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Continuum Model of a Shale Shaker

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Abstract

Shale shakers are widely used in industry to separate coarse particles from liquid slurries. Existing models of shakers have limitations in predictability. The aim of this work is to apply a new approach to model a shale shaker in two parts, a cake filtration section and a drainage section. This paper discusses the cake filtration section of the model

Introduction

Vibratory screens are widely used in the oil and petroleum drilling operations to separate particulates from drilling fluids. Screens are typically the first separation device that drilling fluids encounter as they are pumped out of the wellbore. The screens consist of different layers of mesh depending on the slurries they filter and are vibrated in order to improve their performance. Drilling fluid, also known as "mud", contains particulate matter such as sand and shale, which must be removed before the fluids are sent for further processing.

The vibrations of the screen aid in moving the particles uphill over the screen and the drilling fluid, now free of the particles, is collected at the underside of the screen. Properties and operating parameters for an API 140 screen obtained from M-I SWACO are used to set up the model. The API 140 screen consists of 3 mesh layers of thickness 0.00069 meters (this value also accounts for the weave up/down bending of the wires). The drilling mud flows onto the screen at a rate of 45.36 kg/s (6000 lbs/min) and eventually forms a cake of certain permeability and porosity, and moves uphill along the screen. The screen movement follows an elliptical path due to the vertical and horizontal components of motion.

Model Description and Assumptions

Figure 1 shows a shaker diagram with some of the key features. The drilling mud with particles to be removed enters the screen area on the left hand side. The mud moves downward by gravity. The solid particles build up a cake layer on the surface of the screen. The cake layer moves to the right up the inclined screen due to the vibrations of the screen. As the cake moves, more mud collects on the cake surface causing the cake depth to increase as the cake moves to the right. Eventually, the surface of the cake reaches the mud-air surface and the cake stops growing in thickness. To the right of the intersection of the mud-air surface and the top of the cake, the process becomes one of liquid drainage from the cake material.

Different phenomena occur in each section requiring different mechanistic models. Hence, the model is divided into two sections. The first (on the left) is the cake-formation section and a cake-filtration model is used to describe the performance. The second (on the right) is the liquid-drainage section in which the drilling mud drains from the cake and is controlled by capillary forces as the fluid is pulled downward by gravity. This paper discusses the derivation of the cake-filtration model and how that model is used to estimate the length of the screen needed for the cake-filtration section.

Cake formation on the vibrating screen is a complex and dynamic process. The following assumptions are made to simplify the analysis:

- The drilling mud (excluding the particles that are separated) has constant and uniform density and viscosity; the concentration of fine particles that make up the mud is constant and uniform.
- The solid particles to be separated from the drilling mud are of constant and uniform diameter; agglomeration of particles is neglected (in this version of the model), and the particles have a constant density.
- The process is isothermal.
- There are no chemical reactions and no phase change.
- The cake is incompressible.
- The cake moves to the right in plug flow, the cake velocity is not a function of vertical position.
- The solid particles are uniformly distributed in the mud with a constant concentration.
- The inertial and wall shear forces are negligible in the cake in the vertical direction.
- The overall process operates in a steady state motion in that the screen vibrations are rapid enough that unsteady oscillations in the screen motions are negligible in the unsteady state terms in the model. However, the screen motions can affect the left to right velocity of the cake and can affect the particle packing in the cake.

Governing Equations for Cake Formation

The primary equations used to model cake filtration are the mass and momentum balances for the solid (particles forming the cake) and fluid (drilling mud) phases. Rectangular coordinates are used to set up the model with the x-axis parallel to the screen surface and the y-axis normal to the screen surface, as shown in Figure 2. The general mass and momentum balances from volume averaging theory for flows through a porous media are used as the starting equations[1,2]. Applying the assumptions listed above, the mass and momentum balances for the solid (s) and liquid (L) phases can be simplified. The most useful equations are:

Mass Balance: $\frac{dv_y^\alpha}{dy} = 0$ where $\alpha = s, L$ (1)

Hence $v_y^s = 0$, $v_y^L = f(x)$, and $v_x^L = v_x^S$ (2,3,4)

Momentum Balance, y-component, Liquid phase:

$$0 = -\varepsilon^L \frac{\partial P}{\partial y} - F_y^L + \varepsilon^L \rho^L g_y \quad (5)$$

To model the cake growth, consider a thin slice of the cake of thickness Δx as shown in Figure 3. The air pressure is atmospheric at the free surface of the mud and at the underside of the screen. The pressure at the mud-cake interface is P_c and the pressure at the cake-screen interface is P_o.

