AADE-15-NTCE-34

Fracture-Based Wellbore Strengthening Design and Application: An Analytical Approach

Mojtaba P. Shahri, Reza Safari and Uno Mutlu, Weatherford;

Reza Majidi, BP

Abstract

Wellbore strengthening techniques can be effectively utilized to reduce mud loss by increasing the safe mud weight window in depleted sections. Although the underlying mechanisms for wellbore strengthening are still a subject of research, several field applications show successful implementation of this technique in depleted zones.

For conventional drilling purposes, strengthening is achieved via drilling fluid and the particulate system (Lost Circulation Material-LCM) within. In other applications, such as casing while drilling, mechanical contact of pipe with the wellbore also helps the strengthening performance. This contact can build a layer of low permeability material on wellbore wall (external mud cake), create a non-porous layer of rock (internal mud cake), induce thermal/compressive stresses and smear particles into induced fractures to effectively isolate the fracture tip and inhibiting further propagation.

Most analytical industry standard strengthening design tools are decoupled from near-wellbore stress and stability analysis. For example, these models ignore the effect of near wellbore stress perturbations due to a combination of wellbore deviation, stress anisotropy, temperature variations, mud-cake and pore pressure changes. The aim of this study is to propose a fast-running integrated geomechanical tool based on analytical models to analyze near-wellbore stresses, stability and strengthening. Utilizing a fully analytical workflow coupled with stress concentration around the wellbore; fracture width distribution and fracture re-initiation pressure (FRIP) after plugging can be modeled. Proposed modeling technique allows us to quantify degree of strengthening and identify conditions where near wellbore region experiences stable vs. unstable fracture growth. The applicability of the integrated tool for drilling depleted zones is shown using different examples.

Introduction

One of the major problems encountered during drilling operations in challenging wells, such as deep-water depleted reservoirs, is lost circulation. Unstable fracture propagation can cause extensive loss of drilling fluids. Several methods can be utilized to mitigate fracture initiation and propagation. For example, Casing while Drilling (CwD) technology is being used to successfully drill through the depleted zones with lowest possible fluid loss rate while the wellbore strengthens. For an effective wellbore strengthening operation, induced fracture characteristics, (such as width, stable length, FRIP), need to be predicted under in-situ stress conditions. This ensures an accurate determination of the required Lost Circulation Material (LCM) size and type as they plug the induced fractures. Plugged fractures not only prevent excessive fluid loss into the formation but also inhibit further fracture propagation as the fracture re-initiation pressure (FRIP) increases and wellbore strengthens.

Current industry standard analytical models for wellbore strengthening design in general ignore near wellbore stress perturbations (i.e. due to a combination of wellbore deviation and far field stress anisotropy). This is an oversimplification of the real process and can lead to an inaccurate strengthening design. In addition, most strengthening models assume a fix stable fracture length for strengthening calculations (Alberty and McLean, 2004, Guo et al. 2011) which might lead to an inaccurate estimation of the wellbore strength and FRIP.

A detailed review of available literature provides several examples for current wellbore strengthening techniques utilized in the field. Most of these techniques focus/rely on induced-fracture(s) created around the wellbore and emphasize "Fracture-Based Strengthening" techniques. For example, Alberty and McLean (2004) introduced the concept of stress cage by explaining hoop stress augmentation around the wellbore due to propped and plugged induced-fractures. According to Alberty and McLean (2004), the increase in hoop-stress around the wellbore mitigates new fracture initiation and might help with wellbore strengthening. The proposed model does not allow re-initiation from the existing/plugged fracture tip (i.e. assumes that new fractures will initiate from other locations/flaws around the wellbore). Other researchers (Morita 1990, Fuh 1992, Fuh 2007, Van Oort et al. 2011) investigated existing induced-fracture stability by fracture tip isolation. In this approach, also known as "fracture tip screen-out" or "Fracture Propagation Resistance", further fracture propagation can be eliminated by isolating the area close to the fracture tip. In addition, Dupriest (2005) proposed another theory for wellbore strengthening technique and based on "Fracture Closure Stress". In this approach, integrity can be built by increasing fracture opening to get sufficient closure stress above wellbore ECD. According to Dupriest (2005), this could be achieved by creating an immobile mass inside the fracture. For example, for low permeability reservoirs, hesitation squeeze can be applied to create layers of LCM and widen the fracture. Several numerical models have also been proposed to simulate strengthening mechanisms (Alberty and McLean, 2004, Wang et al. 2007 and 2009, Guo et al. 2011). However, these models are computationally extensive.

Although different theories have been proposed in the past, there is still a need for a comprehensive model that can quantify degree of strengthening under a variety of stress and wellbore conditions. The aim of this study is to utilize an advanced analytical model for simulating near-wellbore fracture propagation and strengthening phenomena.

Our analytical engine (i.e., poro-thermo-elastic) is embedded within a wellbore stress, stability, strengthening and loss circulation mitigation workflow. The overall workflow is illustrated in Figure (1).

Figure 1: Integrated Workflow

We first calculate formation breakdown pressure and identify scenarios (along a pre-defined wellbore trajectory or among multiple wellbore trajectories) where risk of loss circulation (through induced fractures) is relatively high/low. Risk analysis is followed by Fracture Trace Angle (FTA) and Fracture Stability calculations. FTA analysis provides information on (i) fracture plane deviation(s) w.r.t. the wellbore axis and (ii) coalescence potential of near wellbore fractures. Stability calculations determine the "stable" fracture length, for a given wellbore pressure, formation properties and fluid/plug characteristics, which then feeds into the fracture width calculations. Note that if the analysis indicates that there is risk of unstable fracture propagation: wellbore pressure, fluid/plug characteristics and/or wellbore trajectory is modified to stabilize fractures. Calculated fracture width distribution (along the stable fracture length) provides an important data point for LCM particle size design. Once the fracture is plugged with the optimized LCM, Fracture Re-initiation Pressure (FRIP) analysis can be calculated to quantify the amount of wellbore strengthening and adjust mud weight.

