

Methodology for Testing Drilling Fluids under Extreme HP/HT Conditions

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Abstract

Designing a fit-for-purpose drilling fluid for extreme high-pressure, high-temperature (XHP/HT) operations is one of the greatest technological challenges facing the oil and gas industry today. Under XHP/HT conditions, well control issues become more complicated. Also current logging tools are at best not reliable because the anticipated bottom-hole temperature is often well above their operating limit. The literature shows limited experimental data on drilling fluid properties beyond 350°F and 20,000 psig. The practice of extrapolation of fluid properties at some moderate level to XHP/HT conditions is obsolete and could result in significant inaccuracies in wellbore hydraulics calculations.

A methodology for testing drilling fluids at XHP/HT conditions using an automated viscometer is presented in this paper. This state-of-the-art viscometer is capable of accurately measuring the rheological properties of drilling fluids up to 600°F and 40,000 psig. Following this methodology, a series of factorial experiments were performed on typical XHP/HT oil-based drilling fluids to investigate the change in rheology at these extreme conditions (200 to 600°F and 15,000 to 40,000 psig). Detailed statistical analyses involving: analysis of variance, hypothesis testing, evaluation of residuals and multiple linear regression are implemented using data from the laboratory experiments.

Introduction

As conventional resources decline, and global demand for energy continues to increase, there is need to explore and produce from more challenging/unconventional oil and gas reservoirs. Some of these assets are typically in high temperature and high pressure environments. Proper drilling fluids design is critical to the success of operating in these harsh environments. At present a systematic approach for evaluating the main and interaction effects of high temperature and pressure on a drilling fluids rheology is lacking.

In non-HP/HT wells, the effects of high pressure and temperature on a drilling fluid's viscosity are minimal and can be ignored. However, for HP/HT and XHP/HT wells these effects increase exponentially and can not be overlooked. For example, the volume of an HP/HT gas kick remains virtually unchanged as it rises in the annulus from 14,000 to 10,000 ft. From 10,000 to 2,000 ft its volume triples. But from 2,000 ft to the surface, there is a 100-fold expansion.¹

The methodology presented in this paper is shown to be an effective means of quantitatively estimating the effects of temperature and pressure on the rheological properties of drilling fluids. This technique could be applied in the characterization of other fluid types (e.g. completions fluids, cement slurries) used in XHP/HT operations.

Methodology

The methodology presented in this paper can be summarized in three stages as follows:

1. **Design Stage:** This involves the initial design of the factorial experiments within the range of temperature and pressure to be investigated.
2. **Experimentation:** This starts with fluid sampling/preparation and then performing a series of factorial tests on the drilling fluid using the automated XHP/HT viscometer. At the early stage of each test (150°F), the Fann 35 (or its equivalent) is run simultaneously as a control experiment. See Fig. 1 for details.
3. **Analyses / Interpretation:** To say the least experiments, will be of little value without proper analyses and interpretation of the information thereof. This stage covers statistical calculations and interpretation of results. To facilitate this process a visual basic program (called *FluidStats*) was developed.

We shall proceed to discuss the various aspects of our methodology with emphasis on the statistical analyses / interpretation using the program.

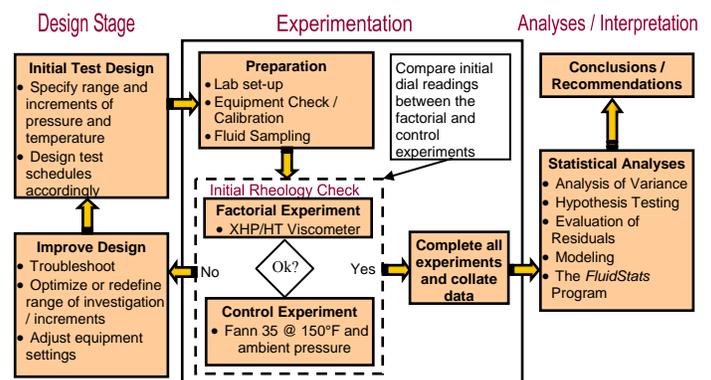


Fig. 1- Summary of methodology

Design of Experiments

In general factorial designs are the most efficient way of investigating experiments that involve the study of the effects of two or more factors. Primarily factorial designs allow the effects of a factor to be estimated at several levels of other factors yielding results that are valid over a range of experimental conditions. A factorial design is necessary especially when interactions may be present to avoid misleading conclusions. Temperature and pressure are the two factors being investigated in this work.

Factorial Experiment: This consists of two or more factors, each with discrete possible values or "levels", and whose experimental units take on all possible combinations of these levels across all such factors.² Such an experiment allows studying the effect of each factor on the response variable, as well as the effects of interactions between factors on the response variable. This research by nature is a typical case of a two-factor factorial experiment as will be discussed in details later. The analysis of variance, hypothesis and residuals calculations will be made and results will be compared with output from the *FluidStats* program.

Fluid Sampling / Preparation

A mineral oil-based drilling fluid formulation was used to perform the experiments. This fluid has a density of 18.0 ppg and 93/7 oil/water ratio, weighted with barite. Viscosifiers and emulsifiers were added. Here it is referred to as "Fluid Type B" for confidentiality.

Freshly-mixed samples were heat-aged at 400°F for 16 hours before sampling. Prior to testing the fluid with the XHP/HT viscometer, the following steps were followed:

1. The whole fluid in the bucket is stirred first with a paddle then with two Hamilton Beach mixers set at 70 RPM for 5 minutes to ensure a homogeneous mixture.
2. A sample (2 lab barrels) is then poured out into a cup and uniformly sheared for 10 minutes using a Hamilton Beach mixers set at low speed.
3. 25 ml is extracted using a syringe and 175 ml poured into the thermowell of the XHP/HT viscometer.
4. The remaining fluid sample is poured into the thermo-cup for initial rheology check with the Fann 35.

Experimentation

The laboratory experiments were executed in two phases using the factorial design concept discussed earlier. The first phase of testing involved the constant pressure increasing temperature schedules and the fluid's plastic viscosity, yield point and gel strengths (10 sec. /10 min.) where determined at each increment. The second phase was the reverse; constant temperature, increasing pressure schedules. Nominal 50°F temperature increments were used initially, however significant changes in viscosity warranted smaller increments (25°F) from 400 to 600°F in the first phase of testing. For each test, the initial rheology check was performed at 150°F and ambient pressure (14.7 psia) using both the XHP/HT and Fann 35 viscometers. Using at least two viscometers was necessary

for proper quality control and to ensure consistency in results. If and when discrepancies occurred, steps were taken to investigate and identify the cause(s) and where necessary recalibrate the affected viscometer(s) according to standard API procedures.

