

## Mathematical Model for Pressure Transmission in Drilling Fluids

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### Abstract

This work puts forward a mathematical model for pressure transmission taking place in drilling fluids pressurized within closed drillpipe-wellbore geometries. The drilling fluid motion is considered as one-dimensional, isothermal and weakly compressible. The governing equations of continuity and momentum balance constitute the model and are solved iteratively by the method of characteristics. The drilling fluids are assumed to behave as Bingham fluids and the friction factor approach is used to account for the viscous effect. The results are validated against experimental data for water and two drilling fluids. Both the magnitudes and the oscillation frequencies of measured and computed values agree quite well for either water or the drilling fluids. In contrast with water, both measured and computed values of pressures for drilling fluids do not stabilize uniformly along the well after compression. In other words, pressure is not fully transmitted because of the strength of the yield stress of viscoplastic fluids. The sooner the pressure gradient along the pipe is unable to overcome the yield stress, the fluid stops moving and the pressure stabilizes.

### Introduction

Pressure transmission in drilling fluids eventually presents abnormal behavior which can be detrimental to several operations. In some situations, pressure fluctuations in bottomhole conditions are not immediately reflected in surface due to non-Newtonian behavior of drilling fluids. This issue can cause delays in influx detection generating safety problems. Additionally, pressure imposed at the surface may, in some conditions, be not fully transmitted to the bottom of the hole. In some situations, such as in use of hydraulically actuated completion valves, an expensive fluid substitution process may be required to guarantee pressure transmission.

Although important the pressure transmission in viscoplastic fluids has not received much attention and investigations should be carried out. Some works found in the literature have been dedicated to model flow start-ups of viscoplastic and even thixotropic materials (Sestak et al., 1987, Cawkwell and Charles, 1987, Chang et al., 1999 and Davidson et al., 2004, Oliveira et al. 2010, Negrão et al., 2011, Vinay et al., 2006, Vinay et al., 2007, Wachs et al., 2009). In flow start-ups, the fluid is pumped at the inlet and the pressure travels along the pipe length before the fluid begins to flow.

Despite the pressure transmission that takes place in flow start-ups none of above works has addressed the described problem, as the pipe outlet is completely open.

Water hammer is another case of pressure transmission that has been extensively studied (Ghidaoui et al., 2005, Bergant et al., 2006 and Martin, 1983). In such case, a valve is suddenly closed at a pipeline end causing pressure oscillations due to wave propagation. However, none of the above reviews (Ghidaoui et al., 2005, Bergant et al., 2006) reported that such problem was studied previously with a viscoplastic fluid.

Only recently, Oliveira et al. (2012) have studied numerically the pressure transmission problem of a Bingham fluid in a closed pipe. Differently from Newtonian fluids, they showed that a constant pressure imposed at one pipe end filled with viscoplastic fluid is not fully transmitted to the other pipe end because of the fluid yield stress. As soon as the pressure gradient along the pipe is not high enough to surpass the yield stress the fluid stops moving and the pressure does not propagate anymore. They also concluded that the final pressure distribution along the pipe depends not only on the Bingham number but also on the relation between the pipe aspect ratio, Reynolds and Mach numbers. In contrast with real applications where constant flow rates are imposed at pipe inlets, Oliveira et al. (2012) studied a problem with constant pressure established at the inlet. Besides, they have never corroborated their results with measurements.

