Abstract
The influences of osmosis, ion transfer, and thermal effects on stability of shale have been investigated. Stress and pore pressure distributions around a borehole have been calculated using a robust and practical physicochemical theory that describes shale deformation. The constitutive equations are developed using a poroelastic theory that couples ion transfer to the formation stresses and pore pressure. The governing field equations are solved analytically to yield the solute mass fraction, pore pressure, and the stress distributions around the borehole. It is shown that osmosis alters the pore pressure and the total tangential stress around the borehole. However, ion diffusion reduces the effects of osmosis by lowering the chemical potential gradient and establishing a pore pressure field characteristic of a hydraulic flow regime. Also, ion transfer into the rock adversely impacts hole stability by inducing tensile stresses. The role of thermal osmosis in fluid/shale interactions has also been considered. Results indicate that thermal osmosis is controlled by the nature of the fluid and the membrane properties of shale.

Introduction
Maintaining wellbore stability is of paramount importance when drilling for oil and gas productions. Shale deterioration and borehole instability are significantly influenced by the amount and distribution of water within the shale. The influx of mud filtrate into a shale formation increases water content and pore pressure near the borehole, alters the solid structure, and reduces shale strength. When in contact with a drilling fluid, shale absorbs water into its clay matrix between aggregates, particles, and basal planes of crystals. The sorption of water is accompanied by a change in the interlayer spacings between clay surfaces, which is manifested as a swelling of the rock. In addition to the chemical composition of the drilling fluid, shale reactivity is influenced by the chemical composition, amount, and distribution of water within the shale itself (Santos et al. 1996). Therefore, the influx of mud filtrate into the formation is a major factor contributing to shale deterioration and instability. Because shales have a very low permeability (order of nano-darcy), hydraulic transport is not the dominant form of fluid movement into the formation. In fact, hydraulic fluid transport is often several times smaller than the contribution of chemical and temperature potentials. The significance of water flux due to a chemical potential is well established (Young and Low, 1965; Kemper and Rollins, 1966; Olsen, 1969; Fritz, 1986). Its impact on hole stability has also been studied and several closed-form and numerical solutions for stress analysis around a borehole have been developed (Chenevert, 1970; Mody and Hale, 1993). Sherwood (1993) developed a coupled, Biot-like theory in which the presence of more than one salt is allowed. This theory considers only the differences in chemical potential between the bulk water of the shale and the bulk water of the drilling mud. In applying this theory, Sherwood and Bailey (1994) developed a plane strain solution for stress and pore pressure around a borehole in a hydrostatic stress state. Ghassemi et al. (1998) extended this solution to complete plane strain useful for inclined wellbores. Although these studies included the macroscopic transport processes driven by hydraulic and chemical potential gradients, coupling of ion diffusion to pore pressure and rock deformation was neglected. These investigations share the assumption that shale is an ideal ion exclusion membrane. However, it is known that shales are not ideal membranes and do allow transfer of ions (Fritz, 1986), hence, ion transfer and associated effects should be considered. Using arguments of irreversible thermodynamics, Heidug and Wong (1996) proposed a fully-coupled Biot-like model that considered ion transfer. This continuum model is an advancement beyond the traditional chemical potential formulations, both coupled and uncoupled. It is based on the extended versions of equations of poroelasticity (Biot, 1941) and Darcy's transport law, which are derived on the basis of non-equilibrium thermodynamics. Although this theory has the advantage of permitting ion transfer between the drilling mud and the pore fluid, its governing equations encumber development of analytical solutions for the
stress-strain response of shale. Analytical solutions are most useful for field applications, for gaining a fundamental understanding of the physical phenomena, and for validating numerical solutions. Thus, it is desirable to further develop the theory and to derive analytical solutions. The theoretical development proposed by Heidug and Wong is reformulated herein to yield a robust and practical physico-chemical theory. Ion transfer in the mud/shale system is coupled to stress and pore pressure within the framework of a Biot-like isotropic poroelastic theory. Field equations are developed within the proposed theoretical framework that allow determination of the mechanical response of shales to chemical, stress, and pore pressure disequilibria induced by drilling. Specifically, the governing field equations of the theory are solved analytically to yield the solute mass fraction, pore pressure, and the stress distributions around a borehole.