From cake filtration literature, the rate of cake volume growth to the filtrate volume flow rate (through this section of cake) is given by [3]

$$\frac{\dot{V}_{cake}}{Q_{cake}} = \frac{1-\varepsilon^m}{\varepsilon^m - \varepsilon^L} \quad (6)$$

The rate of cake volume growth and the filtrate flow rates are also given by

$$\dot{V}_{cake} = b\Delta x \frac{dh_c}{dt} \quad (7)$$

$$Q_{cake} = -b\Delta x \varepsilon^L v_y^L \quad (8)$$

where b is the width of the screen in the z-direction.

At steady state operation of the screen the rate of cake height increase is only a function of position. The time rate of change of the cake height is converted to position by applying chain rule in calculus to obtain

$$\frac{dh_c}{dx} = -\frac{v_y^L}{v_x^S} \frac{\varepsilon^L(1-\varepsilon^m)}{(\varepsilon^m - \varepsilon^L)} \quad (9)$$

To obtain an expression for the liquid velocity in the y direction in Eq.(9) we turn to the momentum balance. The pressure at the top of the cake can be obtained from the static head in the mud. Using the dimensions indicated in Figure 3, the pressure at $y = h_c$ is :

$$P_c = P_{atm} + \rho^m g cos\beta \ (h_m - h_c) \quad (10)$$

where the bulk density of the mud ρ^m is the sum of the volume fractions of the phases times the phase intrinsic densities, given by

$$\rho^m = \varepsilon^m \rho^L + (1 - \varepsilon^m)\rho^S \quad (11)$$

The height from the surface of the screen to the surface of the mud, h_m, is determined from the geometry in Figure 2, assuming the surface of the mud is horizontal.

$$h_m = h_o - x \ tan(\beta) \quad (12)$$

Darcy's law may be used to represent the drag force, F_y^L, in Eq.(5). Assuming constant properties, then Eq.(5) may be integrated over the cake to obtain

$$P_o = P_c + \left(\varepsilon^L \frac{\mu}{k_c} v_y^L + \rho^L g cos\beta\right) h_c \quad (13)$$

where k_c is the permeability of the cake. The permeability may be determined from experiments or estimated using a correlation such as Ergun's equation [4]. A similar balance can be written for the pressure drop across the screen. The mass continuity requires the product of the velocity times the void volume for the cake and the screen to be the same. Combining the pressure relations yields the expression

$$P_c - P_{atm} = -\varepsilon^L v_y^L \mu \left(\frac{h_c}{k_c} + \frac{h_{SCR}}{k_{SCR}}\right) - \rho^L g \cos\beta (h_c + h_{SCR}) \quad (14)$$

where k_{SCR} and h_{SCR} are the permeability and thickness of the screen. Combining equations (14) and (10) the velocity of liquid in the cake is :

$$v_y^L = \frac{-g \cos\beta\left(\rho^L(h_c + h_{scr}) + \rho^m(h_m - h_c)\right)}{\varepsilon^L \mu\left(\frac{h_c}{k_c} + \frac{h_{scr}}{k_{scr}}\right)} \quad (15)$$

Equation (14) is combined with Eq.(9) to calculate the cake height as a function of position. The position in x where the cake height equals the mud height given by Eq.(12) determines the length of screen used in the cake section of the vibrating screen. To get the total volumetric flow rate of the drilling mud passing through the screen in the cake section is obtained by integrating the velocity over position x in the cake section from 0 to L

$$Q = b \int_0^L \varepsilon^L v_y^L dx \quad (16)$$

Model Results and Sample Calculations

Parametric studies were performed to determine the effects of process parameters on screen operation. Table 1 shows the parameter values and constants used for the calculations. Figure 4 shows the effect of particle size and porosity on volumetric flow rate of liquid through the cake. The model shows that the flow rate increases at higher porosities and larger particle diameters. This is because the permeability of the cake increases at higher porosities and larger particles. The tilt angle of the screen was maintained constant at 3 degrees for these calculations.

