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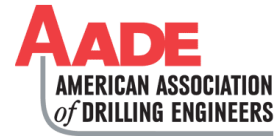
Transitional and Turbulent Flow of Drilling Fluids in Pipes: An Experimental Investigation

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Abstract

Precise prediction of frictional pressure loss of drilling fluids, which usually exhibit non-Newtonian behavior, plays a vital role in drilling hydraulics. A transitional or turbulent flow regime is often anticipated in the drillstring during drilling/circulating operations. Semi-analytical solutions that are extensively used in the industry rely on empirical correlations to predict the frictional pressure loss in transitional or turbulent flow. However, this approach has been proven to be inappropriate for many types of drilling fluids used nowadays.

This paper presents an extensive experimental study to better evaluate industry standard hydraulic models for different types of drilling fluids. Five drilling fluids are examined in this study: a bentonite clay mud, three polymer-based muds, and a synthetic-based mud. An experimental setup was constructed to measure frictional pressure loss of these fluids in a 0.9525 cm (3/8") pipe under laminar, transitional, and turbulent flow regimes. Then, experimental results were compared to the model predictions.

Our study shows that although the historical models present relatively accurate results for the bentonite clay mud, significant discrepancies in frictional pressure loss were observed for polymer-based muds in transitional and turbulent flow. It is believed this phenomenon occurs due to the presence of long-chain polymers, which have a tendency to delay the transition to turbulence and reduce friction. With the extensive use of polymer fluids in the drilling industry, this study provides vital insight for superior hydraulics modeling, particularly for applications such as managed pressure drilling (MPD) that rely heavily on accurate hydraulics models.

Introduction

Most fluids used to drill and cement present-day oil/gas wells exhibit non-Newtonian, shear-thinning behavior. Prediction of friction pressure losses plays a vital role in modeling the hydraulics of such non-Newtonian well construction fluids. This is particularly important when encountering a narrow mud window (i.e. the difference between the fracture gradient and either the pore pressure or the mud weight required for borehole stability, whichever of the two is greater), such as in (ultra-)deepwater wells, and/or dealing with high frictional pressures within the available window, such as on extended reach (ERD) wells. Exceeding the boundaries of the mud window usually results in significant well trouble (e.g., well control incidents, lost circulation, borehole instability, stuck pipe, etc.) and associated non-productive time and recovery costs (see e.g. Karimi Vajargah and van Oort, 2015; Subramanian and Azar, 2000). Precise prediction of pressure losses is therefore crucial to properly manage downhole fluid pressures within the available mud window and establish if advanced drilling technologies such as MPD and dual gradient drilling (DGD) are necessary to help manage such pressures within the available window.

Well-established and relatively accurate analytical methods are available to predict the frictional pressure loss of Newtonian fluids in both laminar and turbulent flow. However, the development of such methods is very cumbersome for non-Newtonian fluids due to their complex and diverse behavior. There are well-established analytical methods for laminar flow of some non-Newtonian fluids, but for turbulent flow, the existing models introduced so far are not yet reliable (Chilton and Stainsby, 1998).

Several factors, such as rheological complexity of the fluid and turbulent eddies, make mathematical modeling of turbulent flow of non-Newtonian fluids very complicated. To overcome this complexity, an empirical friction factor term is usually introduced. Several correlations have been proposed to obtain the friction factor for turbulent flow of Newtonian fluids in pipes (e.g. Colebrook, 1939; Blasius 1913). These equations present acceptable accuracy for a variety of practical applications. However, only a few robust experimental studies have been carried out for non-Newtonian fluids. In addition, transitional flow is encountered in drilling applications, and there is currently no well-established model to calculate the friction factors for non-Newtonian fluids in this flow regime.

Dodge and Metzner (1959) proposed a semi-empirical friction factor correlation for turbulent flow of non-Newtonian, time-independent, non-elastic fluids in smooth pipes. As will be discussed further in this study, the Dodge and Metzner correlation exhibits acceptable accuracy for some drilling fluids and hence is extensively used in the drilling industry. However, the use of certain additives in drilling fluids makes this correlation unreliable. For instance, long-chain polymeric

additives are extensively used in drilling fluids as viscosifiers and fluid loss agents. Their presence in drilling fluid may delay the transition to turbulent flow. Knowledge of the critical Reynolds number (characterizing the transition from laminar flow to transitional flow) is very important in hydraulic planning and cuttings transport. In addition, due to inherent friction reduction qualities of polymers, the observed pressure loss in polymeric fluids can be significantly less than the predictions in turbulent flow (e.g., Subramanian and Azar, 2000; Graham, 2004; Dosunmu and Shah, 2013; Karimi Vajargah et al., 2016). This can lead to significant overestimation of pump pressure and equivalent circulation density (ECD) in the transitional / turbulent flow regime.

