A Leak-Off Model for Critical Permeability in Wellbore Strengthening Applications

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Abstract
Fracture-based strengthening techniques have been used in geologically complex reservoirs to strengthen wellbores during the past two decades. These techniques efficiently reduced the cost and risks caused by lost circulation especially in the depleted sections that have narrow mud weight window (MWW). The industry experience is that wellbore strengthening works well for conventional reservoirs (e.g., sandstone formation) but not well for unconventional reservoirs (e.g., shale formation). However, no rigorous models were available so far to characterize the critical permeability. If the formation permeability is larger than the critical permeability, conventional wellbore strengthening techniques are suitable. In this paper, an analytical model is developed to find the leak-off time of fracturing fluid in the plug zone to surrounding formation after wellbore strengthening materials (WSM) plugging. The results show that the influencing factors for leak-off time are plugged fracture area, WSM plug location, fracturing fluid viscosity, differential pressure between wellbore pressure and pore pressure, formation porosity, and permeability. A case study is conducted with varying formation permeability (0.0001 md to 50 md). For permeable formation (permeability >1 md), the leak-off time is only a few seconds. However, the leak-off time can increase to several hours for low permeability formation (permeability < 0.001 md). A critical permeability zone is identified and the results are in good agreement with gained field experience. The analytical model is a strong tool for wellbore strengthening methods selection and accurate particle size distribution (PSD) for wellbore strengthening applications.

Introduction
Drilling through depleted reservoirs is challenging because the MWW may be very narrow due to the drop of pore pressure and horizontal stresses in the formation. In depleted reservoirs, the depletion of pore pressure due to production may induce land subsidence (Geertsmama 1973; Zhang et al. 2016b) and can affect the horizontal stresses, i.e. stress path (Addis 1997; Shahri and Miska 2013). Consequently, the fracture gradient is decreased and the available MWW narrows at the upper limit (Aadnoy 1991; Shahri et al. 2013). On the other hand, as the neighboring formations maintain their pore pressure, a kick zone is easily formed. The lost circulation events will occur once the equivalent circulating density (ECD) exceeds the fracture gradient (Morita et al. 1990; Chen et al. 2014; Chen et al. 2015). The estimated industry cost for lost circulation in the Gulf of Mexico is $1 billion per year and the worldwide is $2-4 billion per year (Growcock 2010). To address the problem, field applications of wellbore strengthening techniques have shown the ability to reduce mud loss and increase the breakdown pressure limit.

Techniques with different working mechanisms have been proposed and widely used in the industry, and mainly include fracture propagation resistance (FPR) (Fuh et al. 1992; van Oort et al. 2011), hoop stress enhancement (Alberty and Mclean 2004), which is also called “stress cage”, and fracture closure stress (FCS) (Dupriest 2005). In this paper, we are mainly focused on the stress cage method. The main process of stress cage method is depositing WSM at or close to a newly formed fracture mouth. Then WSM will act as a seal isolating the fluid in the fracture. The fluid beyond the seal will dissipate into the surrounding medium and the fluid pressure will decrease to pore pressure. After fluid dissipation, the hoop stress will exceed its original value around the wellbore.

Field operations have demonstrated the success of stress cage method (Aston et al. 2004; Song and Rojas 2006; Whitfill et al. 2006). On the other hand, a number of researchers have conducted theoretical (Mehrabian et al. 2015; Feng and Gray 2016a) and numerical studies (Wang et al. 2006; Guo et al. 2011; Feng et al. 2015; Feng and Gray 2016b; Zhong et al. 2017). Both field applications and lab experiments (Guo et al. 2014) found that it is easier to strengthen a sandstone formation than a shale formation. Chemical strengthening methods are commonly used for low permeability formations (Aston et al. 2007; Growcock et al. 2009). However, there are no rigorous models available for characterizing the critical permeability for the stress cage method. Thus, it is very important to develop a reliable model to distinguish the critical permeability for drilling to avoid the unnecessary cost and risks.