There is evidence that thermal osmosis can play a similarly significant role in filtration transport and hence shale behavior. Significant mass transport through clay membranes due to a temperature gradient has been observed in laboratory experiments (Dirksen, 1969). Based on the experimental results of Srivastava and Avasthi (1975), it has been estimated that thermo-osmotic volume flow through kaolinite can be 800 times larger than Darcian flow (Carnahan, 1984). The phenomenon of thermo-osmosis has long been recognized in the soil science community (Gray 1966; Dirksen, 1969; Groenevelt and Kay, 1974). However, the role of thermal membrane processes in shale instability has not been adequately addressed. Ghassemi et al. (1999) recognized the potential significance of thermal osmosis, and have included it in borehole stability analysis in shale (Ghassemi and Diek, 2000). Prior research has addressed the effects of fluid transport due to hydraulic pressure gradient and chemical osmosis on shale stability (Mody and Hale, 1993; Onaisi et al. 1993; Sherwood and Bailey, 1994; van Oort et al. 1996; Ghassemi et al. 1998; Ghassemi and Diek, 1999). This paper also considers the effects of the macroscopic process of osmosis due to a thermal potential. In assessing the contribution of thermal osmosis to fluid flux and rock deformation, solutions are developed to calculate pore pressure and stress distributions around a wellbore drilled in shale. These solutions are generated by using a generalized plane strain approach formulated within the framework of the non-isothermal poroelastic theory of Biot (1941), which includes thermal osmosis. Consequently, both thermoporoelastic and thermo-osmotic processes are included in the problem formulation and solution. Furthermore, a theoretical formulation has been developed for investigating the combined effects of chemical and thermal osmosis (Ghassemi and Diek, 2000; Diek and Ghassemi, 2001). Some results are discussed. This work is particularly useful in mud design for drilling through shale sections in high temperature environments.

**Chemical Osmosis and Ion Diffusion**

Knowledge of prevailing stresses and pore pressure distributions around the wellbore is important to stability analysis. The presence of a fluid in the formation strongly affects the physico-chemical response of the rock, the pore pressure, and the stress field. The shale-fluid system consists of a solid matrix that contains two kinds of connected voids with different size scales: (1) pore space that is filled with a freely diffusing fluid, and (2) interlayer space between individual clay platelets occupied by bound water. The bound water is defined here as the water that is subject to intermolecular surface forces (e.g., van der Waals, hydration, and electric double layer forces), and which possesses physical properties that deviate from those of the bulk (Derjaguin et al. 1987). On the other hand, the pore water is not subject to these forces because it occupies voids having dimensions that are much larger than distances over which intermolecular surface forces are effective <10 nm (Diek et. al., 1994; Diek and Santos, 1996). Both types of water need to be considered in developing constitutive equations for shale. Equations for describing the linear, isothermal deformation of shale are developed by defining a strain potential or free energy density for the shale-fluid system. This is done such that the differential of the potential represents the work associated with the elastic deformation and the work done to alter the masses of all fluid components in an infinitesimal macroscopic transformation (Heidug and Wong, 1996).

**Theory.** The fundamental constitutive relations for the temporal evolution of stresses and pore volume fraction in terms of the solid strains, pore pressure and chemical potentials of the solute and the diluent can be derived from the free energy density of the wetted mineral matrix containing bound water (Heidug and Wong, 1996). This free energy density is obtained from the difference between the free energy density of the shale-fluid system and that of the pore fluid. The isotropic and linearized constitutive equations for the temporal evolutions of the total stresses $\sigma_i$ and the variation of fluid content per unit referential volume $\xi$ in terms of the solid strains $\varepsilon_i$, pore pressure $p$ and solute mass fraction $C$ are then given by (Diek et al. 2001):

$$\sigma_i = 2G\varepsilon_i + \left[ K - \frac{2G}{3} \right] \varepsilon_i - \alpha' \chi + \omega \chi \xi$$

$$\xi \xi = \alpha' \chi \xi + \left( \beta + A' \right) \chi + A \omega \chi \xi$$

where $K$ and $G$ are the bulk and shear moduli,
is the universal gas constant, and \(T\) is the absolute temperature, \(M^s\) is the molar mass of the solute, \(\phi\) is the porosity, and \(\rho_f\) and \(K_f\) are, respectively, the intrinsic density and bulk modulus of the fluid.