In this study, we investigate the effect of drilling fluid composition on frictional pressure loss in more depth. Several types of drilling fluid are tested in this study and the obtained results are compared to hydraulic models that are widely in use.

Background

Dodge and Metzner (1959) developed a semi-empirical equation for friction factor in fully developed turbulent flow of time-independent, purely viscous non-Newtonian fluids in smooth pipes. They applied Prandtl's mixing length theory, obtaining suitable values for the empirical constants from the experimental data. The Dodge and Metzner correlation has been widely accepted and is routinely quoted in books on non-Newtonian fluid technology (e.g. Chabra and Richardson, 1999; Skelland, 1967; Steffe, 1996; Ahmed and Miska, 2009). Similar to the Dodge and Metzner equation, several other friction factor correlations were introduced for pipe flow of power law fluids (e.g., Tomita, 1959; Clapp, 1961; Trinh, 1969; Shah, 1984; Desouky, and El-Emam, 1990; El-Emam et al., 2003).

Torrance (1963) extended Dodge and Metzner's work to be applicable to yield pseudo-plastics and to account for pipe roughness. Only a few correlations addressing relative roughness for non-Newtonian fluids can be found in literature (Szilas et al., 1981; Shah, 1990; Reed and Pilehvari, 1993).

As a common practice for non-Newtonian drilling fluids, the end of stable laminar flow is considered to occur when the Reynolds number is approximately 2100 (Ahmed and Miska, 2009). Although this is a reasonable assumption for several drilling fluids, it cannot be generalized for all types of non-Newtonian drilling fluids. For these, the critical Reynolds number is not constant but a function of generalized flow behavior index (Dodge and Metzner, 1959; Kelessidis et al., 2011).

It is also possible to predict the transition from laminar to non-laminar flow based on stability analysis. Ryan and Johnson (1959) developed a stability criterion based on the ratio of input energy to energy dissipation for an element fluid volume that depends on local parameters. Several other transition criteria have been proposed to predict the critical Reynolds number (e.g., Mishra and Tripathi, 1971; Hanks and Ricks, 1974; Desouky 1991; Merlo, et al., 1995; Slatter, 1999; Kalayci et al., 2013).

Experimental Set up

A flow loop was constructed at The University of Texas at Austin to collect the experimental data for this study. This flow loop is approximately 5.5 m (18 ft) long and consists of two pipe sections, 1.27 cm (0.5") and 0.9525 cm (0.375") in diameter. Wall thickness for both pipes is 0.89 mm (0.035"). Pressure data was obtained using two differential pressure transducers. The test section between pressure transducers for both pipes is 3.048 m (10 ft) long. Entrance and exit lengths were estimated based on empirical correlations from literature (Collins and Schowalter, 1963) in order to minimize flow anomalies associated with these. Note that only results from the small pipe (0.9525 cm), which covers laminar, transitional, and turbulent flow regimes, are presented here. **Figs. 1a** and **b** show the flow loop and related schematics.

a)

b)
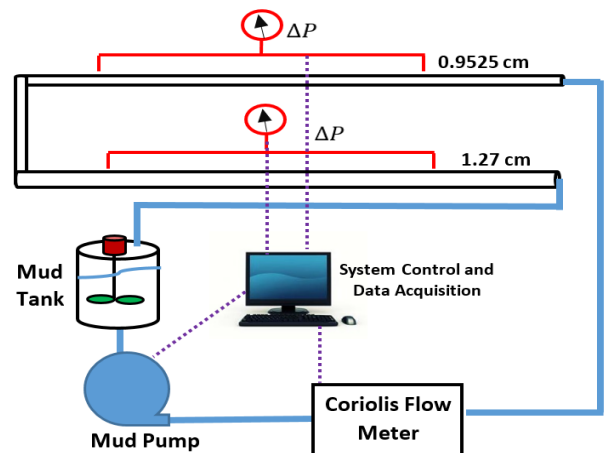
Figure 1: (a) Flow loop used to conduct the experiments; (b) related schematics.