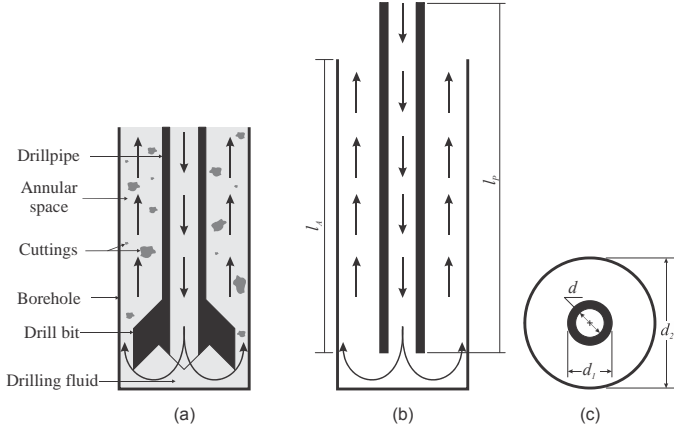
The current work puts forward a mathematical model to simulate the pressure transmission in drilling fluid pumped throughout a closed well. Differently from the prior work of Oliveira et al. (2012), the present study considers constant flow rates as inlet boundary condition and the model results are compared with pressure values measured in an experimental plant.

### Mathematical Model

#### Problem Description

Fig. 1a depicts a schematic representation of the drilling operation at the wellbore in which a drilling fluid is pumped into the drillpipe and then returns to the wellhead through the annular space. In the current study, the fluid is pushed into the drillpipe but the annulus is closed so that the fluid is displaced in the domain but is not allowed to flow. Despite the changes, the cross sectional areas of the drillpipe and annular space are

taken to be constant, as shown in Fig. 1b. According to Fig. 1c, the drillpipe internal diameter is identified as  $d$  and the internal and external diameters of the annular space, as  $d_1$  and  $d_2$ , respectively. As the drill bit was removed and the region below the drillpipe is disregarded in the modeling, the fluid is displaced directly from the drillpipe to the annular space. The fluid motion in both drillpipe and annular space is considered to be compressible, isothermal and one-dimensional in the axial direction. Besides, the drillpipe and the well structure are regarded as completely rigid. The shear stress at the drillpipe and annular space walls is evaluated by using the friction factor approach and the drilling fluid is modeled as a Bingham fluid.



**Fig. 1. (a) Illustration of the drilling fluid circulation in the wellbore. (b) Longitudinal view and (c) cross section of drillpipe and annular space.**

### Governing Equations

By applying the above hypotheses, the mass and momentum balance equations can be written for either the drillpipe or annular space as:

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial v}{\partial z} + v \frac{\partial \rho}{\partial z} = 0 \quad (1)$$

$$\rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial z} - \frac{4}{d_h} \tau_w + \rho g \quad (2)$$

where  $v$  is the average velocity across the geometry cross section,  $\rho$  is the fluid density,  $p$ , the pressure,  $g$  is the gravity acceleration,  $\tau_w$ , the average value of the shear stress at the pipe or annular space wall,  $d_h$  is the hydraulic diameter of the drillpipe ( $d$ ) or annular space ( $d_1-d_2$ ),  $t$ , time and  $z$ , the axial coordinate. A constant isothermal compressibility is assumed for the fluid, which is defined according to Anderson (1990) as:

$$\alpha = \frac{1}{\rho} \left. \frac{\partial \rho}{\partial p} \right|_{\theta} = \frac{1}{\rho c^2} \quad (3)$$

where  $c$  is the pressure wave speed and  $\theta$  is the temperature.

Similar to fully developed flows, a linear variation of shear stress is admitted across the pipe sectional area (Oliveira et al., 2012) and therefore,  $\tau_w$  can be computed by using the Fanning friction factor approach:

$$\tau_w = \frac{f \rho v^2}{2} \quad (4)$$

As discussed by Oliveira et al. (2010), the non-linear terms of the momentum and mass balance equations can be disregarded for weakly compressible flows, such as those that take place in drilling fluids. As the drillpipe and annular space average velocities are different,  $v$  is replaced by the volumetric flow rate ( $=q/A$ ) in Eq. (1) and (2). Finally, Eq. (3) is substituted into Eq. (1) and  $\tau_w$  is replaced by Eq. (4) in Eq. (2), resulting in the following:

$$\frac{\partial p}{\partial t} + \frac{\rho c^2}{A} \frac{\partial q}{\partial z} = 0 \quad (5)$$

$$\frac{1}{A} \frac{\partial q}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{2f|q|}{A^2 d_h} - g = 0 \quad (6)$$