The XHP/HT Viscometer

The Chandler model 7600 Ultra-High Pressure High Temperature viscometer is a concentric cylinder (Couette) viscometer that uses a rotor and bob geometry. This is in compliance with the requirements set forth in ISO and API standards for viscosity measurement of completion fluids at high pressure and high temperature.

For this viscometer, the shear stress (torque) created between the bob and rotor is measured using a precision torsion spring and high resolution encoder. Known sample shear rates are created between the bob and the rotor using precision defined bob/rotor geometry and a stepper motor sub-system providing rotational speeds ranging from 0 to 900 RPM (0 to 1533 sec⁻¹). Suspended solids in the sample are circulated during the test using a helical screw on the outside diameter of the rotor as illustrated in **Fig. 2**. Other unique features are listed below³.

- External digital torque measurement
- Fluid tests up to 600°F and 40,000 psig
- Sample/Oil separation zone
- High strength, corrosion resistant, steel super-alloys
- Programmable Temperature and Pressure Controllers
- Maximum shear stress: 1533 dyne/cm²
- Shear stress accuracy: ±0.50%
- Maximum shear rate: 1022-sec⁻¹ (600 RPM with B1/R1 Bob and Rotor) ±0.1 RPM accuracy.

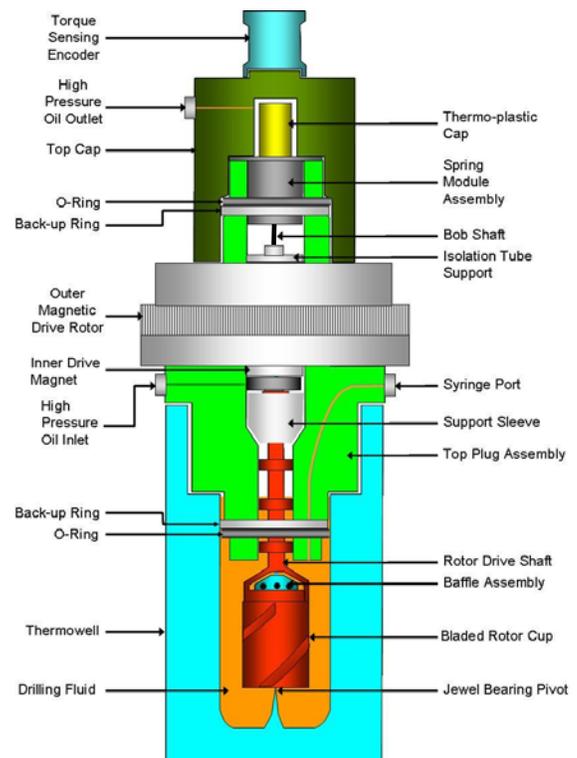


Fig. 2 - Test cell schematic.

Statistical Analyses

Two-Factor Factorial Design

This is most appropriate for this study because only two factors or sets of treatments are being investigated. Given that there are a levels of factor A (pressure) and b levels of factor B (temperature), all arranged in a factorial design; that is, each of the n replicates of the experiment contains ab treatment combinations. Let y_{ijk} be the observed response when factor A is at the i th level ($i = 1, 2, \dots, a$) and factor B is at the j th level ($j = 1, 2, \dots, b$) for the k th replicate ($k = 1, 2, \dots, n$). The order in which the abn observations are taken is selected at random so that the design is a completely randomized design. The observations can be described by the linear statistical model,²:

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk} \begin{cases} i = 1, 2, \dots, a \\ j = 1, 2, \dots, b \\ k = 1, 2, \dots, n \end{cases} \dots\dots\dots (1)$$

Where μ is the overall mean effect, τ_i is the effect of the i th level of the row factor A (pressure), β_j is the effect of the j th level of the column factor B (temperature), $(\tau\beta)_{ij}$ is the effect of the interaction between τ_i and β_j , ε_{ijk} is a random error component having a normal distribution with a mean zero and variance, σ^2 . We are interested in testing the hypotheses of no main effect for factor A , no main effect for B , and no AB interaction effect.

Analysis of Variance

The analysis of variance or ANOVA is used to test hypotheses about the main factor effects of A and B and the AB interaction. Let $y_{i..}$ represent the total of the observations taken at the i th level of factor A ; $y_{.j.}$ denote the total of the observations taken at the j th level of factor B ; $y_{ij.}$ denote the total of the observations in the ij th cell as shown in **Table 1** and $y_{...}$ denote the grand total of all the observations. Define $\bar{y}_{i..}$, $\bar{y}_{.j.}$, $\bar{y}_{ij.}$ and $\bar{y}_{...}$ as the corresponding row, column, cell, and grand averages. That is:

$$\left. \begin{aligned} y_{i..} &= \sum_{j=1}^b \sum_{k=1}^n y_{ijk} & \bar{y}_{i..} &= \frac{y_{i..}}{bn} & i &= 1, 2, \dots, a \\ y_{.j.} &= \sum_{i=1}^a \sum_{k=1}^n y_{ijk} & \bar{y}_{.j.} &= \frac{y_{.j.}}{an} & j &= 1, 2, \dots, b \\ y_{ij.} &= \sum_{k=1}^n y_{ijk} & \bar{y}_{ij.} &= \frac{y_{ij.}}{n} & i &= 1, 2, \dots, a \\ & & & & \text{and } j &= 1, 2, \dots, b \end{aligned} \right\} \dots\dots\dots (2)$$