A positive displacement pump was used for fluid circulation. Fluid volume for circulation was handled in a 40-liter (10.57 gallon) reservoir tank. A Coriolis flow meter installed at the inlet of the supply line measured fluid density, flow rate, and temperature. A fully automated control and data acquisition system was installed for this flow loop. Parameters

such as flow rate, temperature, differential pressure, pump frequency, etc. were monitored during the tests. All experiments were conducted at room temperature.

Five non-Newtonian drilling fluids (labeled mud A, mud B, mud C, mud D, and mud E) were used in this study. Mud A was a bentonite clay suspension with zero polymer content. Muds B, C, and D were polymer-based, and mud E was a synthetic based drilling fluid. Only muds A and B were prepared in the lab; their composition is presented in **Table 1**. Mud C was water-based mud based on a concentrated mixed cesium / potassium formate brine with xanthan polymer added. Three different polymer concentrations were used (0.25 lb/bbl, 0.5 lb/bbl and 1.0 lb/bbl) to investigate the effect of polymer concentrations. Muds D and E were field-based water-based and synthetic-based muds, with proprietary formulations. Long-chain polymers, however were used in field mud D. A rotational viscometer (**Fig. 2**) was used to obtain the rheological parameters in accordance with a yield-power-law (YPL) rheological model. A Coriolis mass flow meter was used to record the density of each fluid. Rheological parameters and density for each fluid are presented in **Table 2**.

Table 1: Compositions of muds A and B. Concentrations are presented in lb/bbl (gr/350 cc).

Component	Mud A	Mud B
	Concentration	Concentration
Water	322.14	328.61
Bentonite	9.17	0
Xanvis	0	0.33
PAC R	0	0.67
Barite	10.36	12.06
Drilling cuttings	8.33	8.33

Table 2: YPL rheological properties (yield stress, τ_y, consistency index, K, and flow behavior index, m) and density for each fluid (note that mud C is formulated with 1 lb/bbl xanthan gum).

Fluids	τ_y (pa)	K (Pa.s^m)	m	Specific Gravity
Mud A	3.244	0.1109	0.7506	1.14
Mud B	2.139	0.5992	0.4679	1.12
Mud C	0.7919	0.286	0.6526	1.87
Mud D	0	0.4079	0.5383	1.26
Mud E	0.9512	0.0318	0.9611	1.186

Figure 2: Rotational viscometer used to obtain rheological parameters of drilling fluids.

Theory

The non-linear three-parameter model proposed by Herschel and Bulkley (1926), also known as the YPL model, was used in this study to define the rheological characteristics of the test fluids:

$$\tau = \tau_y + K(-\frac{dv}{dr})^m \tag{1}$$

where τ_y is the yield stress, K is the consistency index, and m is the fluid behavior index. When yield stress is negligible or zero, the YPL model reduces to the Power Law model. Additionally, when m is equal to one, the YPL model reduces to the Bingham Plastic model. The YPL model has been extensively used in the petroleum industry (e.g. Kelessidis et al., 2007 and 2011; Bailey and Peden, 2000; Hemphil et al., 1993; Mehrabi et al., 2012; Zamora, 2005).

The following equation represents the relationship between the wall shear stress, τ_w and pressure loss, $\frac{dp}{dl}$ in a circular pipe:

$$\frac{dp}{dl} = \frac{4\tau_w}{D} \tag{2}$$

Therefore, in order to calculate pressure loss, one needs to obtain wall shear stress. It can be shown that for 1-D, steady-state, fully-developed, incompressible, isothermal flow of time-independent YPL fluids with no slip at the wall, wall shear stress can be obtained from the following equation:

$$\frac{8v}{D} = \frac{(\tau_w - \tau_y)^{\frac{1+m}{m}}}{K^{\frac{1}{m}}\tau_w^3}\left(\frac{4m}{3m+1}\right)\left[\tau_w^2 + \frac{2m}{1+2m}\tau_y\tau_w + \frac{2m^2}{(1+m)(1+2m)}\tau_y^2\right] \tag{3}$$