The aim of the present work is to develop an analytical model to find the critical permeability for the stress cage method. It considers the time for fracturing fluid in the plug zone leaking...
off to surrounding formation. Based on the model, the influencing factors are found to be the plugged fracture area, WSM plugging location, fracturing fluid viscosity, differential pressure between wellbore pressure and pore pressure, porosity and formation permeability. A fast-running static fracture model based on dislocation method (Warren 1982; Shahri et al. 2014) and an empirical model were (Zhang et al. 2016a) employed to obtain the fracture area. A case study with varying formation permeability was conducted. The results were in good agreement with field observations and a critical permeability zone was targeted. The model is important for wellbore strengthening methods selection and accurate PSD design during drilling in depleted reservoirs.

Mathematical Modeling

In this section, an analytical model is first developed to characterize the fluid leak-off after WSM plugging. Then, two methods are introduced to obtain the fracture geometry.

First let us consider a scenario with WSM plugging. A schematic of fluid leak-off after WSM plugging is shown in Fig. 1. Assume the WSM plug is located at $L_1$ and the total fracture length is $L$. Fluid inside the plug zone will leak off to the surrounding formation as shown. Different fluid leak-off mechanisms are involved in the fracture, which mainly include viscosity controlled, formation fluid controlled and wall-building effects. In this paper, only viscosity controlled mechanism is considered. It is assumed that the fracturing fluid has high viscosity and no mud cake is considered. For viscosity controlled fracturing fluid, the leak-off coefficient is (Craft et al. 1962)

$$C_v = 0.0469 \left( \frac{k\Delta p \phi}{\mu} \right)^{1/2}$$

(1)

The leak-off velocity in the plug zone is (Howard and Fast 1957)

$$u = \frac{2C_v}{\sqrt{T}}$$

(2)

We assume that the fracture cross-sectional area in the vertical direction has elliptical shape, which is the same assumption used for PKN fracture model. For 3D model, equal the fluid inside the plug zone to the fluid leaking-off to the surrounding formation for time $T$,

$$H \int_0^T u dt \ast (L - L_1) = \frac{\pi}{4} H \int_{L_1}^L w(x) dx$$

(3)

Substituting Eq. 2 into Eq. 3 and canceling $H$, we have

$$\int_0^T 2C_v \sqrt{T} dt \ast (L - L_1) = \frac{\pi}{4} \int_{L_1}^L w(x) dx$$

(4)

Integrating Eq. 5 yields

$$4C_v \sqrt{T} \int_0^T (L - L_1) = \frac{\pi}{4} \int_{L_1}^L w(x) dx$$

(5)

Separate $T$ from the left hand side of Eq. 6,

$$T = \left( \frac{\pi \int_{L_1}^L w(x) dx}{16C_v(L - L_1)} \right)^2$$

(6)

Substituting Eq. 1 into Eq. 7, gives

$$T = \left( \frac{\pi \int_{L_1}^L w(x) dx}{0.7504 \left( \frac{k\Delta p \phi}{\mu} \right)^{1/2} \ast (L - L_1)} \right)^2$$

(7)

Finally, the equation can be simplified as

$$T = \left( \frac{\pi \int_{L_1}^L w(x) dx}{0.7504(L - L_1)} \right)^2 \frac{\mu}{k\Delta p \phi}$$

(8)

Based on Eq. 8, the influencing factors of leak-off time for a fixed-length fracture are: 1. Plugged fracture area ($L_1 w(x) dx$); 2. WSM plug location ($L_1$); 3. Fracturing fluid viscosity ($\mu$); 4. Effective formation permeability ($k$); 5. Differential pressure between wellbore pressure and pore pressure ($\Delta p$); 6. Formation porosity ($\phi$). As we know, the leak-off time should be small enough to let the seal hold the pressure fluctuation during drilling. In other words, the fluid inside of the plug zone should drain quickly to avoid instability issues during pressure ramps and the WSM plug can build up additional hoop stress in time.

The leak-off time has a positive relationship with fracturing fluid viscosity and an inverse relationship with formation permeability, porosity and differential pressure. Because this equation also involves the fracture area, the leak off time will also be affected by other fluid property (i.e., wellbore pressure), rock properties (Young’s modulus, horizontal stress anisotropy) and wellbore conditions (wellbore inclination, azimuth and radius). For field applications, a comprehensive parametric study should be done for favorable operations.