The strengthening model presented in this paper is an improvement over industry standard analytical models. In this model, fracture characteristics are investigated under the effect of near wellbore perturbed stress field. Accordingly, key parameters affecting near wellbore stresses, such as wellbore trajectory, in-situ stress anisotropy, stable-unstable propagation regime(s) are taken into account and their effect(s) on fracture based strengthening can be quantified.

The applicability of the proposed model in wellbore strengthening design during drilling depleted reservoirs is demonstrated using a typical field data of depleted reservoirs. Proposed analytical model provides a suitable tool for designing Lost Circulation Materials (LCM) at a reduced computational cost and improved accuracy as compared to other numerical methods.

Mathematical Model

Shahri et al. (2014) proposed a fast-running semi-analytical model for wellbore strengthening design in depleted reservoirs. Kolossoff and Muskhelishvili complex functions of the plane-strain theory (Muskhelishvili 1953) have been combined with the near-wellbore stress distribution to find induced-fracture width distribution and fracture tip stress intensity factor. Utilizing superposition principle, effect of perturbed stress zone around the wellbore on the induced-fracture characteristics can be quantified more accurately using this model. Wellbore strengthening can be achieved by plugging the induced-fracture, mitigate further fracture propagation and increase FRIP. Accordingly, LCM particle size distribution need to be designed based on fracture opening under near-wellbore stress condition. Upon plugging, amount of wellbore strengthening can be quantified by calculating the stress intensity factor at the tip against fracture toughness under different wellbore pressure. Induced-fractures (and based on plug efficiency) start propagating once (if) the stress intensity factor at the tip reaches the fracture toughness. If the stress state and wellbore pressure dictate that unstable propagation will occur, induced fractures propagate greater lengths into the reservoir with significant fluid losses into the formation. Otherwise, and if stable propagation occur, induced fractures are arrested near wellbore region. Stress intensity factor calculations can be utilized to determine the transition between stable and unstable propagation. This helps going beyond some of the misconceptions (6 inch constant fracture length) as utilized in some of the industry standard wellbore strengthening calculations. These outputs are discussed using different examples in the following section. Based on the semi-analytical model, fracture width distribution and stress intensity factor at the tip can be calculated as follows: (Shahri et al. 2014)

$$W(t) = \frac{\pi(b-a)}{2m}\frac{4(1-v^2)}{E}\sum_{k=1}^{N} F(t_k) \tag{1}$$

$$K_I = \sqrt{2\pi(b-a)}\frac{1}{2m}\sum_{k=1}^{m}(-1)^{k-1}F(t_k)\left(\frac{1+t_k}{1-t_k}\right)^{\frac{1}{2}} \tag{2}$$

Utilizing superposition principle, effect of near wellbore stress perturbation around the wellbore and its effect on the induced-fracture characteristics can also be quantified. Shahri et al. (2014) explained the formulation for vertical wellbore, wherein the normal stress component on the fracture surface corresponds to the tangential (hoop) stress, as shown in Eqn. (3). This condition can be generalized by replacing the tangential stress with the normal stress component along the fracture surface and for different wellbore trajectories. As an example, for a deviated wellbore, stress field can be resolved based on far-field stress components and wellbore trajectory to find the normal stress component along the fracture surface. Accordingly, we can replace the near-wellbore stress distribution (derived based on classical Kirsch solution for a deviated wellbore) as normal stress component along the fracture surface in the following equation:

$$\sigma_{yy}(x) = \frac{\sigma_H-\sigma_h}{2}\left(1+\frac{3a^4}{x^4}\right) - \frac{\sigma_h+\sigma_H}{2}\left(1+\frac{a^2}{x^2}\right) - P_w\frac{a^2}{x^2} - P_f(x) \tag{3}$$

It is important to note that one of the assumptions used to develop the proposed analytical model is in-plane development of fractures (i.e. plane-strain assumption, where induced fractures are assumed to propagate parallel to the wellbore axis). This assumption works very well for vertical wells. However, for a deviated wellbore, fractures might initiate and propagate at an angle with respect to the borehole axis, known as Fracture Trace Angle (FTA). This is primarily due to the rotation of principle stresses caused by the shear stress component acting on the deviated wellbore wall. This is shown in Figure (2). Depending on key parameters; e.g., stress state, wellbore trajectory and pore pressure, FTA could deviate (i.e. by a few degrees to tens of degrees) from the in-plane assumption. In order to check the validity of this assumption for depleted zones, we investigate several scenarios and show the effect of depletion on the resulting FTA. In general, depletion (i.e. a reduction in pore pressure) reduces FTA and relaxes the constraint on the plane strain assumption for deviated wells.

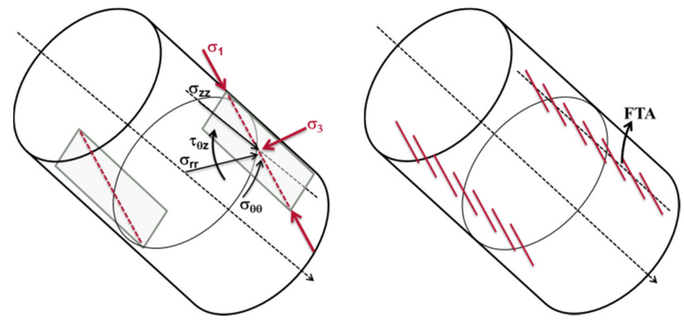

Figure 2: Stress Perturbation around Deviated Wellbore

Along these lines, we extend the classical Kirsch solution to calculate stress perturbation/distribution around the wellbore for deviated wells and as published by Fjar et al. (2008). FTA is then calculated under different stress regimes and wellbore trajectories as explained by Peska and Zoback (1995). Table (1) shows the input stress and pore pressure data used to determine FTA under different conditions. These input data can be considered as typical values for different stress regimes under normal pore pressure.