Various irreversible transport processes occur in shales. The presence of hydraulic pressure and chemical potential gradients engenders both direct and coupled flows. Direct flows are in the form of hydraulic conduction and chemical solute diffusion, whereas coupled flows are in the form of chemical osmosis and conduction and chemical solute diffusion, whereas mechanical equilibrium condition requiring the total stresses in (1) to satisfy the conservation equation of fluid mass: 

\[
\rho_v \frac{\partial \mathbf{u}}{\partial t} + \rho_v \mathbf{u} \cdot \nabla \mathbf{u} + \rho_v \mathbf{f}_v - \frac{1}{3} \rho_v \nabla \mathbf{p} - \rho_v \mathbf{g} = 0
\]

(7)

where \(\mathbf{u}\) denotes the solid displacements and \(\mathbf{v}\) is the drained effective stress. The fluid pressure or fluid diffusion equation is obtained by requiring the time evolution of the fluid content \(\xi^f\) in (2) and the fluid flux \(\mathbf{J}^f\) in (5) to satisfy the conservation of fluid mass:

\[
\rho_f \frac{\partial \xi^f}{\partial t} + \nabla \cdot \mathbf{J}^f = 0
\]

(8)

The linearized solute diffusion equation or last field equation is obtained from the conservation equation of the solute:

\[
\phi \rho_f \frac{\partial \xi^s}{\partial t} + \nabla \cdot \mathbf{J}^s = 0
\]

where the convection term \(\mathbf{J}^f \cdot \nabla \xi^s\) was ignored due to the low permeability of shales.

**Example.** To study the impact of ion transfer on the pore pressure/stress field around a wellbore, consider a vertical well drilled in shale with material properties as well as relevant drilling fluid properties shown in Table 1. It should be noted that the results are presented using the geomechanics sign convention (compression positive). Figure 1 illustrates the pore pressure profile around the wellbore for various times. It can be observed that chemical osmosis significantly modifies the pore pressure profile. It can also be seen that ion transfer causes the chemical-osmosis to be time dependent. Note that as time increases, the transfer of ions leads to osmotic pressure dissipation and re-establishment of a pore pressure regime characteristic of hydraulic flow. This has significant implications for borehole stability and can cause delayed borehole failures. Figure 2 shows the distribution of effective radial stress as a function of distance from the borehole wall. Note that although a mud pressure of 15 MPa supports the wellbore, the rock experiences a tensile effective radial stress of about 5.2 MPa. This induced tensile stress is due to solute invasion into the formation; its magnitude largely depends on the parameter \(\omega_0\). This parameter can be viewed as a swelling or chemical stress coefficient for the solid and is assumed 4.0 MPa in accordance with some experimental results (Krantz et al. 1989). The zone of the induced tensile radial stress expands as time increases, but it does not extend far into the rock. The distribution of tangential effective stress is illustrated in Figure 3. Note that the tangential effective stress is reduced at the borehole wall; however, this reduction is initially overcome by the larger increase resulting from the reduction of pore pressure near the wellbore. The increase in tangential effective stress enhances the potential for shear failure inside the formation. With these results, it is safe to say that although the osmotically-induced reduction in the pore pressure tends to stabilize the borehole, ion transfer can cause the formation to fail in tension or compression. Therefore, the salt concentration and chemistry need to be optimized. It can also be expected that the impact of ion diffusion will vary with the shale’s reflection coefficient as well as with its diffusion coefficient.