Note that there is a plugged fracture area in Eq.8. To calculate that, we should know the fracture area under static loading. A schematic of the static fracture model is shown in Fig. 2a. It is a classical bi-wing fracture model but only a single fracture is shown here. The maximum horizontal stress is $\sigma_h$ and the minimum horizontal stress is $\sigma_v$. It is assumed that the pressure inside the fracture is equal to wellbore pressure ($P_f = P_w$) before WSM plugging. Fig. 2b shows the corresponding finite element model. Due to symmetry, only a quarter of the whole model is considered. For finite element modeling, all boundaries should be assigned to either essential boundary conditions or natural boundary conditions. The detailed boundary conditions are shown in the figure. Notice that the wellbore pressure is applied on the fracture surface (red line).

According to the semi-analytical model in Shahri et al. 2014,
the fracture width distribution is

\[ w(t) = \frac{\pi(b-a)}{2m} \frac{4(1-v^2)}{E} \sum_{k=1}^{N} F(t_k) \]  \(\text{(9)}\)

Where \(F(t_k)\) is a distribution function. To evaluate the model, a base case is conducted with input parameters as shown in Table 1. The wellbore pressure is 9200 psi, which is slightly higher than the minimum horizontal stress. The fracture profile using the semi-analytical model is shown in black line in Fig. 3. It has an excellent match with finite element simulation results (red line). Thus, the semi-analytical model can accurately predict the fracture geometry with given conditions. The plugged fracture area can be obtained by using Eq. 9, as shown in Fig. 4. Area of each small part can be calculated and added together to obtain the plugged fracture area.

Other methods using empirical relationships can also be used to estimate the fracture area. For instance, Zhang et al. 2016a obtained the following equations for fracture width distribution,

\[ w(x_c) = \frac{4(1-v^2)}{E} \left( p_w - \sigma_h + c(\sigma_h - \sigma_h) \right) \sqrt{\frac{L+a)^2 - x_c^2}{x_c}} \]  \(\text{(10)}\)

\[ c = \frac{0.368a^{1/2}}{(L+3a)^{1/3}} \]  \(\text{(11)}\)

Note that the equations are modified to be consistent with parameters used in this paper. The fracture length \(L\) and wellbore radius \(a\) have unit of inches.

**Case Study**

A case study is conducted in this section. The input parameters are shown in Table 2. The fracture length is assumed to be 6 in. and the WSM plug location is 1/6 ft (2 in.) away from the fracture mouth.

The fracture profile using the semi-analytical model is shown in Fig. 5. The fracture mouth opening is about 1450 \(\mu\)m and the plugged fracture area is calculated as 9.413 \(\times\) 10^\(-4\) ft^2. The WSM plug is shown in orange in the figure.

To investigate the influence of formation permeability, the permeability varies from 0.01 md to 50 md. The results are shown in Fig. 6. For better observation, a semi-log plot is used. As we can see, the leak-off time has an inverse relationship with the formation permeability. For permeable formation (permeability >1 md), the leak-off time is only a few seconds.

It means that the fluid inside the plug zone can leak-off to the surrounding formation quickly after WSM plugging. However, for formation permeability smaller than 0.1 md, the leak-off time increases rapidly from 0.8 mins to 8 mins. The surge pressure caused by drill string movement may break the WSM plug because the fluid cannot drain quickly enough to build the addition hoop stress. For this specific case, the critical permeability zone is targeted as from 0.01 md to 0.1 md. Of course, the true critical permeability zone may not be the same as calculated values for field applications because the stabilization of WSM plug is influenced by many parameters (pressure fluctuation rate, fracture breakdown pressure, WSM properties, etc.). Thus, for simplicity, it is reasonable to use a single parameter (leak-off time) to estimate the critical permeability.

If the permeability is further decreased to 1 \(\times\) 10^-4 md, the leak-off time will increase to almost 13 hours (799 mins) as shown in Fig. 7. Such a long time will inevitably prohibit the stabilization of WSM plug and cause non-productive time (NPT). In other words, the stress cage method may not work well for unconventional reservoirs (permeability<0.01 md) in this case.