Table 1: Different Stress Regimes Input Data

	σ_v (SG)	σ_H (SG)	σ_h (SG)	P_o (SG)
Normal Faulting	1	0.9	0.8	0.5
Strike-Slip Faulting	1	1.2	0.7	0.5
Reverse Faulting	1	1.3	1.1	0.5

FTA(s) for different wellbore trajectories and stress regime; i.e., normal, strike-slip and reverse faulting, and under normal pore pressure condition are shown in Figures (3), (5) and (7), respectively.

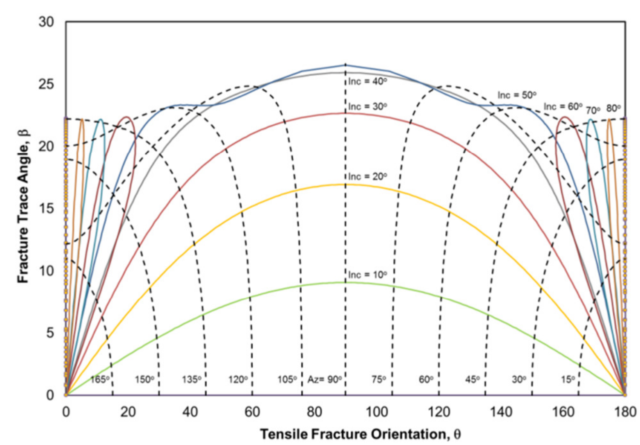

Figure 3: Normal Faulting Regime, Normal Pore Pressure

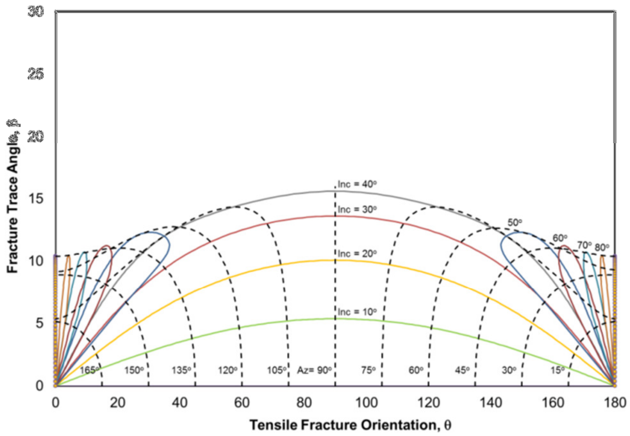

Figure 4: Normal Faulting Regime, Depleted Reservoir

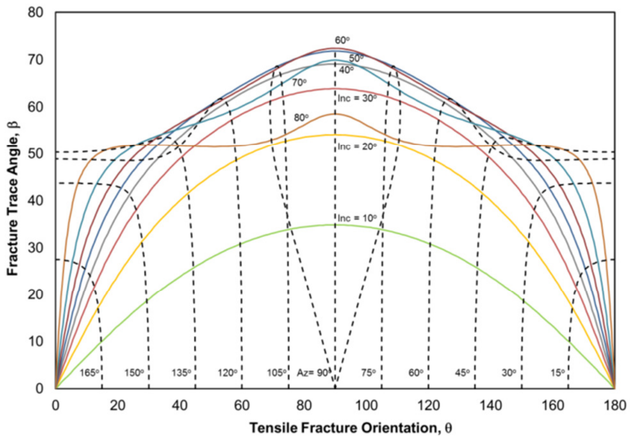

Figure 5: Strike-Slip Faulting Regime, Normal Pore Pressure

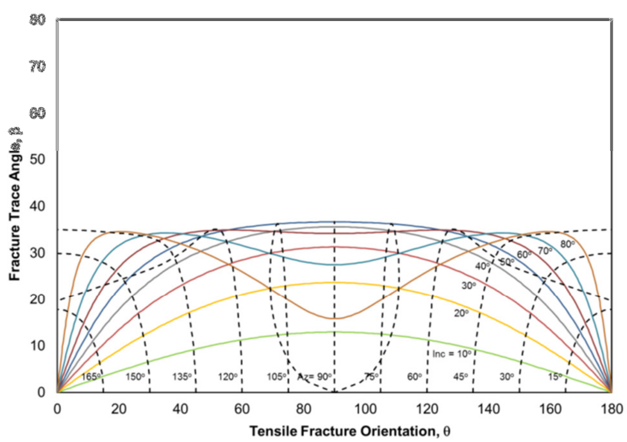

Figure 6: Strike-Slip Faulting Regime, Depleted Reservoir

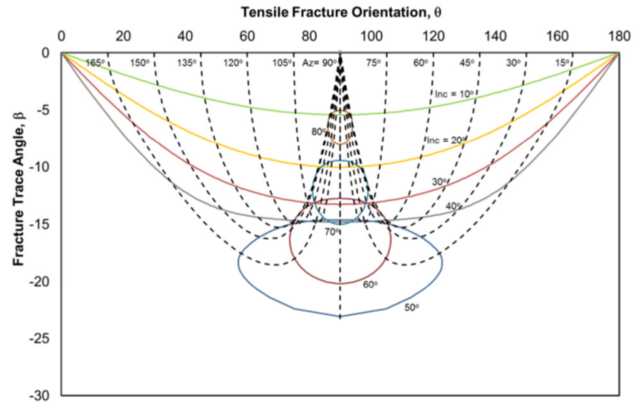

Figure 7: Reverse Faulting Regime, Normal Pore Pressure

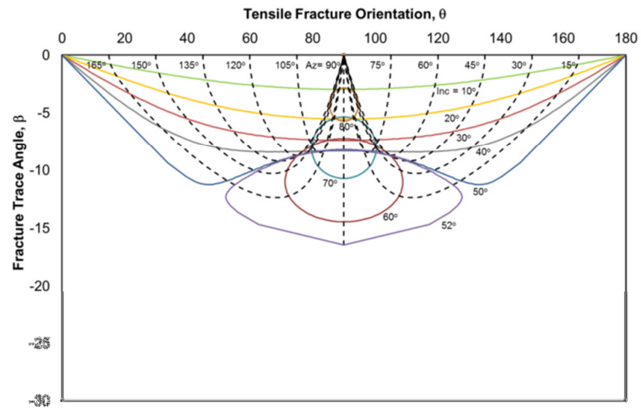

Figure 8: Reverse Faulting Regime, Depleted Reservoir

In order to show the effect of depletion on FTA, pore pressure is reduced from 0.5 SG to 0.2 SG. FTA(s) for different stress regime; i.e., normal, strike-slip and reverse faulting, and depleted pore pressure condition is shown in Figures (4), (6) and (8), respectively. According to the results, depletion reduces the deviation from plane-strain assumption where FTA starts aligning with the wellbore as induced-fractures initiate in a plane parallel to the wellbore axis. For example, maximum FTA under specified strike-slip faulting regime is equal to about 70 degree under normal pore pressure condition as shown in Figure (5). In this case, pore pressure depletion causes the reduction in FTA up to 35 degree (see Figure (6)). Similarly, the same effect reduces maximum FTA under normal faulting regime from about 27 to 15 degree, further aligning FTA with wellbore axis. Note that as FTA reduces, fractures (Figure (2), in red) tend to coalesce as they propagate. This phenomenon forces multiple fractures to coalesce into a single fracture and align along the wellbore axis. (Aadony and Looyeh 2000)

Therefore, although the plane-strain assumption “for deviated well fracture(s)” is a simplification, dictated by the limitations of the analytical model, for depleted reservoirs (i.e. the focus of this paper) this constraint can be relaxed by FTA analysis.