**Thermal Osmosis**

In the presence of a temperature gradient across the shale-mud interface, thermal osmosis occurs when the mud/shale system exerts selectivity with respect to hot (high specific enthalpy) and cold (low specific enthalpy) molecules. The presence of a temperature gradient...
causes preferred transport of molecules, which increases the heat content of the system at the lower temperature side or lowers the heat content of the higher temperature side. As shown by Groenevelt and Bolt (1969), in the presence of hydraulic and thermal potentials, fluid volume and heat fluxes can be related to the corresponding thermodynamic driving forces through measurable phenomenological coefficients $L_{ij}$:

$$q^f = -L_{ij} \nabla p - \frac{L_{ij}}{T} \nabla T$$  \hspace{1cm} (10)

$$q^T = -L_{ij} \nabla p - \frac{L_{ij}}{T} \nabla T$$  \hspace{1cm} (11)

where $L_{ij}$ and $\left( L_{ij} / T \right)$ represent the coefficients corresponding to hydraulic and thermo-osmotic processes, respectively. $L_{ij}$ and $\left( L_{ij} / T \right)$ represent the coefficients corresponding to pressure thermal effects and heat conduction, respectively. The contribution of thermal osmosis to fluid flow and rock deformation may be considered within the framework of a linear non-isothermal poroelastic theory (e.g., McTigue, 1986; Kurashige, 1989). This provides a basis for consideration of thermo-poroelastic and thermo-osmotic processes. For completeness, a brief description of thermo-poroelasticity is presented next. Then, an example of a borehole problem drilled in shale is considered and solved to illustrate the influence of thermal osmosis.

**Theory.** The isotropic and linearized thermo-poroelastic constitutive laws used in this study are:

$$\sigma_{ij} = 2G\epsilon_{ij} + \left[ \frac{2G\nu}{1-2\nu} \right] \varepsilon_{kk} - \alpha f \rho - \frac{2G(1+\nu)}{3(1-2\nu)} \alpha_m T \delta_{ij}$$  \hspace{1cm} (12)

$$\zeta = \frac{\alpha_m (1-2\nu)}{2G(1+\nu)} \left[ \varepsilon_{kk} + \frac{3}{B} \right] - \phi (\alpha_f - \alpha_m) T$$  \hspace{1cm} (13)

where $B$ is the Skempton’s pore pressure coefficient, and $\alpha_m$ and $\alpha_f$ represent the volumetric thermal expansion coefficient of the solid matrix and the pore fluid, respectively.

When considering thermal osmosis, Darcy’s law can be written as:

$$q^f = -\frac{k^f \nabla p}{\eta} + K^T \nabla T$$  \hspace{1cm} (14)

where $K^T = -\left( L_{ij} / T \right)$ is the thermo-osmotic coefficient. It is assumed that thermal pressure effects are negligible ($L_{ij} = 0$) and that the heat flow obeys Fourier’s law:

$$q^T = k^T \nabla T$$  \hspace{1cm} (15)

where $k^T$ is the coefficient of thermal conductivity.

The mechanical equilibrium condition generates the Navier-type field equations in terms of the solid displacements:

$$G\nabla^2 u_i + \frac{G}{1-2\nu} u_{kk,i} - \alpha f \rho_i - \frac{2G(1+\nu)}{3(1-2\nu)} \alpha_m T_i = 0$$  \hspace{1cm} (16)

The other field relations are the fluid and heat diffusion equations:

$$\zeta \nabla^2 \zeta = \kappa^f \nabla^2 T$$  \hspace{1cm} (17)

$$\Phi = \kappa^T \nabla^2 T$$  \hspace{1cm} (18)

where

$$\kappa^f = \left[ \frac{2\alpha_m (\nu - \nu_f) c^f}{B(1+\nu_f)(1-\nu_f)} \right] + \phi c^f (\alpha_f - \alpha_m) T - K^T$$  \hspace{1cm} (19)

and $c^f$ and $c^T$ are the fluid and thermal diffusivity coefficients, respectively. Note that convective heat transport is neglected in the heat flow equation. This is justified for a low permeability system such as shale.

