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Power Law Model Hydraulic Calculations Can Be Made More Accurate (Part I)

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Abstract

When characterizing fluid flow in drilling fluids, the standard procedure is to either calculate the shear rate and the shear stress using a graph known as a rheogram, or use any of the three classical rheological models: Bingham Plastic, Power Law, or Herschel-Bulkley. The contention of this paper is that using the rheogram in conjunction with the Power Law model is more accurate than other rheological methods. The curve of the shear rate vs. shear stress on the rheogram is made up of five segments with each segment composed of two points of shear rate and shear stress.

For a determined shear rate, the shear stress reading from the rheogram is always maintained between the two closest shear rate points. Since the shear rate varies at different sections in the annulus, the two closest shear rate points of the rheogram which are the RPM, will likewise vary. Consequently, by following the same logic based on these two closest points, the Power Law is reasonably the best mathematical model to use as it can be applied using two shear rate points applicable to the annular section of interest, just like the rheogram. This paper explains how to use the Power Law model using the variable two closest shear rate points to determine the shear stress for a certain shear rate within the annulus.

In this – the first of two corresponding papers – the authors will explain how to use the Power Law model using the variable two closest shear rate points to determine the shear stress for a certain shear rate. The follow-up paper, "*AADE-13-FTCE-02: Power Law Model Hydraulic Calculations Can Be Made More Accurate (Part II)*", will compare the calculations of Effective Viscosity, Reynolds Number, and Critical Reynolds Number using the different rheological models.

Introduction

The objective of this paper is to lay the foundation for a standardized methodology for hydraulic calculations, using the simplest and most accurate mathematical model based on the closest pair of shear rate points available from the viscometer readings. Further, the calculations will confirm that it is unnecessary to go beyond the range of the closest pair of points applicable to the annular section of interest. Table 1 details the method for determining the closest pair of shear rate points.

Since it requires three shear-rate points, the Herschel-Bulkley model cannot be applied. Moreover, its critical velocity equation, among others, is more complicated than required. The Bingham Plastic model is fixed between 300 and 600 rpm, even though actual shear rates vary beyond that range (or range from 3 to 300 rpm). The ultimate effect of not using the two closest shear rate points, or rather applying a mathematical model using an unnecessary wider spread of two or three shear-rate points, is damaging to the original resolution of the actual curve of shear rate and shear stress as it dampens the curve and reduces its accuracy. Consequently, the ultimate objective of this paper is to standardize the calculation of the Power Law Model's index, "n", in the simplest and most accurate way, making it a useful and easily understood tool for unifying hydraulic results for the drilling industry.

Typical hydraulic calculations always start by determining the shear rate, followed by its corresponding shear stress rheological data. These two data define the viscosity (shear stress/shear rate) of the drilling fluid for a given flow rate, flow geometry, and rheological profile. The shear rate data is related to the rpm of the viscometer, while shear stress data is related to dial reading of the viscometer (Fig. 1). Determining the shear stress for a certain shear rate can be accomplished using two basic methodologies: the Graphical and Mathematical methods.

Graphical Method

For each standard shear rate (3, 6, 100, 200, 300, and 600 rpm), viscometer shear stress dial readings are plotted on logarithmic scales (rheogram), as shown in Figures 1, 2 and 3. These plot the rheology of two drilling fluids.

Mathematical Model Method

Although the rheogram graph is cumbersome and tedious, it is more accurate than the Bingham Plastic, Power Law or the Modified Power Law (Herschel-Bulkley) mathematical models. The disparity in accuracy is a result of the curve of the shear rate vs. shear stress on the rheogram being made up of five segments with each segment comprising two points of shear stress and shear rate, as Fig. 1 illustrates.

For the purposes of this examination, a six-speed viscometer was used, which provides six points of shear rate that include the 3, 6, 100, 200, 300 and 600 rpm readings.