Fracture-Based Strengthening

The aim of this study is to apply the analytical model on a typical input data set from Gulf of Mexico depleted reservoir and quantify the degree of strengthening for a variety of geologically plausible scenarios. Input data, shown in Table (2), are used to simulate the characteristic of induced-fractures and to quantify the strengthening after plugging under different conditions. In this section we will not go through the entire workflow steps as illustrated in Figure (1), but rather focus on fracture width, stable length and FRIP calculations.

Table 2: Typical Input Data – Gulf of Mexico

Attribute	Value	Attribute	Value
TVD (ft)	16590	K_{Ic} (psi.in$^{0.5}$)	500
Hole Size (in)	12.25	E (psi)	1.6 E+6
φ (Degree)	17	v	0.2
P_{oi} (ppg)	10.8	ECD (ppg)	12.4
P_{of} (ppg)	10	Perm (mD)	700-900
σ_v (ppg)	14.2	∅ (%)	20-25
σ_h (ppg)	11.7	FPC	70%

Eqns. (1) and (2) can be used to determine fracture opening and stress intensity factor at the tip under different stress conditions. To do so, normal stress component along the fracture surface should be determined corresponding to wellbore trajectory and in-situ stress condition. Four different scenarios are assumed in this example corresponding to isotropic stress state, and 5%, 15% and 25% in-situ stress anisotropy (i.e. we vary σ_H while keeping σ_h constant to impose the desired stress anisotropy). Figure (9) shows the effect of stress anisotropy and wellbore trajectory on the formation breakdown pressure (fracture initiation pressure). Wellbore with 17 degree inclination (shown by the white/square data point) experiences higher stress concentration around the wellbore for the isotropic case as compared to the anisotropic condition. This increases the fracture breakdown pressure under isotropic stress condition. Using classical Kirsch solution for elastic material, normal stress component along the fracture surface is computed for this specific wellbore trajectory and under different stress state. Then, this normal stress component is used in Eqn. (3) to calculate the required parameters; i.e., fracture opening and stress intensity factor at the tip.