Table 1–Data Arrangement for a Two-Factor Factorial design

| | | Factor B (Temperature) | | | | Totals | Ave. |
|---------------------|---|------------------------------------|------------------------------------|-----|------------------------------------|-----------|-----------------|
| | | 1 | 2 | ... | b | | |
| Factor A (Pressure) | 1 | $y_{111}, y_{112}, \dots, y_{11n}$ | $y_{121}, y_{122}, \dots, y_{12n}$ | | $y_{1b1}, y_{1b2}, \dots, y_{1bn}$ | $y_{1.}$ | $\bar{y}_{1..}$ |
| | 2 | $y_{211}, y_{212}, \dots, y_{21n}$ | $y_{221}, y_{222}, \dots, y_{22n}$ | | $y_{2b1}, y_{2b2}, \dots, y_{2bn}$ | $y_{2.}$ | $\bar{y}_{2..}$ |
| | ⋮ | | | | | | |
| | a | $y_{a11}, y_{a12}, \dots, y_{a1n}$ | $y_{a21}, y_{a22}, \dots, y_{a2n}$ | | $y_{ab1}, y_{ab2}, \dots, y_{abn}$ | $y_{a.}$ | $\bar{y}_{a..}$ |
| Totals | | $y_{.1.}$ | $y_{.2.}$ | | $y_{.b.}$ | $y_{...}$ | |
| Averages | | $\bar{y}_{.1.}$ | $\bar{y}_{.2.}$ | | $\bar{y}_{.b.}$ | | $\bar{y}_{...}$ |

In general, the ANOVA tests the hypotheses of no interaction, and no main effect for each factor by decomposing the total variability in the data into component parts and then comparing the various elements in this decomposition. The total variability is measured by the total sum of squares of the observations given as:

$$\sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^n (y_{ijk} - \bar{y}_{...})^2 = bn \sum_{i=1}^a (\bar{y}_{i..} - \bar{y}_{...})^2 + an \sum_{j=1}^b (\bar{y}_{.j.} - \bar{y}_{...})^2 + n \sum_{i=1}^a \sum_{j=1}^b (\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...})^2 + \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^n (y_{ijk} - \bar{y}_{ij.})^2 \dots\dots\dots (3)$$

Or symbolically as:

$$SS_T = SS_A + SS_B + SS_{AB} + SS_E \dots\dots\dots (4)$$

Where SS_A is the sum of squares for factor A , SS_B is the sum of squares for factor B , SS_{AB} is the sum of squares for the interaction between A and B , and SS_E is the error sum of squares. In summary, the sums of squares in a two-factor ANOVA are computed as follows:

$$SS_T = \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^n y_{ijk}^2 - \frac{y_{...}^2}{abn} \dots\dots\dots (5)$$

$$SS_A = \sum_{i=1}^a \frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn} \dots\dots\dots (6)$$

$$SS_B = \sum_{j=1}^b \frac{y_{.j.}^2}{an} - \frac{y_{...}^2}{abn} \dots\dots\dots (7)$$

$$SS_{AB} = \sum_{i=1}^a \sum_{j=1}^b \frac{y_{ij.}^2}{n} - \frac{y_{...}^2}{abn} - SS_A - SS_B \dots\dots\dots (8)$$

$$SS_E = SS_T - SS_A - SS_B - SS_{AB} \dots\dots\dots (9)$$

For a two-factor ANOVA, there are $abn - 1$ total degrees

of freedom. The main effects A and B have $a-1$ and $b-1$ degrees of freedom, while the interaction effect AB has $(a-1)(b-1)$ degrees of freedom. Within each of the ab cells in Table 1 there are $n-1$ degrees of freedom between the n replicates, and observations in the same cell can differ only because of random error, therefore, there are $ab(n-1)$ degrees of freedom for error.

By dividing each of the sums of squares on the right-hand side of Eq. 4 by the corresponding number of degrees of freedom, we obtain the mean squares for A , B , the interaction and error as presented in Table 2.

Table 2—ANOVA table for a Random-Effects Model²

| Source of Variation | Sum of Squares | Degrees of Freedom | Mean Square | F_0 |
|---------------------|----------------|--------------------|--|------------------------|
| Factor, A | SS_A | $a-1$ | $MS_A = \frac{SS_A}{a-1}$ | $\frac{MS_A}{MS_{AB}}$ |
| Factor, B | SS_B | $b-1$ | $MS_B = \frac{SS_B}{b-1}$ | $\frac{MS_B}{MS_{AB}}$ |
| Interaction | SS_{AB} | $(a-1)(b-1)$ | $MS_{AB} = \frac{SS_{AB}}{(a-1)(b-1)}$ | $\frac{MS_{AB}}{MS_E}$ |
| Error | SS_E | $ab(n-1)$ | $MS_E = \frac{SS_E}{ab(n-1)}$ | |
| Total | SS_T | $abn-1$ | | |

Fixed-Effects Model

Suppose that A and B are fixed factors. That is, the a levels of factor A and the b levels of factor B are specifically chosen by the experimenter, and inferences are confined to these levels only. In this model, it is customary to define the effects τ_i , β_j and $(\tau\beta)_{ij}$ as deviations from the mean, so that

$$\sum_{i=1}^a \tau_i = 0, \sum_{j=1}^b \beta_j = 0, \sum_{i=1}^a (\tau\beta)_{ij} = 0, \text{ and } \sum_{j=1}^b (\tau\beta)_{ij} = 0.$$

Random-Effects Model

Consider a more practical situation as is the case in this research were the levels of both factors A and B are randomly selected from larger populations of factor levels, and we wish to extend our conclusion to the entire population. The observations are represented by the same model as defined in Eq. 1. Here the parameters τ_i , β_j , $(\tau\beta)_{ij}$ and ε_{ijk} are normally and independently distributed random variables with zero means and variances σ_τ^2 , σ_β^2 , $\sigma_{\tau\beta}^2$, and σ^2 respectively. Just like the fixed effects model the basis of analysis of variance remains unchanged; that is, SS_A , SS_B , SS_{AB} , SS_T , and SS_E are calculated as in the fixed effects case. The ANOVA parameters for a two-factor factorial, random-

effects model are presented in Table 2.

Mixed-Effects Model

In the context of this research this could be either of two models: a Random Pressure, Fixed Temperature or a Random Temperature, Fixed Pressure model. Thus one factor is fixed while the other is random. For a given model, the F_0 statistic of the random factor is calculated using MS_{AB} as the denominator whereas MS_E is used as the denominator in estimating the F_0 statistic of the fixed factor. Both the interaction $(\tau\beta)_{ij}$ and error ε_{ijk} terms are random variables having zero mean and variance, σ^2 but the error term is independently distributed.

Hypothesis Testing

The random effects model discussed earlier will be used in hypothesis testing because the data used for illustration was obtained from a larger population. Thus we are treating both pressure and temperature as random factors. The three hypotheses to be evaluated are summarized as follows:

1. There is no main effect of pressure.
2. There is no main effect of temperature.
3. There is no interaction effect between pressure and temperature.