The friction factor depends not only on the fluid properties but also on the geometry of the domain. For laminar flows of Bingham fluids, the Fanning friction factor for both drillpipe and annular space can be written as (Melrose et al., 1958):

$$f_{lam} = \frac{16\zeta}{\gamma Re_{z,t}} \quad (7)$$

where  $\zeta$  is geometric parameter which is 1.0 for pipe flows and is a function of  $d_1/d_2$  for annular flows (see White, 2002),  $\gamma$  is the fluid conductance of the Bingham fluid and  $Re_{z,t}$  is the Reynolds number that is a function of time and axial position ( $Re_{z,t} = \rho v d_h / \eta$ ) and  $\eta$  is the fluid plastic viscosity. Eqs. (8) and (9) are the correlations used to calculate the fluid conductance for the pipe and annular space, respectively, which was obtained from Melrose et al., (1958):

$$\gamma_P = 1 - \frac{\gamma_P B_{z,t}}{6} + \frac{1}{3} \left( \frac{\gamma_P B_{z,t}}{8} \right)^4 \quad (8)$$

$$\gamma_A = 1 - \frac{\gamma_A B_{z,t}}{8} + \frac{1}{2} \left( \frac{\gamma_A B_{z,t}}{12} \right)^3 \quad (9)$$

where  $B_{z,t}$  is the Bingham number ( $B_{z,t} = \tau_y d_h / v \eta$ ) which depends on time and space.  $\tau_y$  is the fluid yield stress and the subscripts  $P$  and  $A$  are the drillpipe and annular space, respectively. Eq. (7) can be reduced to the Newtonian friction factor if the conductance is made equal to one.

According to Melrose et al. (1958), the Bingham fluid flow can be considered to be laminar if  $\gamma Re_{z,t} \leq 2100$ , otherwise is turbulent. For drilling fluid cases shown in the result section,

only laminar flows are investigated. On the other hand, both laminar and turbulent cases are considered for water flows. The turbulent Newtonian flow is admitted to take place if  $Re_{z,t} > 2100$  and the turbulent friction factor adopted is that proposed by Blasius (White, 2002):

$$f_{turb} = 0.079 Re_{z,t}^{-0.25}, \quad Re_{z,t} > 2000 \quad (10)$$

### Solution Algorithm

The governing equations are solved by the method of characteristics (MOC) that is typically used for solving first-order partial differential equations (see Wylie et al., 1993). In comparison to the ordinary finite volume method that is usually employed to solve this kind of equations, the number of grid points that provides accurate results can be considerably smaller. The method consists of simplifying partial differential equations to a family of ordinary differential equations along which the solution can be integrated from some initial data. In the current case, Eqs. (5) and (6) are reduced to two total differential equations which are valid over the characteristic lines. Both drillpipe and annular space are divided into  $N$  equal reaches,  $\Delta z$ , and the total differential equations are discretized according to the finite difference method. The solution of the problem begins with the fluid standing still at time zero, so that  $p$  and  $q$  are known at each grid point for  $t=0$ . The solution consists of finding  $p$  and  $q$  for each grid point along  $t=\Delta t$ , then proceeding to  $t=2\Delta t$ , etc., until the desired time duration has been covered. A grid of 328 points was found to provide accurate results and then it was used for all simulations.

### Description of the Experimental Rig

Experiments conducted by PETROBRAS in an experimental drilling rig are used to validate the current model. The rig consists of a drillpipe with internal and external diameters of 76 mm and 88.9 mm (3.5"), respectively, and a well case of 157 mm of internal diameter, as shows Fig. 2. Two fiber Bragg grating sensors were placed within the drillpipe and six in the annular space to measure pressure and temperature. The results of only three sensors (S1, S2 and S3), as indicated in Fig. 2, are used in the comparisons. The first was located within the drillpipe and the last two in the annular space at 29 m (S1), 1192 m (S2) and 14.64 m (S3) below the wellhead, respectively.