Note that Eq. 3 is only valid for laminar flow. Wall shear stress can be obtained numerically from this equation. Keep in mind that Eq. 3 can also be used for Newtonian, power law and Bingham-plastic fluids. The next step is to determine the flow regime. When wall shear stress is known, Reynolds number can be obtained from the following equation:

$$Re = \frac{8\rho v^2}{\tau_w} \tag{4}$$

where v is the average velocity of the fluid, obtained from:

$$v = \frac{Q}{A} \tag{5}$$

To determine the critical Reynolds number and characterize transitional flow, the following equations are used:

$$Re_1 = 3250 - 1150N \tag{6}$$
$$Re_2 = 4150 - 1150N \tag{7}$$

N is the generalized flow behavior index obtained from Eq. 8. Note that transition points are not fixed but are a function of N.

$$\frac{1}{N} = A + B$$
$$A = \frac{(1 - 2m)\tau_w + 3m\tau_y}{m(\tau_w - \tau_y)}$$
$$B = \frac{2m(1 + m)\left[(1 + 2m)\tau_w^2 + m\tau_y\tau_w\right]}{m(1 + m)(1 + 2m)\tau_w^2 + 2m^2(1 + m)\tau_w\tau_y + 2m^3\tau_y^2}$$

$$(8)$$

When the Reynolds number is lower than Re_1 (Eq.6) the flow regime is assumed to be laminar. In this case, the friction factor can be obtained from Eq. 9 and pressure loss from Eq. 2 accordingly.

$$f = \frac{16}{Re} \qquad (9)$$

When the Reynolds number is larger than Re_2 (Eq.7), the flow regime is assumed to be turbulent and Eq. 10 (Dodge and Metzner, 1959) is applied to obtain the friction factor.

$$\frac{1}{\sqrt{f}} = \frac{4}{N^{0.75}} \log\left(Re \times f^{\left(1 - \frac{N}{2}\right)}\right) - \frac{0.4}{N^{1.2}} \qquad (10)$$

Wall shear stress must then be re-calculated for turbulent flow using Eq. 11. The recalculated wall stress is replaced in Eq. 2 to obtain the pressure loss.

$$\tau_w = \frac{f\rho v^2}{2} \qquad (11)$$

For the transitional flow regime (with a Reynolds number value between Re_1 and Re_2), an averaging technique is used to obtain the friction factor. The approach presented here is relatively simple and hence widely used in the petroleum industry to obtain the frictional pressure loss of non-Newtonian fluids in pipes. Therefore, we applied this approach here to evaluate its performance for different types of non-Newtonian drilling fluids.

Results and Discussion
Calibration Test with Water

Prior to experiments on the drilling muds, calibration tests were conducted with water to verify the pressure loss readings. Then, the test results were compared with the analytical solution for flow of a Newtonian fluid (water) in pipes. The friction factor was obtained using the Colebrook (1939) correlation. As shown in **Fig. 3,** excellent agreement was achieved between the analytical model and experimental results in the test pipe.

Tests with Drilling Fluids

The first test was conducted with mud A, a simple clay mud that contained bentonite, barite, and drill cuttings. The test spanned flow rates from 3.48 to 27.97 liters/min (0.92 to 7.39

gpm) and Reynolds numbers from approximately 470 to 7200. The mud specific gravity was 1.14 (9.50 ppg). **Fig. 4** shows pressure loss vs. flow rate for mud A in the test section, 3.048 m (10 ft) long. According to this figure, very good agreement between the experimental and predicted pressure losses was observed in laminar flow. The Critical Reynolds number was determined by careful examination of the trend of the pressure vs. flow rate curve (Fig. 4). At the end of the laminar flow regime, a sharp increase in pressure loss is expected, which enables us to obtain the critical Reynolds number for each experiment.
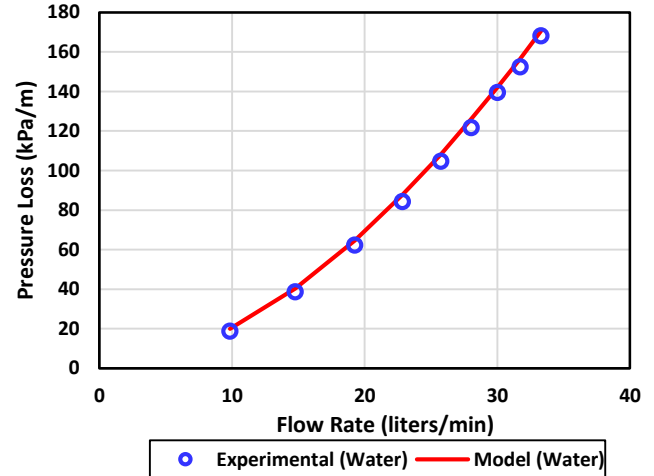