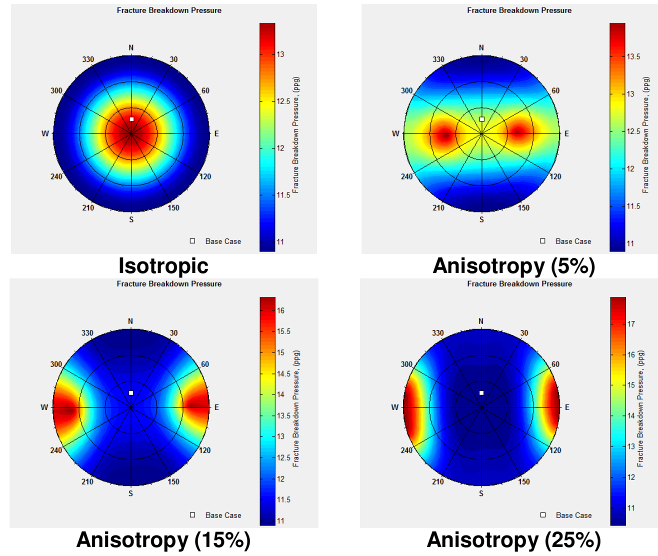

Figure 9: Effect of Anisotropy on Formation Breakdown Pressure

Another important parameter affecting fracture characteristics is the fluid pressure inside the fracture. Since the fracture initiation process happens in a short period of time, pressure inside the fracture is almost equal to the wellbore pressure at the beginning. This pressure exerts on the fracture surface against normal stress component generated due to near-wellbore stress field. Accordingly, wellbore pressure should be used in Eqn. (3) as fracture pressure in order to find fracture width distribution required for LCM particle size distribution design. In order to arrest fracture effectively, induced-fractures should be plugged immediately to mitigate further propagation. Utilizing the aforementioned procedure, fracture width distribution is calculated for different in-situ stress anisotropy and as shown in Figure (10). The results are in agreement with breakdown calculations (i.e. based on near wellbore stress field calculations) and for different anisotropy condition. For example, higher breakdown pressures for the isotropic case in Figure (9), indicate increased resistance to fracture opening and potential fracture closure. This is indeed the case in Figure (10): as fracture width calculations show that increased resistance to fracture opening reduces fracture width significantly (i.e., a comparison between isotropic vs. 25% anisotropic case(s) show that fracture width reduces ~50% at the wellbore and for a fracture of 6 inch length) (also see Bridgeman et al. (2015)).

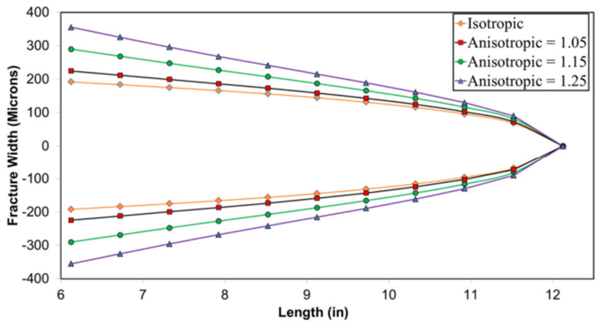

Figure 10: Effect of Stress Anisotropy on Fracture Opening (wellbore is located at 6.125 inches, analysis take into account near wellbore perturbed stress field)

Two other important parameters which need to be quantified for a successful fracture-based wellbore strengthening design are: (i) stable fracture length and (ii) FRIP after fracture plugging. Eqn. (2) can be used to determine stress intensity factor under near-wellbore stress perturbation. If the calculated stress intensity factor is less than fracture toughness, fracture remains stable without further propagation and fracture length can be quantified. On the other hand, higher stress intensity factor (driven by higher pressure inside the fracture) results in unstable fracture growth. If the fracture plug efficiency is relatively low, pressure inside the fracture would be equal to the wellbore pressure which in turn causes unstable fracture growth for a depleted zone. Plug efficiency depends on several factors such as formation permeability and porosity, fluid rheological properties, efficiency of plug, etc. Along these lines, fracture Pressure Coefficient, (FPC), is defined that relates pressure inside the plug region to the wellbore and pore pressures as follows:

$$P_f = P_w - FPC(P_w - P_o) \quad (4)$$

If FPC is equal to 1, pressure inside the plug region is completely cut-off from the wellbore pressure and can diffuse out to the formation pore pressure. On the other hand, for the case of FPC equals to 0, the combination of low permeability formation and low plug efficiency (i.e., direct communication with wellbore pressure) causes the fracture pressure to be equal to wellbore pressure. The step-wise function can be defined to simulate different plug configurations; i.e., near-tip, middle of fracture, whole fracture, as shown in Figure (11).

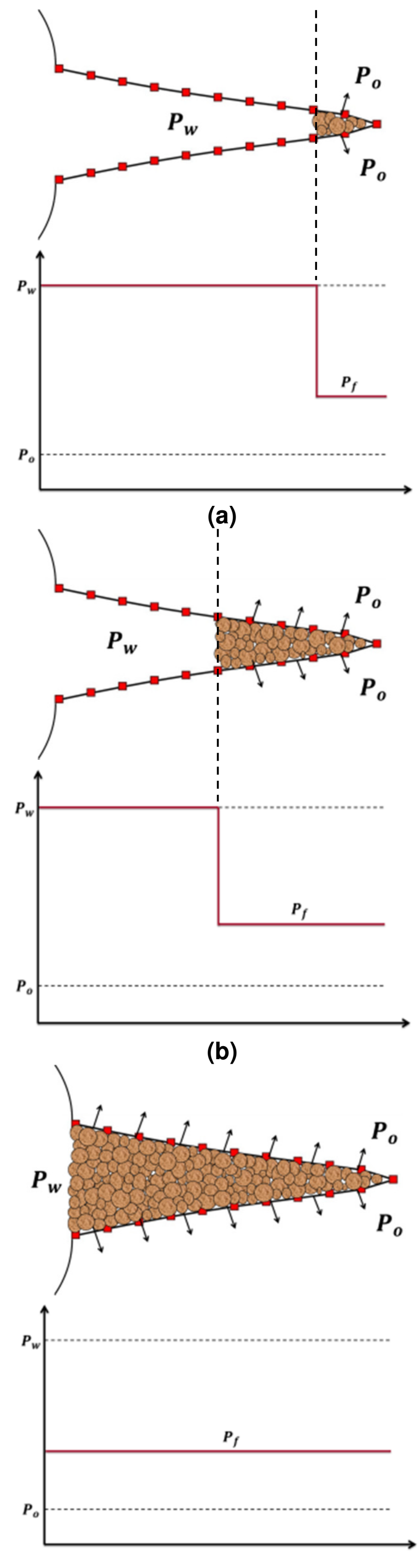

Figure 11: Fracture Plugging; (a) near tip (b) middle of fracture (c) whole fracture

We calculate FRIP to quantify the degree of strengthening under different scenarios. That is, different plugging scenarios are simulated corresponding to their step-wise pressure functions as shown in Figure (11). For each plugging scenario, stress intensity factor is calculated using Eqn. (2) and by varying wellbore pressure. FRIP can be calculated by checking stress intensity factor against fracture toughness corresponding to further fracture propagation/instability. FRIP for different plug locations under different stress anisotropy is shown in Figure (12). These simulations are performed by assuming 70% pressure reduction inside the fracture (i.e., FPC equals to 0.7). According to our results, higher FRIP (or strengthening) can be achieved by plugging induced fracture close to the mouth. In addition, wellbore strengthening application might not be efficient by plugging near-tip or by tip screening. Therefore, fracture should be plugged close to its mouth and pressure should diffuse out behind the plug efficiently.

