

Power Law Model Hydraulic Calculations Can Be Made More Accurate (Part II)

Ricardo A. Jardiolin, Rick Alford, and Tommy Rush, Newpark Drilling Fluids

Copyright 2013, AADE

This paper was prepared for presentation at the 2013 AADE National Technical Conference and Exhibition held at the Cox Convention Center, Oklahoma City, OK, February 26-27, 2013. This conference was sponsored by the American Association of Drilling Engineers. The information presented in this paper does not reflect any position, claim or endorsement made or implied by the American Association of Drilling Engineers, their officers or members. Questions concerning the content of this paper should be directed to the individual(s) listed as author(s) of this work.

Abstract

This is the second of two corresponding papers intended to establish a standardized methodology for drilling fluid hydraulic calculations. The first paper, "AADE-13-FTCE-03: Power Law Model Hydraulic Calculations Can Be Made More Accurate (Part I)", validated the improved accuracy and simplicity of using the rheogram graph in conjunction with the Power Law model to calculate shear rate and shear stress of drilling fluid flow regimes.

The objective of this paper is to compare the hydraulic calculations of effective viscosity, Reynolds number, and critical Reynolds number using the Power Law model discussed in Part I. The equations of effective viscosity and Reynolds number were derived using the previous standard method of derivation. The final equations differ slightly from those the industry has used extensively in the past. In this discussion, the authors will outline the process of derivation and compare the results.

Introduction

The Reynolds number is a mathematical expression that indicates the flow regime (laminar, transition, or turbulent) of circulated drilling fluid at any specific annular section of the wellbore. The effective viscosity is one of the parameters used to determine the Reynolds number. The Critical Reynolds number is the value determined where the regime changes from laminar to transition (or turbulent if transition is disregarded). Table 1 details the methods for verifying the correct critical velocity.

All these hydraulics parameters are functions of the "n" of the Power Law model. However, as the various Power Law models calculate several different "n" values, the results likewise will differ widely. This variance is detailed in Tables 2 and 3, while Tables 4, 5, 6 and 7 compare the different calculated results (effective viscosity, Reynolds Number, critical velocity). Thus, standardization of the "n" calculation would deliver a unified result.

This paper will present the basic derivations and calculations of effective viscosity and Reynolds number applying the two closest rpm points available from the viscometer reading, as explained by R. Jardiolin, et al¹. To maintain both continuity with Part I and the ease of calculations, the variables used in the equations were those expressed in the original paper with respect to viscometer dial

reading and rpm. Therefore, the final equations will differ to some extent from those the industry historically has used.

Later in this presentation, the critical velocity equation will be derived allowing this vital hydraulic parameter to be expressed in a more simplified form. Moreover, a method of calculating the critical velocity will be explained. The methodology to be presented also relies on the two closest rpm points available from the viscometer reading (shear stress and shear rate). These are shown in Fig. 1.

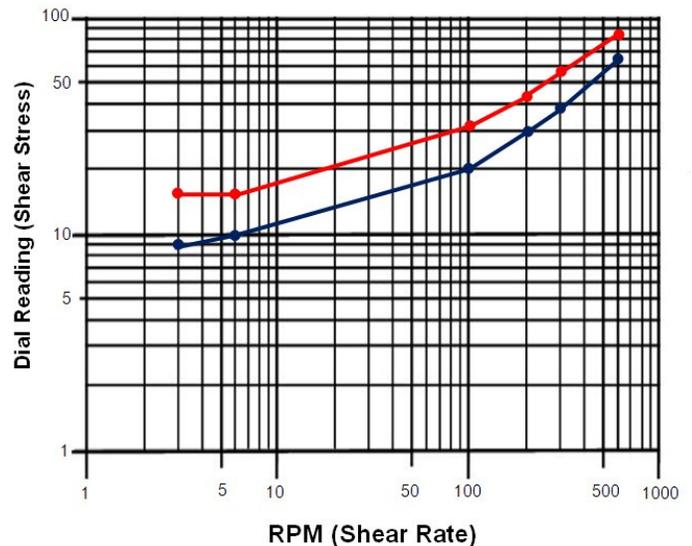


Fig. 1 – Two Rheogram Plots of Viscometer Readings

Power Law Model

The general equation of Power Law Model using the direct reading parameters of the viscometer is, expressed as,

$$\theta = K x (rpm)^n \quad (1)$$

Expressed in engineering units,

$$\tau = (k) x (\gamma)^n \quad (2)$$

Where:

- θ = dial reading (shear stress) of the viscometer
- rpm = rpm (shear rate) of the viscometer, rpm
- K = consistency factor
- τ = shear stress, lb/100 ft²

- γ = shear rate, 1/sec
 k = consistency factor, lb-secⁿ/100 ft²
 n = flow behavior index based from the two closest points of rpm available from the viscometer

Effective Viscosity

The effective viscosity is the ratio of shear stress to shear rate is expressed as,

$$\mu_e'' = \left(\frac{\theta}{rpm} \right) \quad (3)$$

or

$$\mu_e' = \left(\frac{\tau}{\gamma} \right) \quad (4)$$

Where:

- μ_e'' = effective viscosity, dial reading per rpm
 μ_e' = effective viscosity, lb-sec/100 ft²
 θ = dial reading (shear stress) of the viscometer
 rpm = rpm (shear rate) of the viscometer, rpm
 τ = shear stress, lb/100 ft²
 γ = shear rate, 1/secⁿ

The shear stress and shear rate expressed in engineering units,

$$\tau = \theta \times 1.067 \quad (5)$$

$$\gamma = rpm \times 1.703 \quad (6)$$

Substituting τ of equation (5) and γ of equation (6) to μ_e' and μ_e'' of equation (4), the effective viscosity is expressed as,

$$\mu_e = 300 \times \left(\frac{\theta}{rpm} \right) \quad (7)$$

Where:

- μ_e = effective viscosity, cPs
 θ = dial reading (shear stress) of the viscometer
 rpm = rpm (shear rate) of the viscometer, revolution per minute

The annular shear rate (rpm)^[2,3,4] in rpm of the viscometer that matches the annular drilling fluid velocity is expressed as,

$$rpm_a = \frac{1.41 \times Va}{D_h - D_p} \left(\frac{2n+1}{3n} \right) \quad (8)$$

Where:

- rpm_a = annular shear rate (viscometer rpm), revolutions per minute
 Va = average annular velocity at a certain section of the hole, feet per minute
 n = flow behavior index of Power Law fluids, between two closest rpm points
 D_h = diameter of the hole, inches

D_p = outside diameter of the pipe, inches

As discussed in the initial paper the rpm calculation of equation (8) is based on the two closest rpm points available from viscometer readings, as shown in Fig. 2. Since the two closest points have not yet been determined, it is impossible to calculate the “n” of the Power Law. Thus, the process detailed in Part I will be used here to determine the “n” of the Power Law model prior to calculating rpm of equation (8). An example calculation presented later in this paper will add clarity to this process.

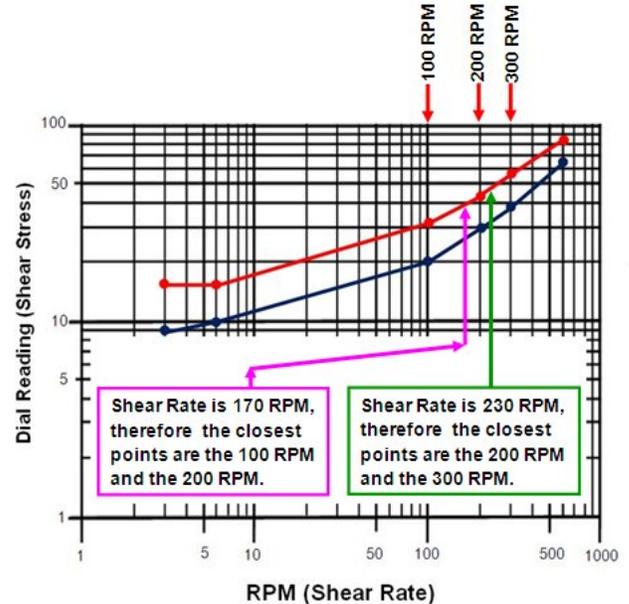


Fig. 2 – Two Rheogram Plots with Two Closest Shear Rates

Thus, based on the two closest points of viscometer as shown in Fig. 2,

Let:

- θ_1 = minimum dial reading (shear stress) of the viscometer based on two closest (chosen) points available from viscometer (e.g., dial reading at 100 rpm if shear rate is 170 rpm. See Fig. 2)
 θ_2 = maximum dial reading (shear stress) of the viscometer based on two closest (chosen) points available from the viscometer (e.g., dial reading at 200 rpm if shear rate is 170 rpm. Fig. 2)
 rpm_1 = minimum rpm of the two closest (chosen) pair of rpm available from the viscometer, corresponding to θ_1 above (e.g., 100 rpm if shear rate is 170 rpm. Fig. 2)
 rpm_2 = maximum rpm (shear stress) of the viscometer based on two closest points, corresponding to θ_2 above (e.g., 200 rpm if shear rate is 170 rpm (Fig. 2).

θ_a = dial reading at the annular section corresponding to rpm_a
 rpm_a = shear rate (viscometer rpm) at the annular section, revolutions per minute calculated using equation (8) based on two closest points (use procedure outlined in Part I)

Apply θ_a and rpm_a to equation (1),

$$\theta_a = K \times (rpm_a)^n \quad (9)$$

Applying also θ_1 , rpm_1 and θ_2 , rpm_2 to equation (1),

$$\theta_1 = K \times (rpm_1)^n \quad (10)$$

$$\theta_2 = K \times (rpm_2)^n \quad (11)$$

Dividing equation (9) by equation (10) and simplifying,

$$\theta_a = \theta_1 \times \left(\frac{rpm_a}{rpm_1}\right)^n \quad (12)$$

Another expression is dividing equation (9) by equation (11) and simplifying,

$$\theta_a = \theta_2 \times \left(\frac{rpm_a}{rpm_2}\right)^n \quad (13)$$

Equation (12) and equation (13) should have the same result.

The effective viscosity at rpm_a and θ_a of equation (7),

$$\mu_e = 300 \times \left(\frac{\theta_a}{rpm_a}\right) \quad (14)$$

Substituting θ_a of equation (13) to θ_a of equation (14) to produce equation (15).

$$\mu_e = 300 \times \left[\frac{\theta_2 \times (rpm_a)^{n-1}}{(rpm_2)^n}\right] \quad (15)$$

or

Substituting θ_a of equation (12) to θ_a of equation (14) to produce equation (16).

$$\mu_e = 300 \times \left[\frac{\theta_1 \times (rpm_a)^{n-1}}{(rpm_1)^n}\right] \quad (16)$$

Equation (15) and equation (16) should have the same result.

Reynolds Number

$$R_n = \frac{15.467 \times (D_h - D_p) \times \rho \times V_a}{\mu_e} \quad (17)$$

Where:

- R_n = Reynolds number
- D_h = diameter of the hole, in.
- D_p = outside diameter of the pipe, in.
- ρ = fluid density, lb/gal
- V_a = fluid annular velocity, ft/min
- μ_e = effective viscosity between two closest rpm points, cPs. Either equation (15) or equation (16) will be applied
- n = flow behavior index of Power Law fluids, between two closest rpm points

Substituting rpm_a of equation (8) with rpm_a in equation (15) is the effective viscosity μ_e . The effective viscosity μ_e can then be substituted for μ_e of equation (17), which becomes equation (18).

$$R_n = \frac{[rpm_2 \times (D_h - D_p)]^n \times \rho \times (V_a)^{2-n}}{19.39 \times \theta_2 \times \alpha} \quad (18)$$

Where:

- R_n = Reynolds number
- D_h = diameter of the hole, in.
- D_p = outside diameter of the pipe, in.
- rpm_2 = the maximum rpm of the two closest (chosen) pair of rpm available from viscometer
- θ_2 = dial reading of rpm_2
- ρ = fluid density, lb/gal
- n = flow behavior index of Power Law fluids, between two closest rpm points
- V_a = fluid annular velocity, ft/min
- $\alpha = \left[\frac{(1.41)(2n+1)}{3n}\right]^{n-1}$

Following is the procedure for determining the Reynolds number, equation (18):

1. Determine the n , rpm_a , and the two closest rpm points using the procedure presented in the first paper.
2. Use the θ_2 and rpm_2 of the 2 closest rpm points from step (1).
3. Finally, calculate the Reynolds number using equation (18).