The test statistics (f_0) for these hypotheses are as defined in Table 2 (for the random effects case). For any of the three options above, the null hypothesis is accepted or rejected based on a comparison with the corresponding F-distribution. The general approach is to first evaluate the hypothesis of no interaction between pressure and temperature. If this hypothesis is rejected, that is, if interaction exists, then the two hypotheses of no main effects are irrelevant, since both factors clearly do affect the response variable through the interaction effect. However if there is no interaction, and if only one of the factor main effects is significant, the two-way ANOVA model reduces to a one-way ANOVA model.

Modeling

Based on the intrinsic linearity existing between rheological properties and pressure-temperature effects the multiple linear regression technique was adopted. This technique is also applicable to models with interaction effects as is typical of this research. The interaction between pressure and temperature can be represented by a cross-product in the model:

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \varepsilon \dots\dots\dots (10)$$

By setting $x_3 = x_1 x_2$ and $\beta_3 = \beta_{12}$, Eq. 10 can be rewritten as:

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon \dots\dots\dots (11)$$

This is a linear model where the parameters β_j , $j = 0, 1, \dots, k$, are the regression coefficients. This model represents a system of n equations that can be expressed in matrix form as²:

$$y = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \dots\dots\dots (12)$$

Where:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & \dots & x_{1k} \\ 1 & x_{21} & x_{22} & \dots & x_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \dots & x_{nk} \end{bmatrix} \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix} \quad \text{and} \quad \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

Here y is a $(n \times 1)$ vector of the observations, \mathbf{X} is a $(n \times p)$ matrix of levels of the independent variables, $\boldsymbol{\beta}$ is a $(p \times 1)$ vector of regression coefficients, and $\boldsymbol{\varepsilon}$ is a $(n \times 1)$ vector of random errors with zero mean and variance, σ^2 . Using the least squares approach, estimate of the coefficients matrix $\boldsymbol{\beta}$ is given as:

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{X})^{-1} \mathbf{X}^T\mathbf{y} \dots\dots\dots (13)$$

Where the superscripts T and -1 stand for transpose and inverse respectively. The fitted regression model is given as:

$$\hat{y} = \mathbf{X}\hat{\boldsymbol{\beta}} \dots\dots\dots (14)$$

From Eq. 21 note that there are $p = k + 1$ normal equations in $p = k + 1$ unknowns or parameters (ie the values of $(\hat{\beta}_0, \hat{\beta}_1, \dots, \hat{\beta}_k)$). Also the matrix $\mathbf{X}^T\mathbf{X}$ is always non-singular with $p \times p$ dimension. Calculations involving matrix operations are often very rigorous and are best done using a computer program. For this reason, *FluidStats* program was developed. Results will be presented later.

Based on fundamental principles and experimental results from this work, pressure and temperature are directly and inversely proportional to viscosity respectively. So a simple algebraic model can be written as:

$$PV = \beta_0 + \beta_1P + \beta_2T^{-1} + \beta_3PT^{-1} \dots\dots\dots (15)$$

Logarithmic, exponential and polynomial model options are also explored to obtain a better fit. These model options are all incorporated in the program with a flexibility to choose the parameter forms.

The FluidStats Program

The *FluidStats* is a Visual Basic two-factor factorial analyses and modeling software. It is user-friendly and very handy for manipulating large factorial data. Other features include:

- Performs ANOVA, hypothesis testing and analysis of residuals calculations at the click of a button for up to 3 sets of 10 by 10 data size.
- Flexibility to implement any of: Fixed-, Random- and Mixed-Effects Models.
- Inbuilt F-distribution and normal probability plotting capability.
- Modeling capability using multiple linear regression technique.
- Check for model adequacy.
- Enables 2- and 3-parameter model options.
- In-built input error checks and controls.

The *FluidStats* program is presented here to show the ease with which these calculations can be done and its potential to be used for the analyses of other two-factor factorial experiments. It is not intended to be a substitute to other more elaborate, statistical programs like SAS or Minitab. Suffice to say results obtained with the program are accurate and have been validated with results from SAS. To illustrate this methodology we present results from a series of factorial experiments performed using the XHP/HT viscometer.

Illustration

Phase 1: This phase of factorial experiments involved the constant temperature, varying pressure tests. Temperature was kept constant while increasing pressure in steps of 5000 psig from 0 to 40,000 psig. **Figs. 3 and 4** are plots of 600 RPM and 300 RPM readings respectively.

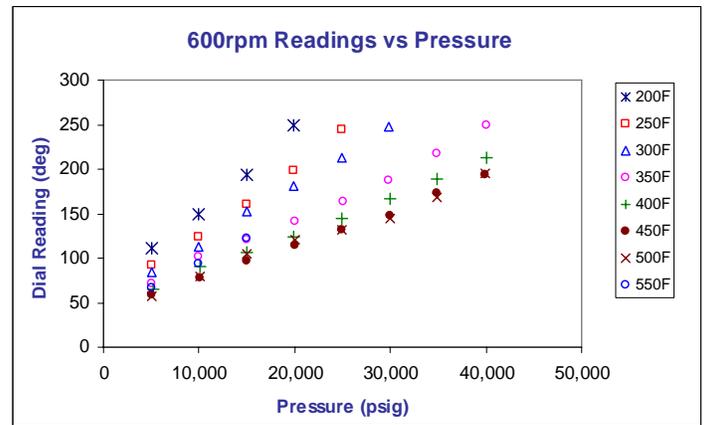


Fig. 3–600 RPM dial readings versus pressure

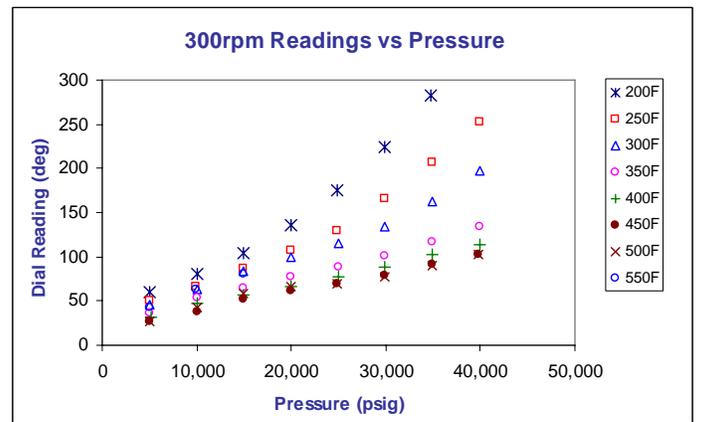


Fig. 4–300 RPM dial readings versus pressure

Phase 2: This involved constant pressure and varying temperature tests. Here pressure was kept constant while varying temperature in steps of 50°F from 150°F to 600°F starting with the first test at 5000 psig. There was a steady decline in viscosity with increase in temperature up to about 450°F where there is a sharp increase. This is thermal

degradation due to a significant change in the composition of the fluid at extreme high temperatures. This behavior was repeated for all the other schedules in this sequence regardless of the set pressure. A total of eight (8) tests were performed in this phase like in the previous phase with an average of 6hrs per test using the XHP/HT viscometer. **Figs 5** and **6** are plots of 600 RPM, 300 RPM dial readings, plastic viscosity and yield point versus temperature respectively.