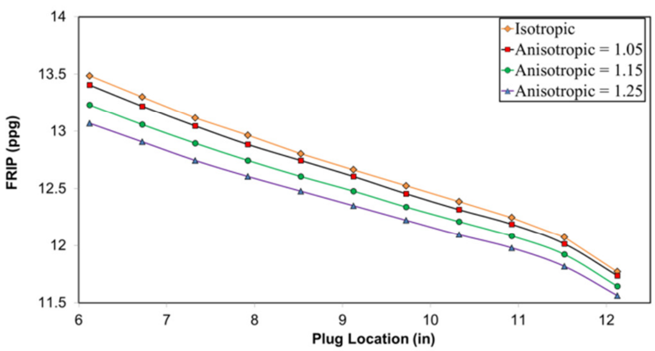

Figure 12: Effect of Anisotropy on FRIP (wellbore is located at 6.125 inches)

FRIP can also be calculated for different FPC (see Figures (13) and (14) and for isotropic and anisotropic cases). Dashed line shows the value of minimum horizontal stress, σ_h, corresponding to fracture propagation pressure. According to these figures, when the plug efficiency is low and FPC is small, induced-fractures FRIP stays close to σ_h and might propagate unstably. Utilizing the proposed methodology, amount of strengthening under different condition(s) can be quantified. For example, for a highly permeable reservoir and good plug efficiency; (i.e., communication between wellbore and fracture pressure is cut-off or FPC equals to 0.9), one could expect 3 ppg strengthening over fracture propagation Pressure, σ_h, by plugging close to the mouth of the fracture. Assuming FPC equals to 0.7, (70% reduction in fracture pressure), expected strengthening value would be equals to 1.7 ppg.

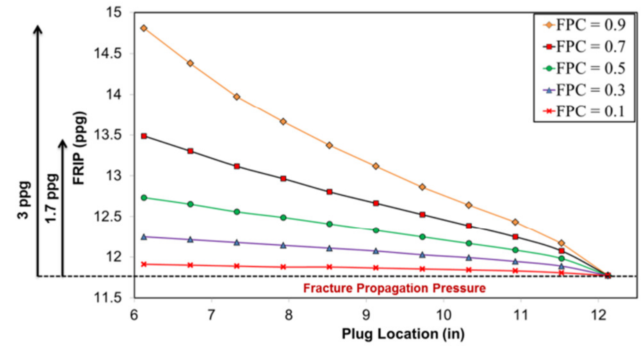

Figure 13: Effect of Fracture Pressure on FRIP – Isotropic (wellbore is located at 6.125 inches)

On the other hand, plugging near fracture tip might result in in-sufficient strengthening as shown in Figures (13) and (14). Even with very high FPC; e.g., even 90% reduction in the fracture pressure, increases FRIP by only about 0.5 ppg above fracture propagation pressure or σ_h. These simulations verify that sufficient strengthening cannot be achieved by plugging the fracture at the tip or by simply relying on a tip-screening process.

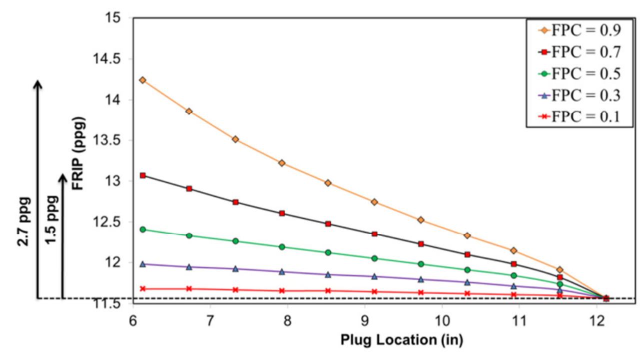

Figure 14: Effect of Fracture Pressure on FRIP – 25% Anisotropy (wellbore is located at 6.125 inches)

One of the assumptions used in all the previous examples/analysis is that the fracture length is constant. Indeed, in most fracture-based wellbore strengthening design applications published in the literature, fracture length is assumed to be equal to 6 inch. However, there is no solid justification explaining the stability of 6 inch fracture and under varying downhole condition(s). Utilizing the stress intensity factor formulation, Eqn. (2), stability of induced-fracture can be investigated under different conditions. As discussed, fracture propagates against normal stress component along the fracture surface. This stress component is shown in Figure (15) and for different stress anisotropy values. For the specific wellbore inclination and azimuth (Table 1), Figure (15) shows that as anisotropy increases resistance to fracture propagation decreases (i.e. normal stress resolved on fracture faces).