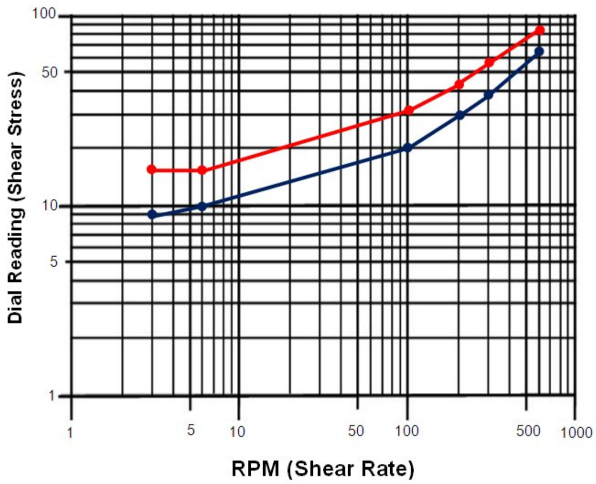

Fig. 1 – Two Rheogram Plots of Viscometer Readings

The slope (related to the"n" of the Power Law model) created by the two points of each segment may have different values, (Fig. 2). For a determined shear rate, the shear stress reading from the rheogram is always maintained between the two closest points, which are the corresponding rpm on either side of the shear rate. This is illustrated more clearly in Fig. 3.

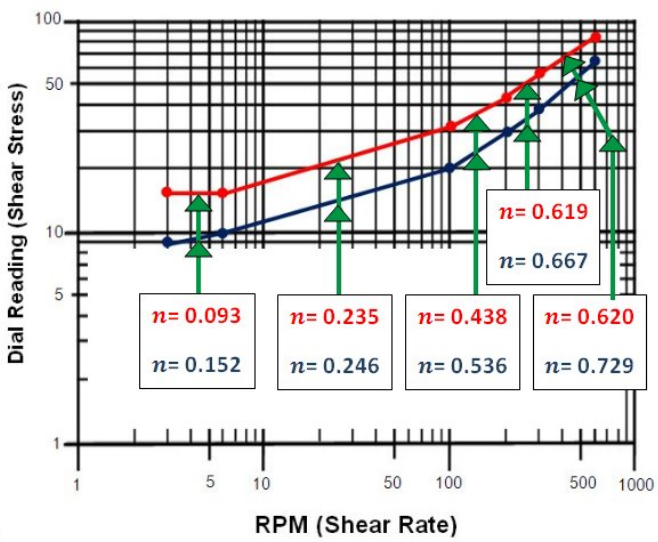

Fig. 2 – Two Rheogram Plots Showing "n"s of Power Law Model

Most of the methods used are the three classic mathematical models, with each having a different "n" calculation result. The value differences result from the mathematical models being expressed differently and all the models being based on different ranges of rheological points. The wide variance is detailed in *Tables 2 & 3—Comparison of Different Rheology Models*. Some of the calculations reside in the turbulent flow regime and some in the laminar flow regime. For example, at certain times in drilling horizontal wells, the flow regime may need to be maintained as close to turbulent as possible, yet still remain laminar. Accordingly, one of the objectives of this work is to present the industry a unified way of calculating the "n" of the Power Law model so the shear rate (rpm) and shear stress (dial reading) calculation results are uniform. This represents the first step in standardizing hydraulic calculations.

If the flow behavior index, "n", in the Power Law model is to be reported, it would then be logical to also mention the corresponding shear rate (rpm) from which it was derived. This effectively would ensure broad consistency in the calculations. A following example problem will demonstrate this calculation.

Power Law Model Simulating the Rheogram Worksheet

To follow the same logic based on the two closest points of the shear rate, the Power Law represents the best mathematical model to use. The two closest points of the shear rate vary and depend upon the annular shear rate. For example, an annular shear rate of 230 falls between 200 rpm and 300 rpm, while an annular shear rate of 170 is between 100 rpm and 200 rpm, as shown in Fig. 3.