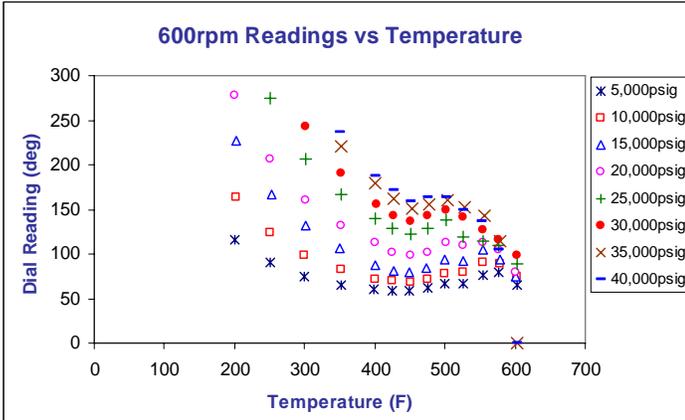


Fig. 5–600 RPM dial readings versus temperature

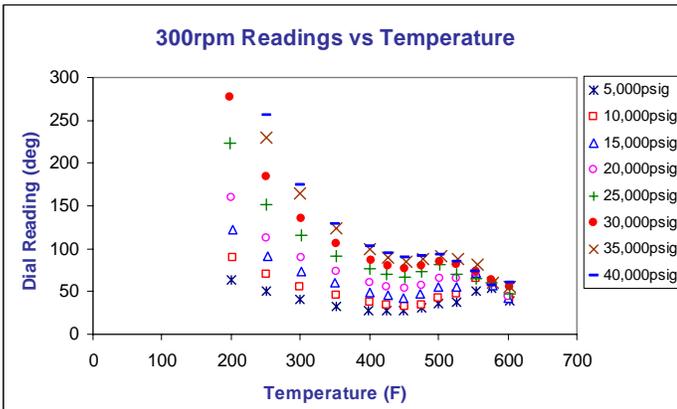


Fig. 6–300 RPM dial readings versus temperature

The plastic viscosity data from both phases of factorial experiments are combined in the **Table 3** below. The first value is from the constant pressure-variable temperature test while the second is from the corresponding constant temperature-variable pressure test. These data were sampled from a larger population of temperature and pressure ranging from 200 to 600°F and 10,000 to 40,000 psig respectively.

For most practical applications in the oil field, the rheology of drilling fluids are usually defined by Bingham Plastic parameters. Bingham Plastic fluids approach Newtonian behavior at very high shear rates and approximate the behavior of a solid at low shear rates. The constitutive equation is:

$$\tau = \tau_y + \mu_p \gamma \dots\dots\dots (16)$$

Where: $\tau_y = \theta_{300} - \mu_p$, yield stress, lbf/100 ft²
 $\mu_p = \theta_{600} - \theta_{300}$, plastic viscosity, cP
 γ = shear rate, sec⁻¹

Table 3–Plastic viscosity data from phases 1 and 2.

| | | Temperature (°F) | | | | |
|-----------------|--------|------------------|-------|-------|-------|-------|
| | | 300 | 350 | 400 | 450 | 500 |
| Pressure (psig) | 10,000 | 43/50 | 37/47 | 34/42 | 36/40 | 35/37 |
| | 15,000 | 57/69 | 46/56 | 39/50 | 36/45 | 38/46 |
| | 20,000 | 71/82 | 59/65 | 52/57 | 45/53 | 46/55 |
| | 25,000 | 92/98 | 74/75 | 62/67 | 55/61 | 57/62 |
| | 30,000 | 108/114 | 84/86 | 69/77 | 61/69 | 64/67 |

From the above data set, $n = 2$ since we have two sets of data. So k will range from 1 to 2. Also $a = b = 5$, representing 5 levels each for pressure and temperature. Using the sums and averages formulas defined earlier in Eq. 2, the following calculations are made:

Totals and Averages

For the first row representing (10,000 psig data):

$$y_{1..} = \sum_{j=1}^5 \sum_{k=1}^2 y_{1jk} = 401 \quad \text{and} \quad \bar{y}_{1..} = \frac{y_{1..}}{bn} = \frac{401}{5 \times 2} = 40.1$$

Similarly for the other rows;

$$y_{2..} = \sum_{j=1}^5 \sum_{k=1}^2 y_{2jk} = 482 \quad \text{and} \quad \bar{y}_{2..} = \frac{y_{2..}}{bn} = \frac{482}{5 \times 2} = 48.2$$

$$y_{3..} = \sum_{j=1}^5 \sum_{k=1}^2 y_{3jk} = 585 \quad \bar{y}_{3..} = \frac{y_{3..}}{bn} = \frac{585}{5 \times 2} = 58.5$$

$$y_{4..} = \sum_{j=1}^5 \sum_{k=1}^2 y_{4jk} = 703 \quad \bar{y}_{4..} = \frac{y_{4..}}{bn} = \frac{703}{5 \times 2} = 70.3$$

$$y_{5..} = \sum_{j=1}^5 \sum_{k=1}^2 y_{5jk} = 799 \quad \bar{y}_{5..} = \frac{y_{5..}}{bn} = \frac{799}{5 \times 2} = 79.9$$

This process is also true for the column operations. So for the first column:

$$y_{.1} = \sum_{i=1}^5 \sum_{k=1}^2 y_{ik1} = 784 \quad \text{and} \quad \bar{y}_{.1} = \frac{y_{.1}}{an} = \frac{784}{5 \times 2} = 78.4$$