An example problem and a comparison of effective viscosity, Reynolds number, and critical Reynolds number calculations for different values of "n" will be shown later.

Critical Velocity and Critical Flow Rate

The critical Reynolds number is the standard reference for determining the turbulent flow regime. The ensuing calculations will be conservative and consider the transition flow regime as a turbulent flow regime. Myriad factors warrant the use of conservative calculations, including the fact that fluid leaves the bit jets already turbulent, rotation of the bit and pipe, irregularity of the hole, eccentricity, the transfer of temperature from the formation to the fluid at the annulus and the heat dissipated by the drill bit, among others.

The calculation is expressed in Equation 19,

$$R_{nc} = 3470 - 1370 (n)^{[2.5]} \quad (19)$$

The annular velocity calculated from the critical Reynolds number is defined as the critical velocity. The type of flow regime in the annulus also can be determined by comparing the annular velocity to the critical annular velocity. Thus, if the annular velocity is less than the critical annular velocity, the flow regime is laminar. Otherwise, if the transitional flow regime is disregarded, the flow regime is turbulent.

Equation 20 gives the critical Reynolds number and solves explicitly for critical velocity,

$$V_c = \left[\frac{19.39 \times \theta_2 \times \alpha \times R_{nc}}{[rpm_2 \times (D_h - D_p)]^n \times \rho} \right]^{\frac{1}{2-n}} \quad (20)$$

Given the V_c , Equation (21) is the equivalent critical flow rate of drilling fluids passing through a certain annular section. If the actual flow rate is less than the critical flow rate, the flow regime is laminar. Otherwise, if the transition flow regime is disregarded, it is considered turbulent flow.

$$Q_c = \frac{(D_h^2 - D_p^2) \times V_c}{24.51} \quad (21)$$

Where:

- V_c = critical velocity, ft/min
- Q_c = critical flow rate, gpm
- D_h = diameter of the hole, in.
- D_p = outside diameter of the pipe, in.
- R_{nc} = equation (19)
- rpm_2 = The maximum rpm of the two (chosen) closest pair of rpm available from viscometer
- n = flow behavior index of Power Law fluids, between two closest rpm points
- ρ = fluid density, lb/gal
- α = $\left[\frac{(1.41)(2n+1)}{3n} \right]^{n-1}$

As with the conclusions in “AADE-13-FTCE-03: Power Law Model Hydraulic Calculations Can Be Made More Accurate (Part I)”¹, the critical velocity from equation (20)

cannot be solved directly as the “ n ” of the Power Law model is not yet known.

The very first step is to determine the approximate critical velocity by using the 200 and 300 rpm as the two closest rpm points using the following equation (22). Afterwards, the two closest points of shear rate (rpm) chosen will be checked to confirm these points are correct. If the points are deemed inaccurate, a correction will be made later.

Approximate Critical Velocity as the Starting Value

$$V_c \text{ approx} = \left[\frac{19.39 \times \theta_{300} \times \alpha \times R_{nc}}{[300 \times (D_h - D_p)]^n \times \rho} \right]^{\frac{1}{2-n}} \quad (22)$$

Where:

$$n = 5.679 \times \log \left(\frac{\theta_{300}}{\theta_{200}} \right) \quad (23)$$

The remainder of the nomenclature is identical to those expressed in equation (20).

The “ n ” of the Power Law Model Based on Two Closest rpm Points

$$n = \frac{\log \left(\frac{\theta_2}{\theta_1} \right)}{\log \left(\frac{rpm_2}{rpm_1} \right)} \quad (24)$$

Where:

- θ_1 = minimum dial reading of the viscometer based on two closest (chosen) points available from viscometer
- θ_2 = maximum dial reading of the viscometer based on two closest (chosen) points available from viscometer
- rpm_1 = minimum rpm of the two closest (chosen) pair of rpm available from viscometer
- rpm_2 = maximum rpm of the two closest (chosen) pair of rpm available from viscometer
- n = flow behavior index of Power Law fluids, between two closest rpm points

Table 1 – Method of Verifying the Correct Critical Velocity

Tier	Method
A	<ol style="list-style-type: none"> Calculate the “n” using equation (23). Determine the $V_c \text{ Approx}$ using equation (22). Calculate the rpm_a using equation (8). Use $V_c \text{ approx}$ for V_a. If the rpm_a of step (2) is between the 200 rpm and 300 rpm, the $V_c \text{ Approx}$ of step (2) is correct. Therefore, the $V_c \text{ approx}$ is the critical velocity. The verification process ends here. If incorrect, proceed to step (4).