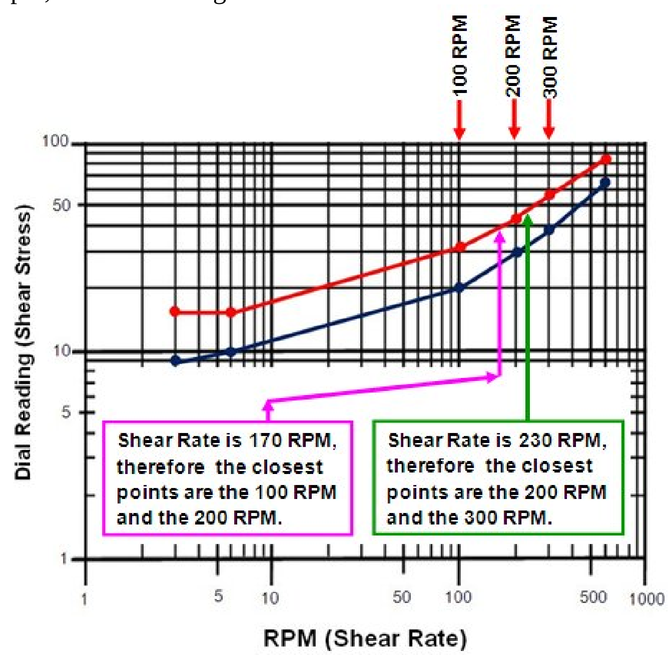

Fig. 3 – Two Rheogram Plots with 2 Closest Shear Rates

Annular Shear Rate Calculation

The equation of the annular shear rate, derived by several authors [1, 2] and held as the industry standard, determines the shear rate and its corresponding shear stress between its two closest shear rate points. However, the calculations used in this paper focus only on the annular sections of the well while drilling.

The annular shear rate (γ_a) in reciprocal seconds, that matches the annular drilling fluid velocity is expressed as,

$$\gamma_a = \frac{2.4\ x\ Va}{D_h - D_p}\left(\frac{2n+1}{3n}\right) \quad (1)$$

The annular shear rate (rpm) [1, 2] in revolution per minute of a viscometer that matches the annular drilling fluid velocity is expressed as,

$$rpm_a = \frac{1.41\ x\ Va}{D_h - D_p}\left(\frac{2n+1}{3n}\right) \quad (2)$$

Where:

- γ_a = *Annular shear rate, 1/sec*
- Va = *Average annular velocity at a certain section of the hole, ft per min*
- rpm_a = *Annular shear rate (viscometer rpm), revolutions per minute*
- n = *Flow behavior index of Power Law model*
- D_h = *Diameter of the hole, in.*
- D_p = *Outside diameter of the pipe, inches*

Since the two closest points have not yet been determined, the " n "of the Power Law model cannot be calculated, therefore rpm_a of equation (2) likewise could not be determined. The rpm_a [2] for a starting value can be approximated by using n[2] = 0.7 in equation (2) with about a 10% error factor, which is corrected later.

The approximated annular shear rate (rpm_{aprx}) [2] deduced from equation (2) is expressed as,

$$rpm_{aprx} = \frac{1.61\ x\ Va}{D_h - D_p} \quad (3)$$

Where:

- rpm_{aprx} = *The approximated shear rate (viscometer rpm), revolutions per minute*

The average annular velocity of a certain annular section of the well is expressed as,

$$V_a = \frac{24.51\ x\ Q}{D_h^2 - D_p^2} \quad (4)$$

Where:

- Va = *Average annular velocity, ft per min*
- Q = *Flow Rate, gals per min*
- D_h = *Diameter of the hole, in.*
- D_p = *Outside diameter of the pipe or drill collar, in.*

After the rpm_{aprx} is determined by equation (3), the two closest points of shear rate (rpm) can be chosen (refer to Fig. 3). Afterwards, the two closest points of shear rate (rpm) chosen will be checked to confirm they are correct. If not, a correction will be made later. This essentially is the starting point in applying the Power Law model based on two closest points of shear rate (rpm).

Reviewing the basics first, the general equation of Power Law model using the direct reading parameters of the viscometer is expressed as

$$\theta = K\ x\ (rpm)^n \quad (5)$$

Where:

- θ = *Dial reading (shear stress) of the viscometer, deflection*
- rpm = *rpm (shear rate) of the viscometer, revolution per minute*
- K = *Consistency factor*
- n = *Flow behavior index of Power Law model*

The "n" of equation (5) can be determined by using the two closest points.

$$n = \frac{\log\left(\frac{\theta_2}{\theta_1}\right)}{\log\left(\frac{rpm_2}{rpm_1}\right)} \quad (6)$$

The "K" of equation (5) can now be determined. See equation (7) or equation (8).

$$K = \frac{\theta_1}{rpm_1{}^n} \quad (7)$$

or

$$K = \frac{\theta_2}{rpm_2{}^n} \quad (8)$$

Knowing "n" and "K" completes the Power Law model at any two closest shear rate points applicable to the annular section of interest.

