,119 psi for ductile cement. Even if the collapse stress is applied at the cement-casing interface, the deformation of the casing will be constrained by the cement sheath and surrounding formation, increasing the collapse resistance.

For burst conditions, it has been well documented that a cemented casing has higher burst resistance than that of uncemented casing. What has not been well documented is the importance of the confining stress of the surrounding formation. As Figure 11 illustrates, for both soft and hard or brittle cements, there is a nearly inverse linear relation-ship between the effective stress in the casing versus the formation confining stress. The point is, if you have a good cement job, the burst resistance of the casing becomes greatly improved. This factor may be of critical importance in derating the casing for wear or corrosion, if there is a good cement job.

**Eccentric Pipe, Voids, and Cement Channels Effects**

Conventional casing design fails to account for the collapse stresses present in nonuniform loaded casing. Cement channels and voids, formation voids, or eccentrically centered casing can cause nonuniform loading. An underlying assumption in equations defining the collapse
and burst resistance is that the stress is applied uniformly to the outside surface of the casing.

A casing that is cemented into the hole with nonuniform clearance is referred to as eccentric. The degree of eccentricity is described by the equation in Figure 12 and is usually given in percent. Standoff ("STO") is the other measurement and equals 100 minus the eccentricity. Our models showed that eccentric casing, surrounded by a perfect cement sheath, is not significantly affected by the eccentricity with respect to the material properties of the cement sheath and has little negative effect on the collapse and burst resistance of the casing. However, the other previously mentioned effects still hold.

Cement channels and voids in the formation beyond the cement sheath have similar effects on collapse and burst resistance of casing. Consider the cement channels and formation voids in Figures 13 and 14, respectively. Reasons for formation voids are bad cementation or sand production during the life of the well. Nonuniform stress arises because the casing-cement sheath is partially confined by formation stress and partially by pore pressure.

Because of the channels or voids, the casing is not confined around the total circumference and allows the casing to deform into the free space, in response to the formation, grain-to-grain or in situ stress. Declining pore pressure aggravates the maximum stress. The deflection introduces large stresses at the bending points of the casing at the edges of the channel or void. The stress is most severe for very low pore pressures and deep channels and voids. Figure 15 from Berger, et al, illustrates the effect of a cement channel beyond the cement sheath. As the pore pressure declines, the nonuniform loading increases on the casing increases, peaking at 60 degrees. Figure 16 illustrates the same effect for a formation void.

When the pore pressure in the channel or void was reduced to simulate drawdown or depletion of the reservoir, the maximum von Mises stress in the casing due to
collapse loading increased dramatically and the possibility of failure became much more likely. The maximum stresses were virtually the same for hard and soft cement and did not make a difference if there was a cement channel or a formation void. The increase of the maximum resultant stress due to collapse loading was approximately 200% to 250%.

Case History
Platform Hogan was placed in the Santa Barbara Channel in 1969. The platform’s oil wells produce from 2,500’ to 5,500’, from turbidite sandstones ranging from moderately to well consolidated. Most wells were completed with perforated, gas lift completions, effectively commingling production from various producing horizons. Very few wells were gravel packed. During the life of the field, “tight pipe” was noted during workover operations across the field. The tight casing incidents ranged from minor bobbles on the weight indicator during casing scraper runs to major fishing complications during packer retrieval operations, including wellbore loss. These incidents were always noted in perforated intervals in the E-1 and F-1 sand, also suspected to be the major sand producing intervals. The sand production very seldom was measured as more than a trace in surface samples. The cumulative effects of sand production can be illustrated in the following manner: if the well is producing 1,000 BPD at 0.1% sand cut, every day, 1 barrel of sand is removed from the near wellbore and a large void is created.

Most of the collapsed casing was noted after the reservoir pressures had fallen to 1,000 psi or less, well within the collapse resistance of 7”, 23 & 26#, K-55 casing. The most severe casing collapse was noted in wells that were produced with artificial lift, which reduced the flowing BHP to 100 psi with corresponding low pressures in the pores and voids outside the casing.

To investigate results, a model was created with 2,000 psi insitu stress, and 100 psi pore pressure to examine the stresses in 7”, 23#, K-55 casing, under producing conditions, with voids ranging from 10° to 60°. The resultant calculated von Mises stresses in the casing were nonuniform, with localized stresses reaching 200,000 to 400,000 psi in the pipe, effectively deforming the casing in the manner observed in the field. Elimination of the formation void removed the stress concentrations in the casing, and von Mises stresses less than 20,000 psi were observed.

Conclusions
The results of the various finite element models reveal that a uniform cement sheath filling the annulus between the casing and borehole increases the burst resistance of the casing. The amount of burst resistance increase is dependent on the formation insitu stress acting upon the outside of the cement sheath. The larger the confining insitu stress, the greater the increase.

A uniform cement sheath also enhances the collapse resistance of casing if the cement sheath exhibits a softer and more ductile behavior, which can be reached with higher values of Young’s Modulus and Poisson’s Ratio. The modeling proved conventional design equations for both burst and collapse that are utilized under uniform stress conditions yield accurate results. However, under particular producing conditions and unpredicted by conventional design equations, collapse damage of casing at producing intervals can occur. Cement channels and/or formation voids reduce the collapse resistance of casing as predicted by conventional equations by up to 60%.

Conventional casing design is valid under uniform distribution of confining insitu stresses. Consistent, uniform, and homogeneous cement sheath and insitu formation stress acting up the outside of cement sheath with little or no channels or voids create appropriate conditions for conventional criteria design. However, because traditional equations derive from theoretical and empirical correlations where interaction between casing, cement, and formation is not considered, any anomaly in these conditions makes those design equations inaccurate. New experimental or theoretical analysis must be done before equations can be extensively utilized under nonuniform stresses conditions.

Over-designing casing to avoid outright failures or casing damage under nonuniform stress conditions is an unprofitable solution. A better understanding of cement and formation characteristics, as well as the potential nonuniform distribution of the confining insitu stresses while producing (primarily allowing sand production creating formation voids) will permit an effective tubular design. Even though it sounds unfeasible, comprehending the casing-cement-formation interaction by design engineers can add great value to specific long-term projects.

Proper cementing practices, including suitable casing centralization and the use of more ductile cement will increase the possibility of an appropriate cement sheath to augment casing’s burst and collapse resistance. Optimum completion design, sand control techniques, and proper production rates will prevent the existence of voids outside the casing, preventing reduction of casing’s collapse resistance.

When casing is designed for a hydraulic fracturing treatment, an accurate cement bond log will show if severe cement anomalies such as voids or channels are present. Knowing the existence of voids outside the casing will permit a better fracture stimulation design and mitigate against irreparable casing failures.

These conclusions brings the following questions to the light: How accurate are conventional casing design equations? Should design engineers expect casing failures under producing conditions? Should engineers alter early production practices to prevent further casing failures?
Nomenclature

\( p_b \) = Casing burst strength, \( \text{m/L}^2 \), psi
\( Y_p \) = Casing yield point, \( \text{m/L}^2 \), psi
\( h \) = Casing thickness, L, in
\( d \) = Casing diameter, L, in
\( \sigma_{vm} \) = von Mises stress, \( \text{m/L}^2 \), psi
\( \sigma_a \) = axial stress \( \text{m/L}^2 \), psi
\( \sigma_t \) = tangential stress, \( \text{m/L}^2 \), psi
\( \sigma_r \) = radial stress, \( \text{m/L}^2 \), psi

References