**Fig. 8** shows the permeability for different formations. The permeability cut-off line is 0.1 md for conventional and unconventional reservoirs. The permeability of unconventional shale reservoirs can be as low as nano-darcy (Zhong et al. 2014; Gao et al. 2016). For conventional reservoirs, the leak-off time is small and the stress cage method works well. On the other hand, for shale or some tight gas or tight oil reservoirs the leak-off time is larger and the stress cage method may not work well. The results of case study are in accordance with field observations. However, the analytical tool developed in this paper is a more reliable way to determine whether it is appropriate to use the stress cage method because other parameters can also affect the fluid drainage. This model is also useful in hesitation squeeze to determine the healing time.

**Conclusions**

In this paper, an analytical model is developed to find the leak-off time of fracturing fluid in the plug zone to the surrounding formation. The following conclusions can be drawn through this study:

1. The influencing factors of fluid leak-time are found to be the plugged fracture area, WSM plug location, fracturing fluid viscosity, differential pressure between wellbore pressure and pore pressure, porosity and formation permeability. On the other hand, the fracture area is affected by fluid property (wellbore pressure), rock properties (Young’s modulus, horizontal stress anisotropy) and wellbore conditions (wellbore inclination, azimuth and radius). The influence of these parameters can be obtained through previous parametric studies.

2. The leak-off time should be small enough to let the seal hold the pressure fluctuation during drilling. If the leak-off time is large, the dynamic drilling operations may break the WSM seal because the fluid cannot drain quickly enough to build the addition hoop stress.

3. The results of case study are in good agreement with field observations and a critical permeability zone is targeted. For this case, the critical permeability zone is close to cut-off line between the conventional and unconventional reservoirs.

4. The analytical model is a strong tool for wellbore strengthening methods selection and accurate particle size distribution (PSD) during drilling. If the calculated leak-off time is large and stress cage method is not appropriate, other wellbore strengthening methods (e.g., fracture propagation resistance or chemical method) can be applied. This model is also useful in hesitation squeeze to determine the healing time.
Acknowledgments

The authors are grateful to the University of Tulsa Drilling Research Projects (TUDRP) member companies for their technical and financial supports.

Nomenclature

- \(a\) Wellbore radius, in.
- \(b\) Distance from wellbore center to fracture tip, in.
- \(c\) Stress anisotropy factor
- \(C_v\) Leak-off coefficient, \(ft/\sqrt{min}\)
- \(E\) Young’s Modulus, psi
- \(F(t_k)\) Distribution function
- \(H\) Fracture height, ft
- \(k\) Formation permeability, Darcy
- \(L\) Fracture length, ft
- \(L_1\) Distance from fracture mouth WSM plug, ft
- \(m\) Number of discretization points, dimensionless
- \(N\) Number of terms, dimensionless
- \(P_f\) Fluid pressure inside the fracture, psi
- \(P_w\) Wellbore pressure, psi
- \(t\) Time, min
- \(t_k\) Fixed fracture coordinate system, dimensionless
- \(\tau\) Fluid leak-off time, min
- \(u\) Fluid leak-off velocity, \(ft/min\)
- \(x\) Distance from the wellbore, in.
- \(x_c\) Distance from the center of the wellbore, in.
- \(w\) Fracture width, in.
- \(\nu\) Poisson’s ratio, dimensionless
- \(\phi\) Porosity, dimensionless
- \(\mu\) Fracturing fluid viscosity, cp
- \(\sigma_h\) Minimum horizontal stress, psi
- \(\sigma_H\) Maximum horizontal stress, psi
- \(\Delta p\) Differential pressure between wellbore pressure and pore pressure, psi

References


Growcock, F. 2010. How to stabilize and strengthen the wellbore during drilling operations. SPE Distinguished Lecturer Program presentation.


Song, J., Rojas, J.C., 2006. Preventing mud losses by wellbore strengthening. Paper SPE 101593-MS presented at SPE Russian Oil and Gas Technical Conferences and Exhibition, Moscow, Russian, 3-6 October.


Fig 1. Schematic of fluid leak-off after WSM plugging.

Fig 2. (a) Schematic of wellbore strengthening before plugging. (b) The corresponding finite element model.

Table 1. Input parameters for the base case.

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Fig. 3. Fracture profile comparison of base case.

Fig. 4. Fracture area calculation

Table 2. Input parameters

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Fig. 5. Fracture profile with WSM plug

Fig. 6. Leak-off time for different formation permeability.
Fig. 7. Leak-off time for smaller formation permeability.

Fig 8. Permeability for different formations (Courtesy from US Department of Energy).