Similarly

$$y_{.2} = \sum_{i=1}^5 \sum_{k=1}^2 y_{ik2} = 629 \quad \bar{y}_{.2} = \frac{y_{.2}}{an} = \frac{629}{5 \times 2} = 62.9$$

$$y_{.3} = \sum_{i=1}^5 \sum_{k=1}^2 y_{ik3} = 549 \quad \bar{y}_{.3} = \frac{y_{.3}}{an} = \frac{549}{5 \times 2} = 54.9$$

$$y_{.4} = \sum_{i=1}^5 \sum_{k=1}^2 y_{ik4} = 501 \quad \bar{y}_{.4} = \frac{y_{.4}}{an} = \frac{501}{5 \times 2} = 50.1$$

$$y_{.5} = \sum_{i=1}^5 \sum_{k=1}^2 y_{ik5} = 507 \quad \bar{y}_{.5} = \frac{y_{.5}}{an} = \frac{507}{5 \times 2} = 50.7$$

To calculate the sum and average we use;

$$y_{ij} = y_{11} = \sum_{k=1}^n y_{11k} = 93 \quad \text{and} \quad \bar{y}_{11} = \frac{y_{11}}{n} = \frac{93}{2} = 46.5$$

This was done for all the cells and the results used later in the analysis of residuals. Finally, the grand sum and average are given as:

$$y_{\dots} = \sum_{i=1}^5 \sum_{j=1}^2 \sum_{k=1}^2 y_{ijk} = 2970 \quad \text{and} \quad \bar{y}_{\dots} = \frac{y_{\dots}}{abn} = \frac{2970}{5 \times 5 \times 2} = 59.4$$

ANOVA Parameters

Using Eq. 4 through to 9 we have that:

$$SS_T = \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^n y_{ijk}^2 - \frac{y_{\dots}^2}{abn} = 194,110 - \frac{2970^2}{5 \times 5 \times 2} = 17,692$$

$$SS_A = \sum_{i=1}^a \frac{y_{i.}^2}{bn} - \frac{y_{\dots}^2}{abn} = 10,378, \quad MS_A = \frac{SS_A}{a-1} = 2,594.5$$

$$SS_B = \sum_{j=1}^b \frac{y_{.j}^2}{an} - \frac{y_{\dots}^2}{abn} = 5,556.8, \quad MS_B = \frac{SS_B}{b-1} = 1,389.2$$

$$SS_{AB} = \sum_{i=1}^a \sum_{j=1}^b \frac{y_{ij.}^2}{n} - \frac{y_{\dots}^2}{abn} - SS_A - SS_B = 1,072.2$$

$$SS_E = SS_T - SS_A - SS_B - SS_{AB} = 685$$

$$MS_{AB} = \frac{SS_{AB}}{(a-1)(b-1)} = \frac{1,072.2}{(5-1)(5-1)} = 67.0$$

$$MS_E = \frac{SS_E}{ab(n-1)} = \frac{685}{25(2-1)} = 27.4$$

Hypothesis Testing

The random effects model discussed earlier will be used in hypothesis testing because the data used for illustration was obtained from a larger population. Thus we are treating both pressure and temperature as random factors. First we will test the hypothesis of no interaction:

1. $H_0 : (\tau\beta)_{11} = (\tau\beta)_{12} = \dots = (\tau\beta)_{ab} = 0$ (no interaction effect between temperature and pressure. or $H_1 : \text{at least one } (\tau\beta)_{ij} \neq 0$. To test the hypothesis that all interaction effects are equal to zero $H_0 : (\tau\beta)_{ij} = 0$, we would use the ratio:

$$F_0 = \frac{MS_{AB}}{MS_E} = \frac{67.0}{27.4} = 2.45$$

The null hypothesis is rejected at the α level of significance if $f_0 > f_{\alpha, (a-1)(b-1), ab(n-1)}$. Performing the calculations; $f_{\alpha, (a-1)(b-1), ab(n-1)} = f_{0.05, 16, 25} = 2.07$. Clearly for $\alpha = 0.05$ (level of significance) there is strong evidence to conclude that estimates of the slope τ_i , β_j and $(\tau\beta)_{ij}$ are not equal to zero. This implies that the main and interaction effects of pressure and temperature on the plastic viscosity of the fluid are not negligible. We will now evaluate the hypothesis of no main effect of the individual factors.

2. $H_0 : \tau_1 = \tau_2 = \dots = \tau_a = 0$ (no main effect of factor A –

Pressure) or $H_1 : \text{at least one } \tau_i \neq 0$. To test that the effects of pressure are all equal to zero, we would use the ratio:

$$F_0 = \frac{MS_A}{MS_{AB}} = \frac{2594.5}{67.0} = 38.72$$

Assuming a significance level of $\alpha = 0.05$, then $f_{\alpha, a-1, (a-1)(b-1)} = f_{0.05, 4, 16} = 3.01$. Since $38.72 > 3.01$, the null is rejected, meaning that at $\alpha = 0.05$ (ie 95% confidence level), the effects of pressure on plastic viscosity of the fluid is not negligible.

3. $H_0 : \beta_1 = \beta_2 = \dots = \beta_b = 0$ (no main effect of temperature). Or $H_1 : \text{at least one } \beta_j \neq 0$. Similarly, the test statistic will be given as:

$$F_0 = \frac{MS_B}{MS_{AB}} = \frac{1389.2}{67.0} = 20.7$$

For the case at hand, the null hypothesis is still rejected at $\alpha = 0.05$ since; $f_{\alpha, b-1, (a-1)(b-1)} = f_{0.05, 4, 16} = 3.01 < 20.7$

Results from the FluidStats Program

These calculations can be facilitated using the program discussed earlier. Data from Table 3 is entered into the program. A summary of the ANOVA table and test of hypothesis results is shown below.

a) Analysis of Variance

| Parameter | Sum of Squares | Degrees of freedom | Mean Square | F ₀ Statistic |
|-------------|----------------|--------------------|-------------|--------------------------|
| Pressure | 10378 | 4 | 2,595 | 38.72 |
| Temperature | 5557 | 4 | 1,389 | 20.73 |
| Interaction | 1072 | 16 | 67 | 2.45 |
| Error | 685 | 25 | 27 | |
| Total | 17692 | 49 | | |

b) Test of Hypothesis

| Parameter | Level of Significance $\alpha = 0.05$ | | Interpretation |
|-------------|---------------------------------------|--------------------------|--------------------|
| | F _{α, v1, v2} | F ₀ Statistic | |
| Pressure | 3.01 | 38.72 | Very Significant |
| Temperature | 3.01 | 20.73 | Very Significant |
| Interaction | 2.07 | 2.45 | Fairly Significant |