Figure 3: Validation test with water for the test pipe (0.9525 cm outer diameter). Excellent agreement between the experimental and predicted values is observed.
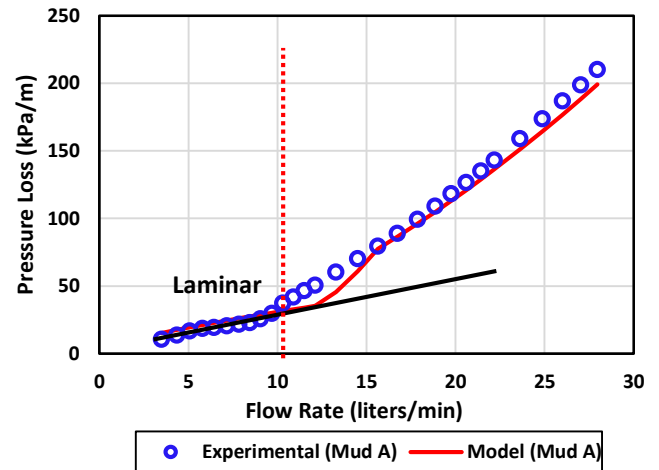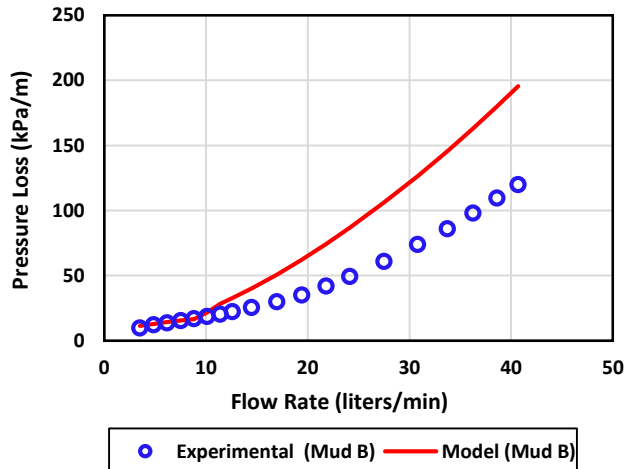

Figure 4: Comparison between values obtained from the experimental data and the model for Mud A (bentonite clay mud).

Although the model fails to predict the pressure loss accurately in the transition region, very good agreement with the Dodge and Metzner (1959) correlation was observed in the turbulent region.

Fig. 5 shows pressure loss vs. flow rate for mud B, a polymer-based mud that was prepared in the lab and contains xanthan gum and polyanionic cellulose (long-chain polymers). Although this figure shows a very good match in the laminar flow region, the model significantly overestimates the frictional pressure loss in the transitional and turbulent regions.
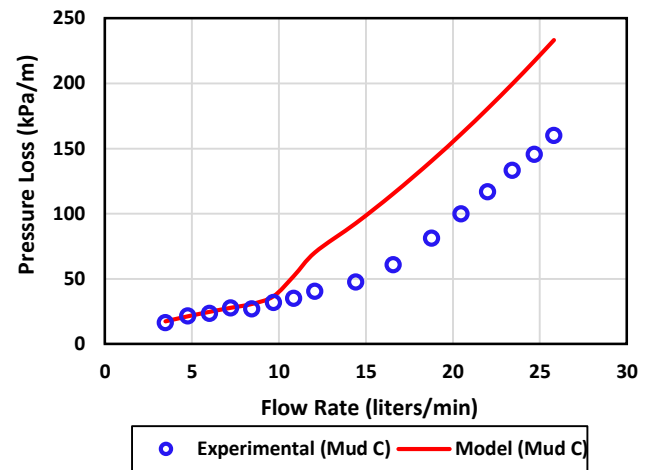

Figure 5: Comparison between values obtained from the experimental data and model for Mud B (polymer-based mud).