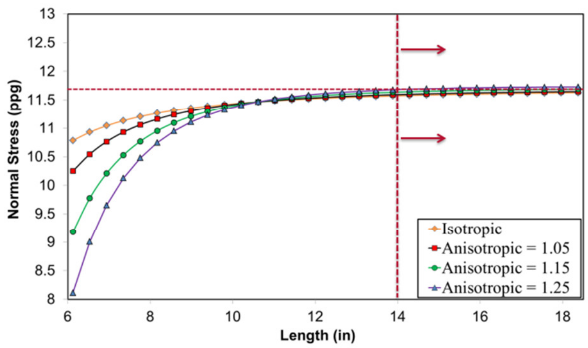

Figure 15: Normal Stress on Fracture Surface (wellbore is located at 6.125 inches)

Figure (16) shows the stable-fracture length under different far-field anisotropy conditions and for different fracture pressures. Note that pressure inside the fracture can vary between pore pressure; i.e., efficient plug or permeable formation (FPC = 1), to wellbore pressure; i.e., non-efficient plug or impermeable formation (FPC = 0). In order to arrest the fracture and mitigate further propagation, fracture pressure should be reduced from the given wellbore pressure (12.4 ppg according to Table 1), to a value lower than 11.7 ppg. Otherwise at values equal or greater than 11.7 ppg, fracture length reaches an asymptotic behavior and will propagate unstably. For example, to maintain a 6 inch "industry standard" stable fracture length, wellbore pressure should be reduced to ~ 11.2 ppg for a highly anisotropic case and for the plug efficiency defined in Table 1. Our analysis confirm that stable fracture length is a function of near-wellbore stress perturbation, in-situ stress anisotropy, wellbore trajectory, wellbore pressure, fracture plug efficiency, fracture toughness and among others, and needs to be quantified for each analysis.

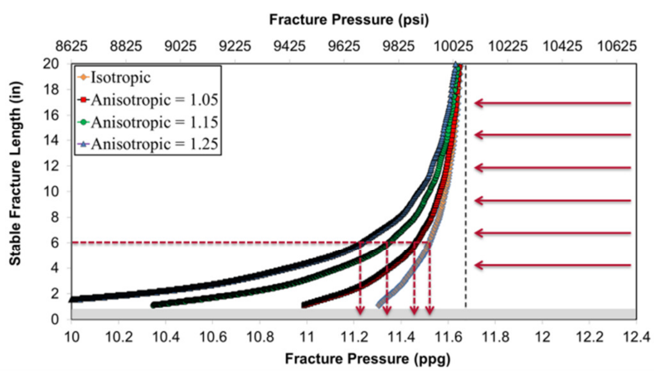

Figure 16: Effect of Anisotropy on Stable Fracture Length

There are other factors controlling fracture propagation under near-wellbore stress field such as fracture toughness. Different simulations have been performed to show the effect of fracture toughness on stable fracture length. Our results show that as fracture toughness increases, a specific stable fracture length requires less pressure reduction inside the fracture. This is shown in Figure (17).

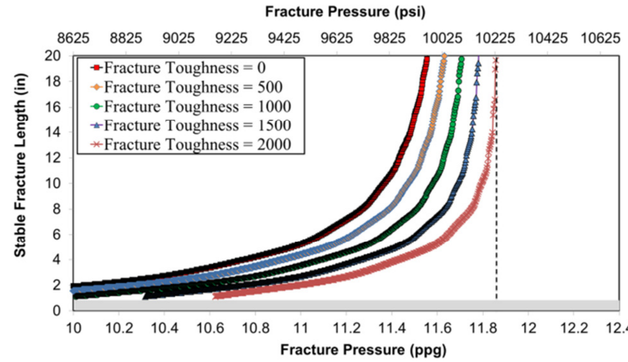

Figure 17: Effect of Fracture Toughness on Stable Fracture Length

Finally and as discussed by Shahri et al. (2015), transient pore pressure changes around the wellbore can be simulated using analytical models as well. Total normal stress component (which includes pore pressure effects) can be calculated using analytical solutions and imported into the strengthening design algorithm via mud cake efficiency. Figure (18) shows the effect of pore pressure build-up around the wellbore on the fracture stable length. Lower mud-cake efficiency causes higher pressure build-up around the wellbore which in turn increase(s) the total stress component on the fracture surface. This higher resistive force might result in stable fracture length even with small pressure drop inside the fracture. Analyses presented in Figure(s) 16-18 assume that linear elastic fracture mechanics govern fracture propagation. Analyses don't take into account fracture propagation in unconsolidated formations or plastic dissipation.

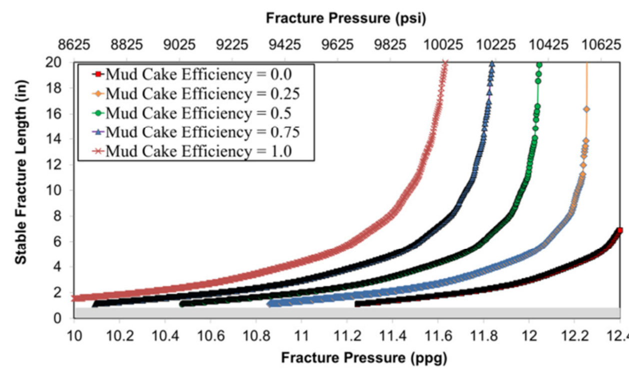

Figure 18: Effect of Mud-cake Efficiency on Stable Fracture Length

Conclusions

We utilize a fast-running analytical model to quantify induced fracture characteristics and wellbore strengthening with implications to drilling through depleted zones. Fracture width distribution, FRIP, stable fracture length can be determined using an advanced analytical engine. Our results show that fracture geometry, stable fracture length and amount of strengthening are controlled by in-situ stress anisotropy, near-wellbore stress perturbation, wellbore trajectory, fracture plug efficiency and location, wellbore pressure, fracture toughness, mud cake efficiency and among other formation/rock properties. Utilizing the analytical engine, stable fracture length can be calculated under different condition and can be used instead of current industry fixed length (e.g., 6 inch) approach. In terms of strengthening, fracture should be plugged close to the mouth to get desired outcome instead of near-tip isolation. Similar analyses and workflow(s) can be utilized (i.e. pre-drill or near real time) to mitigate loss circulation risks while drilling through depleted zones.