Where:

- θ_1 = *Dial reading (shear stress) of the viscometer at the first point of two closest points, deflection*
- θ_2 = *Dial reading (shear stress) of the viscometer at the second point of two closest points, deflection*
- rpm_1 = *rpm (shear rate) of the viscometer at the first of two closest points, revolutions per minute*
- rpm_2 = *rpm (shear rate) of the viscometer at the second point of two closest points, revolutions per minute*
- K = *Consistency factor*
- n = *Flow behavior index*

Table 1-Method of Verifying Correct 2 Closest Points

Tier	Method
A	1. Calculate the "n" using equation (6). after determining the rpm_{aprx} using equation (3) and choosing the 2 closest rpm points. 2. Calculate the annular rpm_a using equation (2). 3. If the annular rpm_a of Step (2) is between the two closest points of shear rate (rpm) previously chosen, then the annular rpm_a of Step (2) is correct. Therefore the previously chosen two closest points is also correct (true). *The verification process ends here in many cases. If incorrect, proceed to Step (4)*
B	4. Choose the next two closest points based on the annular rpm_a of Step (2). 5. Calculate the new "n" using equation (6). 6. Calculate the new annular rpm_a using equation (2). 7. Repeat Step (3). *It is possible that the verification process ends after repeating Step (3) once. In rare cases, the loop may not end, in which case Tier C is employed.*
C	8. Take note of the 2 pairs of closest points previously chosen. 9. Choose the minimum point of the lower rpm pair as rpm_1 and θ_1. 10. Choose the maximum point of the higher rpm pair as rpm_2 and θ_2. 11. Calculate the new "n" using equation (6) 12. *The verification process ends here.*

The following Example Problems will provide a more explicit understanding of this verification method. After the verification process, the Power Law model will be completely defined and can be used for further hydraulic calculations.

Problem Examples

Example 1

Given Data:

D_h, in	=	8.50
D_p, in	=	6.00
Annular velocity, fpm	=	340
θ_{600}	=	83
θ_{300}	=	54
θ_{200}	=	42
θ_{100}	=	31
θ_{6}	=	16
θ_{3}	=	15

Problem:

1. Define the Power Law model based on the two closest shear rate (rpm) points.
2. Determine the "n" and its corresponding shear rate in rpm.

Solution:

The approximated annular shear rate (rpm_{aprx}) using equation (3) is expressed as,

$$rpm_{aprx} = \frac{1.61\ x\ 340}{8.5 - 6.0}$$

$$rpm_{aprx} = 219.0$$

The two closest points of the viscometer reading are 300 rpm and 200 rpm:

$rpm_1 = 200$, $\theta_1 = 42$
$rpm_2 = 300$, $\theta_2 = 54$

Verifying the correct two closest shear rates (rpm), reference Tier A, Step (1) of *Table 1— Verification Method of Two Closest Points* - Using equation (6),

$$n = \frac{\log\left(\frac{\theta_2}{\theta_1}\right)}{\log\left(\frac{rpm_2}{rpm_1}\right)}$$

$$n = \frac{\log\left(\frac{54}{42}\right)}{\log\left(\frac{300}{200}\right)}$$

$$n = 0.62$$

Then, referencing Tier A, Step (2) of *Table 1— Verification Method of Two Closest Points*, using equation (2),

$$rpm_a = \frac{1.41\ x\ 340}{8.5 - 6.0}\left(\frac{2\ x\ 0.62 + 1}{3\ x\ 0.62}\right)$$

$rpm_a = 231$, which is between 200 rpm and 300 rpm of viscometer reading. Therefore, the previously chosen two closest points were accurate, thereby concluding the verification process.

Continuing with the solution, the consistency factor can now be computed, using equation (6),

$$K = \frac{\theta_2}{{rpm_2}^n}$$

$$K = \frac{54}{(300)^{0.62}}$$

K = 1.57

or using equation (7),

$$K = \frac{\theta_1}{rpm_1{}^n}$$

$$K = \frac{42}{(200)^{0.62}}$$

K = 1.57

Both equations (6) and (7) produce the same result. The Power Law model based from the two closest rpm points (200 and 300 rpm), using equation (5) is expressed as,

$$\theta = 1.57\ x\ (rpm)^{0.62}$$

The "n" value and its corresponding shear rate in rpm is expressed as,

n = 0.62 @ 231 rpm

Example 2

Given Data:

Dh, in	=	10.50
Dp, in	=	8.25
Annular Velocity, fpm	=	227
θ_{600}	=	49
θ_{300}	=	36
θ_{200}	=	31
θ_{100}	=	23
θ_6	=	7
θ_3	=	5

Problem:

1. Define the Power Law model based on the two closest shear rate (rpm) points.
2. Determine the "n" and its corresponding shear rate in rpm.