We believe that the lower-than-expected frictional pressure loss values could be due to inherent friction reduction qualities of polymer-based mud. Such effects are not included in the Dodge and Metzner (1959) correlation, and it therefore fails to predict the friction factor accurately for this mud. In addition, the presence of polymers delays the transition from laminar flow and makes distinguishing between the flow regimes more difficult. When polymers are exposed to very high shear (as experienced inside bit nozzles) for prolonged circulation times their polymer chains will break down. This will diminish the friction reduction effect over time if the polymers are not replenished.

Fig. 6 shows frictional pressure loss vs. flow rate for mud C. This drilling fluid was mixed cesium /potassium formate-based with a specific gravity of 1.87 (15.6 ppg) and 1 lb/bbl of xanthan gum for viscosity. Similar to mud B, very good agreement was observed between model predictions and experimental data in laminar flow. However, the model failed to accurately predict critical Reynolds number and frictional pressure loss in turbulent flow, the latter being some 30-35% lower than predicted.

Mud D, also a polymer-based mud used in the field, was provided by a service company. Although the detailed composition and concentration of the polymer(s) remain confidential, it is known that long chain polymers were present in the mud. Flow rates achieved ranged from 3.33 to 30.21 liters/min (0.88 to 7.98 gpm) with Reynolds numbers varying from about 700 to 17000. **Fig. 7** shows pressure loss vs. flow rate for this mud. Although model predictions are closer to experimental values, a major discrepancy is still observed. Note that in reality most drilling fluids show time-dependent

behavior and their properties may change due to several factors such as "aging" or undergoing high shear.
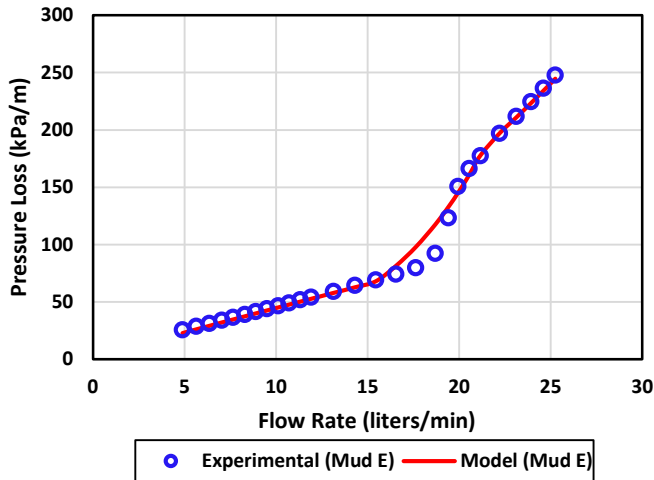

Figure 6: Comparison between values obtained from the experimental data and model for Mud C (Cesium formate mud with xanthan gum).
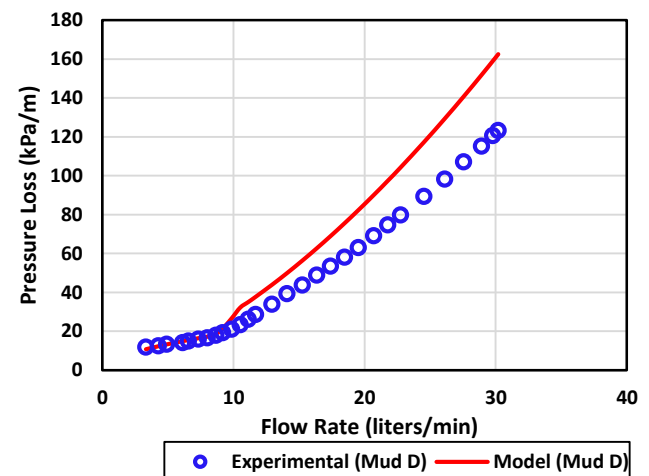
Figure 7: Comparison between values obtained from the experimental data and model for Mud D (polymer-based mud used in the field).

Figure 8 shows pressure loss vs. flow rate for mud E, a reconditioned synthetic based drilling fluid with a specific gravity of 1.19 (9.9 ppg). Since this mud was very viscous, the flow rate could only be varied from 4.88 to 25.25 liters/min (1.29 - 6.67 gpm) which resulted in Reynolds numbers from approximately 600 to 3600. Similar to mud A (bentonite-clay mud), the model closely matched the experimental data in both laminar and turbulent flow. Any presence of polymers in mud E did not result in friction reduction and the Dodge and Metzner (1959) correlation demonstrated very good accuracy in turbulent flow.