Fig. 7–ANOVA table from FluidStats Program

By design, the program allows the user to specify five (5) levels of significance (i.e. $\alpha = 0.01, 0.025, 0.05, 0.10$, and 0.25). After every run, the interpretation column displays either of three (3) pre-programmed categories of significance. Conditions for these categories are defined as follows:

- Very Significant: When the F_0 test statistic is greater than 1.5 times $f_{\alpha, v1, v2}$ calculated from the F-distribution table.
- Fairly Significant: When $f_{\alpha, v1, v2} < F_0 < 1.5 f_{\alpha, v1, v2}$
- Not Significant: When $f_{\alpha, v1, v2} > F_0$

For $\alpha = 0.05$, level of significance (i.e. $100(1 - \alpha) = 95\%$ confidence level), the main effects of temperature and pressure on the plastic viscosity of the fluid is very significant. Though, the interaction effect is fairly significant. When we choose a higher confidence level say 99% (or $\alpha = 0.01$) we notice that the interaction effect becomes less significant. This information is very necessary in deciding which parameter or effects should be included or ignored in adequately fitting a multiple linear regression.

Analysis of Residuals

Residuals from a factorial experiment play an important role in determining the model adequacy. The residual is calculated as the difference between each observation and the corresponding cell averages:

$$e_{ijk} = y_{ijk} - \bar{y}_{ij} \dots \dots \dots (17)$$

Recall \bar{y}_{ij} has been calculated earlier in **Table 1**. The residuals for both phases are given in **Table 4**

Table 4: Residuals of plastic viscosity data

| | | Temperature (°F) | | | | |
|-----------------|--------|------------------|----------|----------|----------|----------|
| | | 300 | 350 | 400 | 450 | 500 |
| Pressure (psig) | 10,000 | -3.5/3.5 | -5/5 | -4/4 | -2/2 | -1/1 |
| | 15,000 | -6/6 | -5/5 | -5.5/5.5 | -4.5/4.5 | -4/4 |
| | 20,000 | -5.5/5.5 | -3/3 | -2.5/2.5 | -4/4 | -4.5/4.5 |
| | 25,000 | -3/3 | -0.5/0.5 | -2.5/2.5 | -3/3 | -2.5/2.5 |
| | 30,000 | -3/3 | -1/1 | -4/4 | -4/4 | -1.5/1.5 |

The normal probability plot of these residuals is shown in **Fig. 8**. The tails do not exactly follow a straight line trend indicating some deviation from normality. There is noticeably a wave-pattern in the distribution of the residuals. However the deviation is not severe.

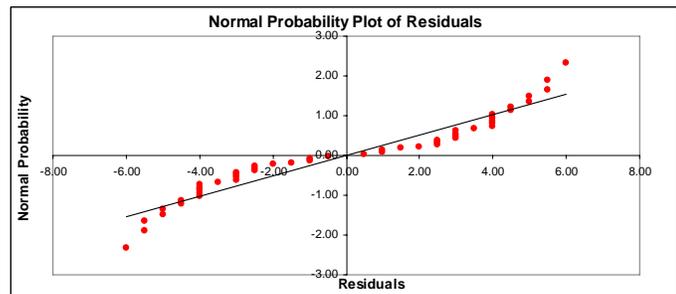


Fig. 8–Normal probability plot of residuals

Figs. 9, 10 and 11 plot the residuals versus pressure, temperature and predicted values respectively.

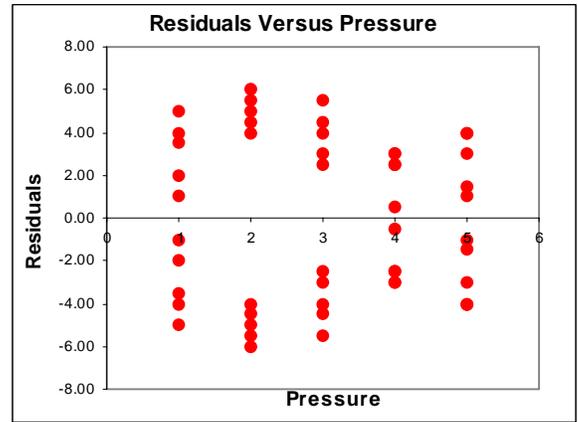


Fig. 9–Residuals versus pressure

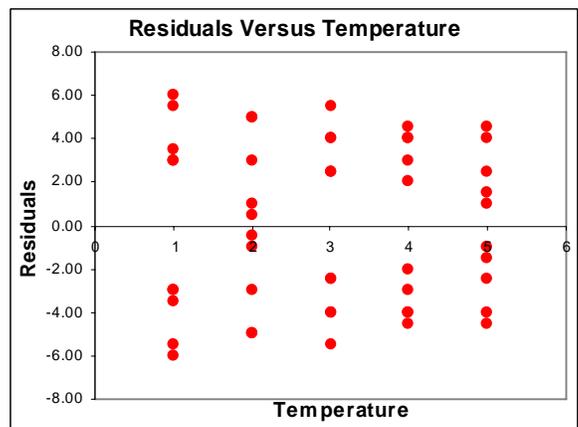


Fig. 10–Residuals versus temperature

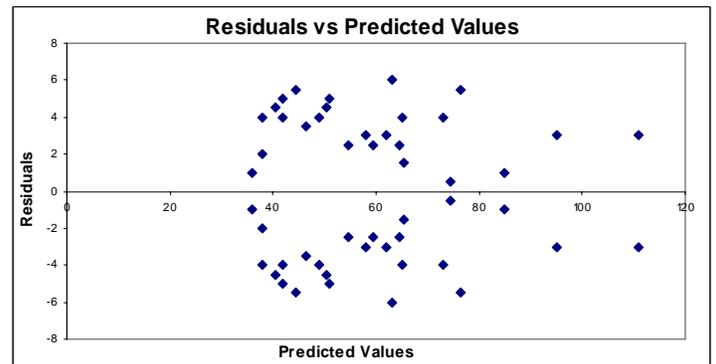


Fig. 11–Residuals versus predicted values

There is more variability in the data measured at 15,000 psig pressure and 300°F. Generally, the variation reduces with increase in temperature and pressure. Measurements taken independently at 25,000 psig and 450°F have the least variations.