Figure 5 shows the effect of particle size on screen length for different values of porosity, ε^L. The model shows that the screen length generally decreases as particle size increases. However, the screen length is not a strong function of the particle size (the screen lengths differed by less than 4mm for particle sizes ranging from 100 to 600 µm). As the porosity increases, the screen length tends to decrease for a particular value of d_p. Irregularities and variations between the curves are attributed to the numerical integration method applied by Polymath [5] but should be investigated in future work.

Porosity values in Figures 4 and 5 were varied from 0.3 to 0.5 as typical values observed in applications. A subscale model is needed to account for the vibration effects of the screen on the cake porosity to improve the model predictions.

The variation of volumetric flow rate with typical tilt angles of the screen of 3 to 5 degrees is shown in Figure 6. The tilt angle does not have much effect on the volumetric flow rate for small particles but as the particle size increases the tilt angle does have a notable effect. For large particles the flow rate is greater at smaller screen angles. Figure 7 shows the effect of tilt angle on length of the screen. The curves lie almost on top of each other indicating no significant effect of tilt angle on screen length. However, the cake velocity v_x^s is held constant in these calculations. An appropriate subscale model that accounts for the dependence of the cake velocity on the tilt angle needs to be incorporated into the model in future work and may change the results shown in Figure 7 to show a stronger dependence of the screen length on the tilt angle. The porosity of the liquid in the cake, ε^L, was held constant at 0.5 for the plots in Figures 6 and 7.

Conclusions

A continuum model has been developed for predicting the screen length for the cake formation section of a vibrating screen separator. A parametric study was conducted to show how the particle diameter, tilt angle, and cake porosity affect the screen performance. These studies showed that particle diameters and porosities have considerable effects on screen performance but the tilt angle of the screen does not have a significant effect as long as the porosity is held constant.

The parametric studies should be expanded to examine other parameters that may affect screen performance. Subscale models are needed for the velocity of the cake in the x-direction and for the cake porosity to further improve the model predictions. A model for the drainage section needs to be developed to predict the total screen length to produce the final cake. The results from the model should be validated with experimental data.

Acknowledgments

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Nomenclature

F_y^L = *drag force of liquid moving through cake*
g_y = *gravity acceleration in y-dir*
h_c, h_m, h_o = *cake thickness, mud depth, initial mud depth*
h_{SCR} = *thickness of screen*
k_c, k_{SCR} = *permeability of cake, screen*
L = *length of the screen in the cake section*
P = *liquid phase pressure*
P_c = *liquid pressure at cake surface*
P_o = *liquid pressure at cake-screen boundary*
P_{atm} = *atmospheric pressure*
Q = *total volumetric flow rate through cake section*
Q_{cake} = *volumetric flow rate though cake slice of* Δx
$\dot{V}_{cake}$ = *rate of cake volume growth*
v_x^L, v_x^S = *liquid phase and solid phase velocities in x-dir*
v_y^L = *liquid phase velocity in y-dir in the cake*
Δx = *thickness of section of cake in x-dir*
β = *tilt angle of the screen relative to horizon*
$\varepsilon^L, \varepsilon^s$ = *liquid, solid volume fractions in cake*
ε^m = *liquid volume fraction in the mud*
ρ^L, ρ^s, ρ^m = *density of liquid, solid, bulk density of mud*
μ = *liquid viscosity*

References

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3. Chase, G.G., and Willis, M.S. "Compressive Cake Filtration," *Chemical Engineering Science*, 47(6), 1373-1381, 1992.
4. McCabe, W.L., Smith, J.C., and Harriott, P. Unit Operations of Chemical Engineering, 7th ed., McGraw-Hill, Boston, 2005.
5. Polymath 6.10 (http://www.polymath-software.com

Tables

Table 1. Parameter values used for the calculations.