There are other mechanisms that contribute to wellbore strengthening: such as transient thermal and/or poro-elastic stress effect(s). Although the proposed analytical engine is capable of simulating such conditions, a detailed investigation of these effects on the strengthening behavior is beyond the scope of this paper.

Acknowledgments

The authors would like to thank Weatherford and BP for permission to publish this paper. The support of Scott Ball and Moji Karimi throughout the study is much appreciated.

Nomenclature

a	Wellbore Radius
b	Fracture Tip Location
E	Young's Modulus
ECD	Equivalent Circulating Density
FPC	Fracture Pressure Coefficient
K_I	Fracture Tip Stress Intensity Factor
K_{IC}	Fracture Toughness
L	Fracture Length
m	Chebyshev Polynomial Terms
P_w	Wellbore Pressure
P_f	Fracture Pressure
P_o	Pore Pressure
v	Poisson's Ratio
W	Fracture Opening
σ_{yy}	Stress Distribution Along Fracture Surface
$\sigma_{\theta\theta}$	Tangential Stress
σ_h	Minimum Horizontal Stress
σ_H	Maximum Horizontal Stress
σ_v	Vertical Stress
φ	Wellbore Inclination Angle
∅	Porosity
θ	Tensile Fracture Orientation Around the Wellbore

References

Aadnoy, B.S., Looyeh, R., 2010. *Petroleum Rock Mechanics: Drilling Operation and Well Design*, Gulf Professional Publishing

Alberty, M., McLean, M., 2004. A Physical Model for Stress Cages. Paper SPE 90493 presented at 2004 SPE Annual Technical Conference and Exhibition, Houston, Texas, 26−29 September

Bridgeman, W., Salminen, C., Rosenberg, S., Karimi, M., Shahri, M.P. Drilling with Liner Technology Enables Successful Sidetrack through Depleted Sands in Shallow Water Gulf of Mexico. IADC/SPE 173046, *IADC/SPE Drilling Conference and Exhibition*, 17–19 March, London, United Kingdom, 2015.

Dupriest, F. 2005. Fracture closure stress (FCS) and lost returns practices. Paper SPE 92192 presented at the SPE/IADC Drilling Conference, Amsterdam, Netherlands, February 23−25

Fjar, Erling, Rachel M. Holt, A. M. Raaen, Rasmus Risnes, and P. Horsrud.*Petroleum related rock mechanics*. Vol. 53. Elsevier, 2008

Fuh, G. et al 1992. A new approach to preventing lost circulation while drilling. SPE 24599 presented at 67th Annual Technical Conference and Exhibition, Washington, 4-7 October.

Fuh, G. et. al 2007. Further development, field testing and application of the wellbore strengthening technique for drilling operations. SPE/IADC paper 105809 presented at 2007 at IADC/SPE drilling conference, Amsterdam, 20-22 February.

Guo, Q., Feng, Y. Z., Jin, Z. H., 2011. Fracture Aperture for Wellbore Strengthening Applications. Paper ARMA 11-378 presented at the 45th U.S. Rock Mechanics/Geomechanics Symposium, San Francisco, California, June 26−29

Morita N. et al. 1990. Theory of lost circulation pressure. SPE paper 20409 presented at the 65th Annual Technical Conference and Exhibition, New Orleans, 23-26 September.

Muskhelishvili, N.I., 1953. *Some Basic Problems of the Mathematical Theory of Elasticity*, 3rd Ed. translated by J.R.M. Radok, P. Noordhoff, Ltd., Groningen, Holland

Peška, P., and Zoback, M. D. 1995. Compressive and tensile failure of inclined well bores and determination of in situ stress and rock strength. *Journal of Geophysical Research: Solid Earth*, 100(B7), 12791-12811.

Shahri, M.P., Safari, R., Oar, T., Karimi, M., Mutlu, O. Advanced Geomechanical Analysis of Wellbore Strengthening for Depleted Reservoir Drilling Applications. SPE-167976, *IADC/SPE Drilling Conference and Exhibition*, Fort Worth, Society of Petroleum Engineers, 2014.

Shahri, M.P., Oar, T., Safari, R., Karimi, M., and Mutlu, U. *In press*. Advanced Semi-Analytical Geomechanical Model of Wellbore Strengthening Applications. *SPE Journal* (accepted 4 February 2015).

Shahri, M.P., Safari, R., Karimi, M., Mutlu, O. An Integrated Analytical Workflow for Analyzing Wellbore Stress, Stability and Strengthening. ARMA 15-82, 49th U.S. Rock Mechanics / Geomechanics Symposium, San Francisco, California, June 28 – July 1.

Van Oort, E., Friedheim, J., Pierce, T., Lee, J., 2011. Avoiding Losses in Depleted and Weak Zones by Constantly Strengthening Wellbores. *SPE Drilling & Completion*, 26(4), 519−530

Wang, H., Towler, B., Soliman, M., 2007. Fractured Wellbore Stress Analysis: Sealing Cracks to Strengthen a Wellbore. Paper SPE/IADC 104947 presented at the 2007 SPE/IADC Drilling Conference, Amsterdam, The Netherlands, 20−22 February

Wang, H., Soliman, M., & Towler, B., 2009. Investigation of Factors for Strengthening a Wellbore by Propping Fractures. *SPE Drilling & Completion*, 24(3), 441−451