B	<p>4. Choose the next two closest points based from the rpm_a of step (2).</p> <p>5. Calculate the new “n” using equation (24).</p> <p>6. Calculate the V_c of equation (20).</p> <p>7. Use V_c for V_a, calculate the rpm_a using equation (8).</p> <p>8. If the rpm_a is between the two closest points of step (4), the critical velocity V_c is correct. <i>The verification process ends here. If not correct, repeat step (4).</i></p> <p>9. <i>In rare cases, the loop may not end. In such cases, proceed to step (10).</i></p>
C	<p>10. Take note of the two pairs of closest points chosen previously.</p> <p>11. Choose the minimum point of the lower rpm pair as rpm_1 and θ_1.</p> <p>12. Choose the maximum point of the higher rpm pair as rpm_2 and θ_2.</p> <p>13. Calculate the new “n” using equation (24).</p> <p>14. Calculate the final V_c of equation (20).</p>

A subsequent Example Problem will more clearly illustrated this verification method.

Example for Calculating the Reynolds Number

Given Data:

ρ (mud wt), lb/gal	=	12.00
D_h , in	=	8.50
D_p , in	=	6.00
Annular velocity, fpm	=	330
θ_{600}	=	83
θ_{300}	=	54
θ_{200}	=	42
θ_{100}	=	31
θ_6	=	16
θ_3	=	15

Problem:

- Determine the Reynolds number.

Solution:

Apply the method described in “AADE-13-FTCE-03: Power Law Model Hydraulic Calculations Can Be Made More Accurate (Part I)”¹.

The approximated annular shear rate (rpm_{Aprx}) using equation (3) of Part I is,

$$rpm_{apr} = \frac{1.61 \times 330}{8.5 - 6.0}$$

$$rpm_{apr} = 254.2$$

The two closest points of the viscometer reading are 300 rpm and 200 rpm (to be confirmed).

$$rpm_1 = 200, \theta_1 = 42$$

$$rpm_2 = 300, \theta_2 = 54$$

Verifying the correct two closest shear rates (rpm), reference Tier A, Step (1) of *Table 1—Verification Method of Two Closest Points* - Using equation (6) of Part 1,

$$n = \frac{\log\left(\frac{\theta_2}{\theta_1}\right)}{\log\left(\frac{rpm_2}{rpm_1}\right)}$$

$$n = \frac{\log\left(\frac{54}{42}\right)}{\log\left(\frac{300}{200}\right)}$$

$$n = 0.619$$

Then, referencing Tier A, Step (2) of *Table—Verification Method of Two Closest Points*, using equation (2) of Part I,

$$rpm_a = \frac{1.41 \times 330}{8.5 - 6.0} \left(\frac{2 \times 0.619 + 1}{3 \times 0.619} \right)$$

$rpm_a = 224$, which obviously is between 200 and 300 rpm of viscometer reading. Therefore the previously chosen two closest points were correct (true) and the verification process is concluded.

$$n = 0.62 @ 224 \text{ rpm}$$

Continuing with the solution for Reynolds number, Equation (18),

$$\alpha = \left[\frac{(1.41)(2n+1)}{3n} \right]^{n-1}$$

$$\alpha = \left[\frac{(1.41)(2 \times 0.62+1)}{3 \times 0.62} \right]^{0.62-1}$$

$$\alpha = 0.818$$

$$R_n = \frac{[RPM_2 \times (D_h - D_p)]^n \times \rho \times (V_a)^{2-n}}{19.39 \times \theta_2 \times \alpha}$$

$$R_n = \frac{[300 \times (8.5 - 6.0)]^{0.62} \times 12.0 \times (330)^{2-0.62}}{19.39 \times 54 \times 0.818}$$

$$R_n = 2,538$$

Table 2 – Comparison of Different Reynolds Number (Given Data)