Figure 8: Comparison between values obtained from the experimental data and model for Mud E (synthetic-based mud used in the field).

Critical Reynolds Number

As mentioned earlier, experimental values for the critical Reynolds number can be determined by carefully examining the pressure vs. flow rate curve for a sharp increase in pressure loss. **Table 3** compares the critical Reynolds number (end of laminar flow) obtained from the model (Eq. 6) with the experimental values. This table indicates that the presence of polymers in mud can significantly delay the transition from laminar flow (e.g. muds B, C and D), which results in erroneous predictions of friction factor and frictional pressure loss. For instance, in the case of mud B, the transition from laminar flow occurs at a Reynolds number of approximately 4750, which is considerably higher than the predicted value (2720). For other drilling fluids, the model demonstrated acceptable accuracy in predicting the critical Reynolds numbers.

Table 3: Comparison of the experimental and predicted critical Reynolds number

Fluids	Experimental	Prediction
Mud A	1950	2440
Mud B	4750	2720
Mud C	5500	2600
Mud D	3610	2630
Mud E	2640	2150

Effect of Polymer Concentration on Frictional Pressure Loss

The experimental results presented here show that mud composition plays a substantial role in friction factor and frictional pressure loss in transitional and turbulent flow. For example, for muds that contained polymeric additives, the observed that friction factor and accordingly pressure loss were noticeably less than the values predicted by the model. This encouraged us to further investigate the effect of polymer concentration on the friction factor and the discrepancy observed between the experimental and predicted frictional

pressure loss. For this purpose, the mixed cesium / potassium formate mud C was reformulated with a xanthan gum concentration of 0.5 lb/bbl for mud C* and a concentration of 0.25 lb/bbl for mud C**. The rheological parameters of these fluids are presented in **Table 4**.

Figure 9 shows pressure loss vs. flow rate for mud C* (0.5 lb/bbl of xanthan gum concentration). It is observed that the gap between predicted and experimental values has been reduced in comparison to mud C. **Figure 10** shows pressure loss vs. flow rate for mud C** in which the xanthan gum concentration has been reduced further to 0.25 lb/bbl. This figure shows a very good agreement between the experimental and predicted values, indicating that polymer concentration has a substantial effect on pressure loss in turbulent flow and therefore must be taken into account. Dependence of friction factor on polymer concentration in these polymer-based drilling fluids complicates the prediction of frictional pressure loss, as a majority of the proposed friction factor correlations do not take this parameter into account. Conducting experiments to observe the possible friction reduction due to mud composition could be helpful in better designing the hydraulic program.

Note that although we ignored the time-dependent behavior of drilling fluids for the sake of simplicity, in reality most drilling fluids show time-dependent behavior, which affects the pressure loss predictions. Therefore, real-time monitoring of drilling fluid properties at the rig site could be very helpful in real-time ECD management. Karimi Vajargah et al. (2016) introduced a method based on the pipe viscometer approach for this purpose.

Table 4: YPL rheological properties (yield stress, τ_y, consistency index, K, and flow behavior index, m) and density for muds C* and C.**