Solution:

The approximated annular shear rate (rpm_{aprx}) using equation (3) is,

$$rpm_{aprx} = \frac{1.61\ x\ 227}{10.5 - 8.25}$$

rpm_{aprx} = 162.4

The two closest points of the viscometer reading are 200 rpm and 100 rpm:

rpm_1 = 100, θ_1 = 23
rpm_2 = 200, θ_2 = 31

Verifying the correct two closest shear rates (rpm), reference Tier A, Step (1) of *Table 1— Verification Method of Two Closest Points* - Using equation (6),

$$n = \frac{\log\left(\frac{\theta_2}{\theta_1}\right)}{\log\left(\frac{rpm_2}{rpm_1}\right)}$$

$$n = \frac{\log\left(\frac{31}{23}\right)}{\log\left(\frac{200}{100}\right)}$$

n = 0.4306

Then, referencing Tier A, Step (2) of *Table 1— Verification Method of Two Closest Points*, using equation (2),

$$rpm_a = \frac{1.41\ x\ 227}{10.5 - 8.25}\left(\frac{2\ x\ 0.4306 + 1}{3\ x\ 0.62}\right)$$

rpm_a = 204.95, which is between 200 rpm and 300 rpm of the viscometer reading. It is important to note that this is *not* the same as the previous two closest shear rate points chosen, which were 100 rpm and 200 rpm. Therefore, it is necessary to proceed to *Tier B*, Step (4) of *Table 1— Verification Method of Two Closest Points.* Using equation (2),

$$n = \frac{\log\left(\frac{\theta_2}{\theta_1}\right)}{\log\left(\frac{rpm_2}{rpm_1}\right)}$$

$$n = \frac{\log\left(\frac{36}{31}\right)}{\log\left(\frac{300}{200}\right)}$$

n = 0.3688

Then, referencing Tier B, Step (6) of *Table 1— Verification Method of Two Closest Points*, using equation (2),

$$rpm_a = \frac{1.41\ x\ 227}{10.5 - 8.25}\left(\frac{2\ x\ 0.3688 + 1}{3\ x\ 0.3688}\right)$$

rpm_a = 233, which is between 200 rpm and 300 rpm Therefore the previously chosen two closest points were correct (true) and the verification process ends.

Continuing with the solution, the consistency factor can now be computed, using equation (6),

$$K = \frac{\theta_2}{rpm_2{}^n}$$

$$K = \frac{36}{(300)^{0.3688}}$$

K = 4.39

or using equation (7),

$$K = \frac{\theta_1}{rpm_1{}^n}$$

$$K = \frac{31}{(200)^{0.3688}}$$

K = 4.39

Both equations (6) and (7) produce the same result.

The Power Law model based from the two closest rpm points (200 and 300 rpm), using equation (5), is expressed as,

$$\theta = 4.39\ x\ (rpm)^{0.3688}$$

The "n" value and its corresponding shear rate in rpm, is:

n = 0.37 @ 233 rpm

Table 2 – Comparison of Different Rheology Models (Given Data)

Properties	**Data A**	**Data B**
D_h, in	8.50	10.5
D_p, in	6.00	8.25
Annular Vel, fpm	340	227
θ_{600}	83	49
θ_{300}	54	36
θ_{200}	42	31
θ_{100}	31	23
θ_6	16	7
θ_3	15	5
θ_{init} = (2xθ_3- θ_6)	14	3

Using different ranges of $rpms$ will result in different "$\boldsymbol{n}$" values and, thus, deliver different hydraulic results.

Table 3 – Comparison of Different Rheology Models (Index of Power Law Model "n")

Rheological Models	Data A "n"	Data B "n"
Power Law model (300 rpm & 3 rpm)	0.278	0.429
Power Law model (100 rpm & 3 rpm)	0.505	0.408
Power Law model (2 Closest Points, 300 rpm & 200 rpm)	0.619	0.369
Herschel-Bulkley (300 rpm, 100 rpm, & θ_{init})	0.779	0.456
Herschel-Bulkley (600 rpm, 300 rpm, & θ_{init})	0.787	0.479

Conclusion

It can be concluded that using the method presented in this paper will make the Power Law model more accurate, since it is based on the two closest shear rate points (viscometer rpms) applicable to the annular section of interest. The paper reinforces the need for standardization of hydraulic calculations [4]. Furthermore, it will make the shear stress and shear rate calculation consistent. The unification of hydraulic calculation begins with "n" of the Power Law model. Thus, it would be logical when reporting the "n", that the corresponding annular shear rate (rpm_a) also be reported.

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