Properties	Data A	Data B
ρ (mud wt), lbs/gal	12.0	11.0
D_h , in	8.50	10.0
D_p , in	6.00	6.5
Annular Vel, fpm	330	200
θ_{600}	83	45
θ_{300}	54	32
θ_{200}	42	26
θ_{100}	31	19
θ_6	16	8
θ_3	15	7
$\theta_{init} = (2x\theta_3 - \theta_6)$	14	6

Table 3 – Comparison of Different Rheology Models (Index of Power Law Model “n”)

Rheological Models	Data A “n”	Data B “n”
Power Law model (300 rpm & 3 rpm)	0.278	0.330
Power Law model (100 rpm & 3 rpm)	0.207	0.285
Power Law model (2 Closest Pts, 300 rpm & 200 rpm)	0.620	0.453
Herschel-Bulkley (300 rpm, 100 rpm, & θ_{init})	0.779	0.631
Herschel-Bulkley (600 rpm, 300 rpm, & θ_{init})	0.787	0.585

Table 4 – Comparison of Shear Rate and Effective Viscosity Using Different “n” of Data A

Rheological Models	Data A *Shear rate (rpm) at Annular Section	Data A **Effective Viscosity (cPs) at Annular Section	Data A “n”
Power Law model (300 rpm & 3 rpm)	347	49	0.278
Power Law model (100 rpm & 3 rpm)	424	30	0.207
Power Law model (2 Closest Pts, 300 rpm & 200 rpm)	224	60	0.619
Herschel-Bulkley (300 rpm, 100 rpm, & θ_{init})	204	59	0.779
Herschel-Bulkley (600 rpm, 300 rpm, & θ_{init})	203	52	0.787
*Equation (8) **Equation (15)			

Table 5 – Comparison of Reynolds Number Using Different “n” of Data A

Rheological Models	Data A *Reynolds Number	Data A **Critical Reynolds Number	Data A “n”
Power Law model (300 rpm & 3 rpm)	3,150	3,089	0.278
Power Law model (100 rpm & 3 rpm)	5,174	3,186	0.207
Power Law model (2 Closest Pts, 300 rpm & 200 rpm)	2,538	2,621	0.619
Herschel-Bulkley (300 rpm, 100 rpm, & θ_{init})	2,603	2,403	0.779
Herschel-Bulkley (600 rpm, 300 rpm, & θ_{init})	2,928	2,392	0.787
*Equation (18) **Equation (19)			

Table 6 – Comparison of Shear Rate and Effective Viscosity Using Different “n” of Data B

Rheological Models	Data B *Shear Rate (rpm) at Annular Section	Data B **Effective Viscosity (cPs) at Annular Section	Data B “n”
Power Law model (300 rpm & 3 rpm)	155	50	0.330
Power Law model (100 rpm & 3 rpm)	170	39	0.285
Power Law model (2 Closest Pts, 300 rpm & 200 rpm)	130	49	0.453
Herschel-Bulkley (300 rpm, 100 rpm, & θ_{init})	111	46	0.631
Herschel-Bulkley (600 rpm, 300 rpm, & θ_{init})	115	45	0.585
*Equation (8) **Equation (15)			

Table 7 – Comparison of Reynolds Number Using Different “n” of Data B

Rheological Models	Data B *Reynolds Number	Data B **Critical Reynolds Number	Data B “n”
Power Law model (300 rpm & 3 rpm)	2,754	3,018	0.330
Power Law model (100 rpm & 3 rpm)	3,515	3,080	0.285
Power Law model (2 Closest Pts, 300 rpm & 200 rpm)	2,774	2,850	0.453
Herschel-Bulkley (300 rpm, 100 rpm, & θ_{init})	3,963	2,606	0.631
Herschel-Bulkley (600 rpm, 300 rpm, & θ_{init})	3,062	2,669	0.585
*Used Equation (18)			
**Used Equation (19)			

Example of Calculating the Critical Velocity and Flowrate

Given Data:

ρ (mud wt), lb/gal	=	12.00
D_h , in	=	8.50
D_p , in	=	6.00
Annular velocity, fpm	=	330
θ_{600}	=	83
θ_{300}	=	54
θ_{200}	=	42
θ_{100}	=	31
θ_6	=	16
θ_3	=	15

Problem:

Determine the critical velocity and the critical flow rate.