**Example.** To study the impact of thermo-osmosis on the pore pressure/stress field around a wellbore, consider a vertical well drilled in a shale with material properties as well as relevant drilling fluid properties shown in Table 2. Figures 4 and 5 illustrate the role of thermal osmosis when drilling with a mud that is warmer and cooler than the formation, respectively. It can be observed that thermo-osmosis significantly modifies the pore pressure near the wellbore at early times. A larger temperature difference between the mud and shale results in a magnification of the thermal osmotic contribution to pore pressure changes in the formation. In general, the extent and nature of the pore pressure change depends not only on the magnitude and on sign of the temperature gradient but also on the magnitude and sign of the coefficient of thermo-osmosis $K^T$. When $K^T < 0$, the osmotic flow direction is from warmer to cooler, whereas when $K^T > 0$, the flow is from cooler to warmer. Consider the shale-mud system, if $K^T$ and $\nabla T$ are either both positive or both negative, then the direction of the osmotic fluid flow is into the shale resulting in an increase of the pore pressure near the wellbore; while if either $K^T$ or $\nabla T$ is negative, then the fluid flows out of the shale reducing the pore pressure. Flow in both directions has been observed in laboratory tests using compacted clays. Therefore, depending on the nature of fluid and the membrane properties of the shale, thermal osmosis can either increase flow into shale, which reduces effective stress and shale strength, or it can increase flow into the wellbore, which may cause excessive shale dehydration. The resulting effective stress distribution corresponding to drilling with a warmer mud is shown in Figure 6.
Conclusions
A fully-coupled chemo-mechanical model has been developed and used to conduct stress analyses for a borehole drilled in shale. The model is based on the analytical solution of a set of field equations for solute mass fraction, pore pressure, and solid displacements. Results indicate that osmosis alters the pore pressure and the total tangential stress around the borehole. It would appear then, that increasing mud salinity could improve hole stability. However, it has been revealed that ion diffusion reduces the effects of osmosis by lowering the chemical potential gradient and establishing a pore pressure field characteristic of a hydraulic flow regime. As expected, the degree to which ion diffusion reduces the osmotic pressure depends on time and the membrane characteristics of the shale: it increases with an increase in time, a decrease in the reflection coefficient, and an increase in the diffusion coefficient. Furthermore, it has been found that ion transfer adversely impacts hole stability by inducing tensile radial stress and increasing the hoop stress. Thus, ion transfer can significantly contribute to borehole failure.

A thermoporoelastic solution for calculating the prevailing stress and pore pressure distribution around a borehole drilled in shale has also been developed. The solution is based on a coupled theory that considers certain thermal processes in drilling fluid/shale interactions. It improves previous studies that have neglected the contribution of thermoporoelastic effects and/or thermo-osmosis. The nature of the fluid and the membrane properties of shale determine whether thermal osmosis increases or reduces the fluid flow into shale. The former produces a reduction in effective stress and shale strength, while the latter may cause excessive shale dehydration and tensile failure. Results suggest that cooling ($\nabla T > 0$) a mud with a lower activity than the shale does not always guarantee a pore pressure reduction near the wellbore. For example, if $K_T$ is positive and significantly large, then thermal osmosis can reduce or even eliminate the effect of chemical osmosis. On the other hand, if $K_T$ is negative and large enough, then thermo-osmosis may intensify the effect of chemical osmosis. Prior studies have shown that cooling the borehole results in reduced potential of shear failure but increases potential of fracturing. However, in optimizing mud cooling, one should also consider its effects on the reflection coefficient in reducing and dissipating the chemical osmotic pressure.

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References
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<th>Table 1. Values of Parameters</th>
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<td>Drained Poisson’s ratio, $\nu$</td>
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<td>Fluid diffusivity, $c_f$</td>
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<td>Fluid mass density, $\rho_f$</td>
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<td>Mud’s solute mass fraction, $C_{mud}$</td>
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<td>Fluid’s volumetric thermal expansion coefficient, $\alpha_f$</td>
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<td>Thermo-osmotic coefficient, $K^T$</td>
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<td>Thermal diffusivity, $c_T$</td>
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Figure 1. Pore pressure distribution around the borehole.

Figure 2. Variation of the effective radial stress with time and distance from the borehole.

Figure 3. Profile of the effective tangential stress around the borehole.

Figure 4. Effect of thermal osmosis on pore pressure around the wellbore (shale cooler than mud).
Figure 5. Effect of thermal osmosis on pore pressure around the wellbore (mud cooler than shale).

Figure 6. Effective stress distribution around the borehole (mud warmer than shale).