Fluids	τ_y (pa)	K $(Pa.s^m)$	m	Specific Gravity
Mud C*	0.5591	0.05246	0.8202	1.87
Mud C**	0.0195	0.03917	0.8191	1.87
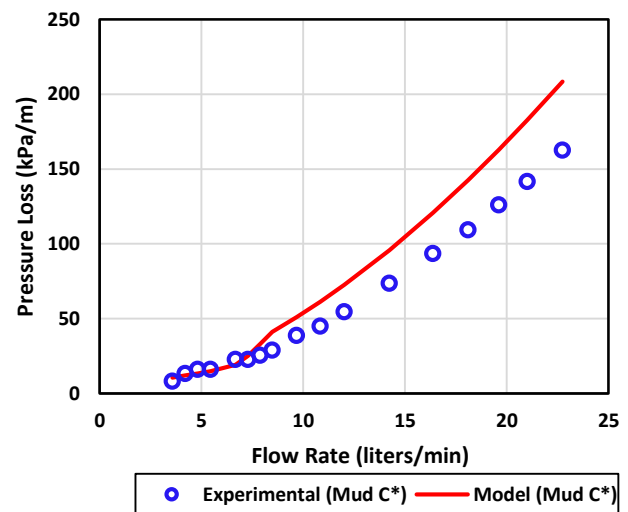
Figure 9: Comparison between values obtained from the experimental data and model for Mud C*.
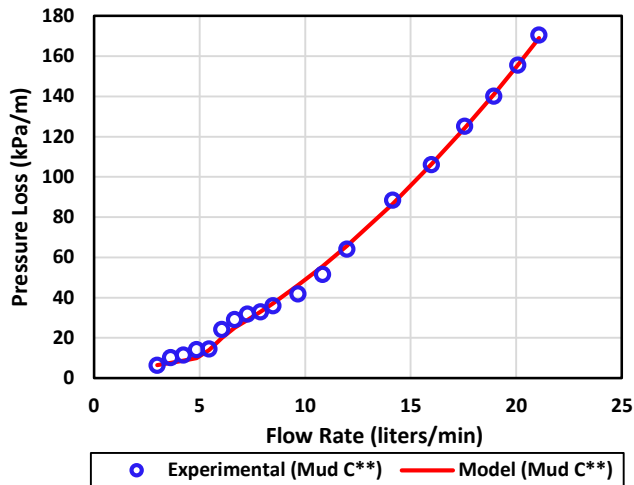
Figure 10: Comparison between values obtained from the experimental data and model for Mud C**.

Conclusions

- In this paper, we investigated the frictional pressure loss of several non-Newtonian drilling fluids in a 0.9525 cm (0.375") pipe under laminar, transitional and turbulent flow. Experimental results are compared to a hydraulic model used widely in the industry.
- Very good agreement was achieved between the predicted and experimental results in laminar flow for all types of drilling fluids used in this study. However, in turbulent flow, acceptable agreement was achieved only for muds A (bentonite clay) and E (synthetic-based mud). The model significantly overestimated frictional pressure loss for muds B, C and D containing long-chain polymers. The authors believe this is due to inherent friction reduction properties of such polymers in water-based mud. Considering the extensive use of these materials in drilling practice nowadays, relying on the industry standard friction factor correlations for turbulent flow is not sufficient.
- In addition to reducing friction, polymer presence in drilling fluid may significantly delay the transition from laminar flow, which makes Eqs. 6 and 7 unreliable for this purpose. Furthermore, a more sophisticated model is required to predict frictional pressure loss in the transitional flow regime, which is quite common in drilling applications.
- Our investigation shows that polymer concentration is a crucial factor in the discrepancy observed between the model predictions and experimental values. A 1.85 SG mixed cesium / potassium formate fluid with 1 lb/bbl xanthan gum showed a friction reduction of 30-35% compared to the modeled results. This reduction dropped to 20-25% and almost 0% for the same density fluid with 0.5 lb/bbl and 0.25 lb/bbl xanthan gum respectively.
- The absence of any noticeable friction reduction in synthetic-based muds should not dissuade the pursuit of effective friction reducers for such muds given the prevalent use of such muds on wells with low drilling margins (e.g. (ultra-)deepwater wells) and/or wells with high ECDs (e.g. ERD wells).
- With the advent of new materials, drilling fluid compositions are becoming more and more complex. This study shows that relying on widely used friction factor correlations may lead to erroneous results and poor hydraulics planning. This is of particular importance for applications such as MPD and DGD in which accurate ECD prediction plays a vital role. Applying real-time fluid characterization techniques (as addressed in this paper) at the rig site can be very helpful for accurate real-time ECD prediction and management.

Nomenclature

A = area, m^2
D = diameter, m
f = friction factor
K = consistency index, Pa.s^m
m = fluid behavior index
N   = generalized flow behavior index
l = length, m
p = pressure, Pa
Q = flow rate, m^3/s
r = radius, m
Re = Reynolds number
v = velocity, m/s
ρ = density, kg/m^3
τ = shear stress, Pa
τ_w = shear stress at the wall, Pa
τ_y = yield stress, Pa

Glossary

ECD   = *Equivalent circulating density*
ERD   = *Extended reach drilling*
DGD   = *Dual gradient drilling*
MPD   = *Managed pressure drilling*
YPL   = *Yield power law*

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