Multiple Linear Regression Results

Four types of models (algebraic, logarithmic, exponential and polynomial) were considered in developing the multiple

regression code in the program. Generally the response (e.g. plastic viscosity or yield point in this case) was found to have a direct and inverse relationship with pressure and temperature respectively. This is in line with the concept discussed earlier. **Fig. 12** shows the results of the four models using the mean response from the factorial data sets. The predicted response is a mathematical expression involving pressure, temperature and interaction terms together with their coefficients and an intercept.

Thus the polynomial model is selected as the best because it has the lowest *MSE* and highest R^2 value. The model is written as:

$$PV = 9.11 + 0.25P^{0.5} + 6.54 \times 10^{20} T^{-8} + 1.48P^2 T^{-3} \dots \dots \dots (18)$$

Where plastic viscosity is in cP, pressure in psig and temperature in °F. Eq. 18 is valid for pressure and temperature ranging from 10,000 to 30,000 psig and 300 to 500°F respectively. To validate the results above, a separate run was made using SAS, a well-known commercial statistical analysis software.

Conclusions

The following conclusions can be made with regards to the results of the experiments contained in this report:

- An effective method for quantitatively estimating the effects of temperature and pressure on the rheological properties of drilling fluids has been developed.
- All experimental data suggest a linear relationship between pressure and viscosity while that of temperature is exponential.
- A truly representative polynomial model has been developed for Fluid Type-B using the *FluidStats* program. The model relates plastic viscosity to pressure and temperature from 10,000 to 30,000 psig and 300 to 500°F respectively.
- Proper evaluation of the pressure-temperature interaction effect on the rheology of a fluid is vital to achieving a good model fit.
- The effect of pressure is strongest at low temperatures.
- The effects of temperature on viscosity of the oil-based fluids have been observed to be dominant at higher pressures. (>20,000 psig) while pressure effects prevail at lower temperatures (<350°F).
- An active factor is more influential in determining the responses in a factorial experiment. For instance when pressure is increased at constant temperature, the viscosity obtained is higher than that at constant pressure and varying temperature.

Other XHP/HT fluid types (WBM, synthetic fluids, formate brines e.t.c) and property variations (density, oil/water ratio, weighting agent, viscosifiers e.t.c) should be investigated using the methodology presented in this report.

Acknowledgments

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Nomenclature

ANOVA = Analysis of Variance
XHP/HT = Extreme High-Pressure, High Temperature

Greek Letters

μ_p, PV = plastic viscosity (cP)
 ρ = fluid density (ppg)
 τ_y = yield stress (dynes/cm²)
 γ = shear rate (sec⁻¹)
 θ = angular deviation (deg)
 θ_{600} = 600RPM dial reading
 θ_{300} = 300RPM dial reading

Statistical Parameters

A = factor A, Pressure
B = factor B, Temperature
a = number of levels of factor A
b = number of levels of factor B
n = number of data replicates
k = number of regressor variables
p = number of parameters ($p = k + 1$)
 μ = sample mean
 σ^2 = variance
 τ_i = effects of factor A at various *i* levels
 β_j = effects of factor B at various *j* levels
 β = vector of regression coefficients
 $\hat{\beta}$ = least squares estimate of β
 $(\tau\beta)_{ij}$ = interaction effects of factors A and B
 ϵ_{ijk} = random error component
 ϵ = random error vector
 e_{ijk} = residual of factorial data
 e_i = residual of fitted model
 R^2 = coefficient of multiple determination
 R^2_{adj} = adjusted R^2
 X = matrix of the levels of independent variables
 y_{ijk} = individual observation (response)
 $y_{i..}$ = total of the observations taken at the *i*th level of factor A (pressure)
 $y_{.j.}$ = total of the observations taken at the *j*th level of factor B (temperature)
 $y_{ij.}$ = total of the observations in the *ij*th cell
 $\bar{y}_{i..}$ = row factor averages

| | |
|--------------------|---|
| $\bar{y}_{.j}$ | = column factor averages |
| $y_{...}$ | = sum of all observations |
| $\bar{y}_{...}$ | = average of all observations |
| \mathbf{y} | = response vector |
| $\hat{\mathbf{y}}$ | = least square estimate of \mathbf{y} |
| SS_A | = sum of squares for the row factor A |
| SS_B | = sum of squares for the row factor B |
| SS_{AB} | = sum of squares for interaction between A and B |
| SS_E | = error sum of squares |
| SS_R | = regression sum of squares |
| MS_A | = mean square for the row factor A |
| MS_B | = mean square for the row factor B |
| MS_{AB} | = mean square for the interaction between A and B |
| MS_E | = error mean squares |
| F_A | = F-ratio for the row factor A |
| F_B | = F-ratio square for the row factor B |
| F_{AB} | = F-ratio for the interaction between A and B |
| F_0 | = F-statistic |
| H_0 | = null hypothesis |
| H_1 | = alternative hypothesis |

Subscripts

| | |
|----|-----------------------------|
| A | = factor A, (Pressure) |
| B | = factor B, (Temperature) |
| AB | = factors A & B interaction |
| i | = range from 1 to a |
| j | = range from 1 to b |
| k | = range from 1 to n |

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XHP/HT FluidStats Program (by Chijioke Ibeh)



Note: The mean response is used in model calculations

Multiple Linear Regression

| Model Type | Response | Pressure PSI | Units | Temperature °F | P & T Interaction | Intercept |
|-------------|----------|-------------------------------|-------|------------------------------|----------------------|--|
| Algebraic | Y = | 0.00085 P | + | 6199.006 1/T | + | 1.563668 P ² /T ³ + 13.10479 |
| Logarithmic | Y = | 31.0081 LogP | + | -26.8738 LogT | + | 1.71343 P ² /T ³ + -17.9218 |
| Exponential | Y = | 0.40837 e ^{P/10,000} | + | -119.781 e ^{-T/100} | + | 243.2141 P/T ² + 24.8282 |
| Polynomial | Y = | 0.24998 P ^{0.50} | + | 6.54E+20 T ⁻⁸ | + | 1.479906 P ² /T ³ + 9.10748 |

| MSE | R ² | Adj. R ² |
|------|----------------|---------------------|
| 6.64 | 0.983595 | 0.981251 |
| 6.99 | 0.982736 | 0.980270 |
| 8.02 | 0.980206 | 0.977378 |
| 3.41 | 0.991587 | 0.990385 |

Model with lowest MSE and highest R² value is the best.

Calculate

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Fig. 12–Multiple linear regression results