Solution:

Determine the approximate critical velocity using the 200 and the 300 rpm shear rates.

$$rpm_1 = 200, \theta_1 = 42$$

$$rpm_2 = 300, \theta_2 = 54$$

$$n = \frac{\log\left(\frac{\theta_2}{\theta_1}\right)}{\log\left(\frac{rpm_2}{rpm_1}\right)}$$

$$n = \frac{\log\left(\frac{54}{42}\right)}{\log\left(\frac{300}{200}\right)}$$

$$n = 0.62$$

$$\alpha = \left[\frac{(1.41)(2n+1)}{3n} \right]^{n-1}$$

$$\alpha = \left[\frac{(1.41)(2 \times 0.62+1)}{3 \times 0.62} \right]^{0.62-1}$$

$$\alpha = 0.8177$$

Using equation (19),

$$R_{nc} = 3470 - 1370(n)$$

$$R_{nc} = 3470 - 1370(0.62)$$

$$R_{nc} = 2,621$$

Using equation (22),

$$V_c \text{ approx} = \left[\frac{19.39 \times \theta_{300} \times \alpha \times R_{nc}}{[300 \times (D_h - D_p)]^n \times \rho} \right]^{\frac{1}{2-n}}$$

$$V_c \text{ approx} =$$

$$\left[\frac{19.39 \times 54 \times 0.8177 \times 2,621}{[300 \times (8.5 - 6.0)]^{0.62} \times 12.0} \right]^{\frac{1}{2-0.62}}$$

$$V_c \text{ approx} = 337.0$$

Verify the approximate critical velocity, using $V_c \text{ approx}$ for V_a and equation (8),

$$rpm_a = \frac{1.41 \times V_a \left(\frac{2n+1}{3n} \right)}{D_h - D_p}$$

$$rpm_a = \frac{1.41 \times 337.0 \left(\frac{2 \times 0.62+1}{3 \times 0.62} \right)}{8.5 - 6.0}$$

$rpm_a = 229.0$, which is between 200 and 300 rpm. Thus, the two closest rpm points are correct. The approximated critical velocity is the critical velocity. Critical velocity therefore is,

$V_c = 337.0$ feet/minute, which is greater than fluid annular velocity (330ft/min).

The critical flow rate, using equation (21),

$$Q_c = \frac{(D_h^2 - D_p^2) \times V_c}{24.51}$$

$$Q_c = \frac{(8.5^2 - 6.0^2) \times 337.0}{24.51}$$

$$Q_c = 498.0 \text{ gal/min}$$

Conclusion

It can be concluded that using the method presented in this paper will make the Power Law model more accurate, since it is based on the two closest rpm points. The paper reinforces the industry need for standardization of hydraulic calculations^[6]. Furthermore, it also delivers consistency to the basic calculations for effective viscosity, Reynolds number, critical Reynolds number, critical velocity, and critical flow rate.

Acknowledgments

The authors thank the management of Newpark Drilling Fluids for permission to publish this paper. Additional thanks to Steve Daniel, Lindsay Fraser, Robert Meyer, David Breeden, John Shelton, Christy Schuepbach, Jené Rockwood, and the Newpark laboratory personnel in Houston, TX.

References

1. Ricardo Jardiolin, Rick Alford, and Tommy Rush, "Power Law Model Hydraulic Calculations Can Be Made More Accurate (Part I)", *AADE Technical Conference 2013*, Feb. 26, 2013.
2. API Recommended Practice 13D, Sixth Edition, May 2010.
3. Moore Preston, *Drilling Practices Manual PennWell Publishing Company*, 2nd Edition 1986.
4. Gene Forbes, Annular Pressure Loss Calculation, *Oil and Gas Journal*, 1972.
5. Schuh, F.J., "Computer makes surge-pressure calculations useful," *Oil and Gas Journal*, Aug. 3, 1965
6. Mario Zamora and David Power, *Making a Case for Hydraulics and Unified Rheological Model*, *AADE Technical Conference 2002*.