Parameter	Value
Gravitational Acceleration, g	9.81 m/s^2
Tilt angle, β	3 to 5°
Porosity of liquid in the cake, ε^L	0.3 to 0.5
Porosity of solids in the mud, ε^m	0.03
Particle Diameters, d_p	60-500 µ
Density of liquid in the cake, ρ^L	1100 kg/m^3
Density of solid particles, ρ^s	2600 kg/m^3
Bulk density of the mud, ρ^m	1145 kg/m^3
Viscosity of the mud, μ	0.024 kg/m/s
Velocity of the cake in the x direction, v_x^s	0.10668 m/s (4.2 inches/s)
Permeability of the cake k_c	Varies with porosity of cake and particle size
Permeability of the screen, k_{scr}	2.5762*10^{-10} m^2
Thickness of the screen, h_{scr}	0.00069 m
Initial depth of the mud, h_o	0.0505 m (2 inches)

Figures

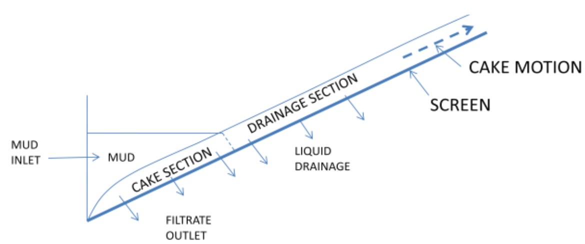

Figure 1: Diagram of the vibrating screen process. The mud with coarse particles enters from the left. A cake of particles forms on the surface of the screen and moves to the right due to the screen vibrations. When the cake moves out of the liquid mud the process becomes one of liquid drainage from the cake. Different phenomena occur in each section requiring different mechanistic models.

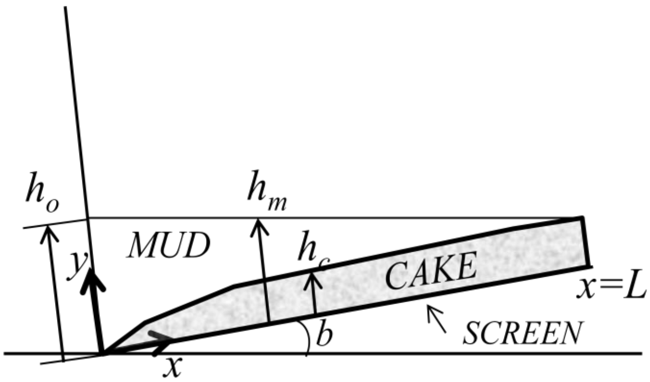

Figure 2. Diagram of the cake section showing the *x* and *y* coordinate origin and directions with the screen at angle β relative to the horizon. The free surface of the mud is height h_m above the screen and is a function of position *x*. The height of the top surface of the cake, h_c, is also a function of position *x*. The mud height at *x=0* is h_o. The screen length where $h_m = h_c$ is $x = L$.

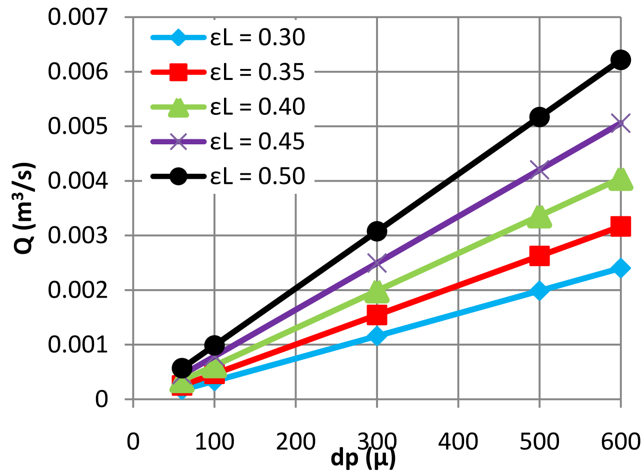

Figure 4: Effect of particle size and porosity on volumetric flow rate (m^3/s per meter width of screen) through the screen in the cake section. (Note: el $=\varepsilon^L$). The tilt angle is 3 degrees.

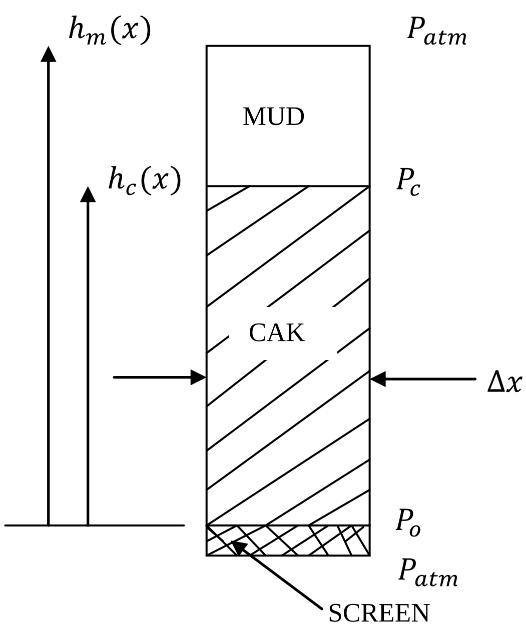

Figure 3: A random slice of the cake section of thickness Δx.

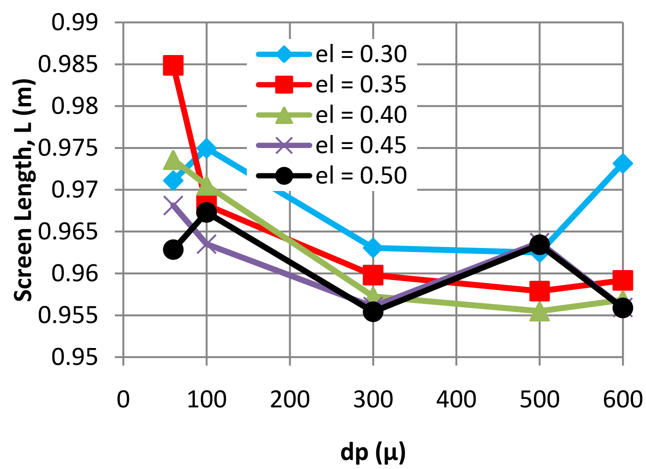

Figure 5: Effect of particle size and porosity on the screen length of the cake section, *L*. (Note: el $=\varepsilon^L$). The tilt angle is 3 degrees.

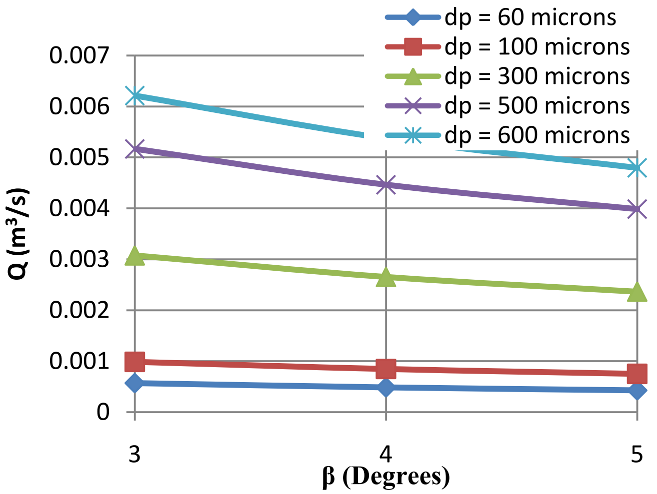

Figure 6: Effect of the tilt angle on volumetric flow rate (m^3/s per meter width of screen) of mud through the screen in the cake section for different particle sizes. The porosity is 0.5.

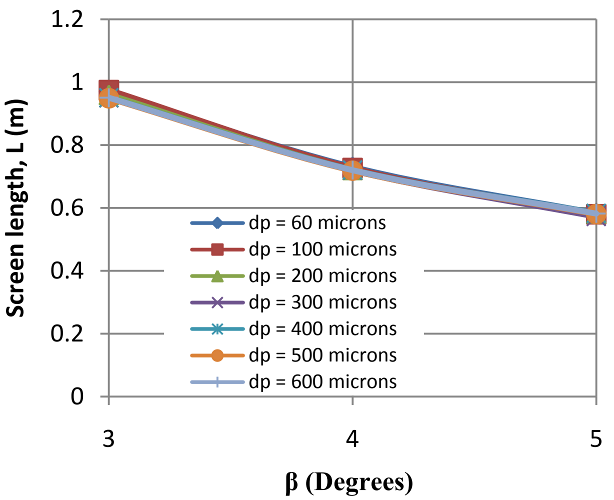

Figure 7: Effect of tilt angle on screen length of the cake section. The porosity is 0.5.