A set of tests was carried out by PETROBRAS to evaluate the pressure transmission in drilling fluids. For such purpose, the drillpipe was assembled without the drill bit and its end was placed very close to the well bottom so that both the drillpipe and the annular space were approximately 1192 m long. The fluid is pumped through the drillpipe and a valve keeps the annular space closed. The vertical arrows shown in Fig. 2 indicate the direction of the fluid motion. The circulation system was connected to the wellhead through a high pressure pipe to control the pump flow rate and the maximum pressure at the pump outlet. The wellhead maximum pressure was established according to the fluid

weight, as the bottomhole pressure cannot exceed 27.6 MPa (4000 psi). To measure the data, an acquisition system compatible with the fiber Bragg technology was used with an acquisition rate of 1.0 Hz.

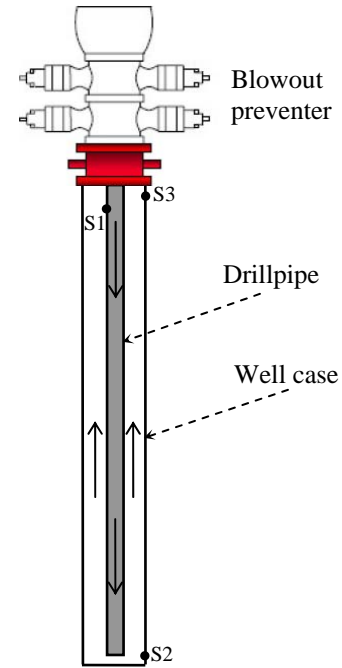


Fig. 2. Schematic representation of the experimental rig.

The tests were performed according to the following: i) the well was first filled with water or drilling fluid and the annulus was closed; ii) the fluid was then pumped into the well with a constant flow rate until the bottom hole pressure reached 27.6 MPa (4000 psi) (maximum working pressure); iii) the pumped was then turned off and the well was kept closed until the pressure stabilized; iv) finally, the well was depressurized by opening the choke.

The tests were run for several fluids but only three were used in the comparison: water and two drilling fluids (fluid A and B). Fluid A is lighter and less viscous in comparison to fluid B. The rheology properties of fluids A and B were obtained by a Fann 35A viscometer and fit to the Bingham model. The wave speeds of the fluids were estimated by rating the distance between two pressure sensors and the time for pressure to be transmitted from the first to the second sensor. The water properties were obtained from the literature (White, 2003). The plastic viscosity (viscosity for water), the yield stress, the wave speed and the density of the fluids are shown in Table 1.

### Model Setup

As an initial condition, the fluid is considered to stand still within the domain (drillpipe and annular space) and consequently, the volumetric flow rate is zero throughout the domain,  $q(z,t=0)=0$ . The initial pressure condition is considered to be the hydrostatic one. Once the choke is

maintained closed during the test, the flow rate is set to zero as the outlet boundary condition,  $q(z=l_p+l_d, t=0)=0$ . Inasmuch as the fluid is injected into a closed domain, the pressure within the well increases continuously until a pre-defined pressure value,  $p_{set}$ , is reached and the pump is turned off. To accomplish this situation, the following boundary condition is established at the inlet:

**Table 1. Fluid properties.**

Property	Water	Fluid A	Fluid B
$\eta$ [Pa.s]	0.001	0.0235	0.0677
$\tau_y$ [Pa]	0	2.166	10.88
$c$ [m/s]	1350	1000	1011
$\rho$ [kg/m <sup>3</sup> ]	1000	1150	1929

$$q(z=0, t) = \begin{cases} q_{in} & \text{if } t \leq t_{set} \\ 0 & \text{if } t > t_{set} \end{cases} \quad (11)$$

where  $t_{set}$  is the time for the inlet pressure to reach  $p_{set}$ . As the pump flow rate was not precisely measured and the experimental tests were mainly conducted to evaluate the pressure transmission after the pump shut down, the boundary value of the pump flow rate was set to match the calculated and measured pressures at the first sensor (S1).

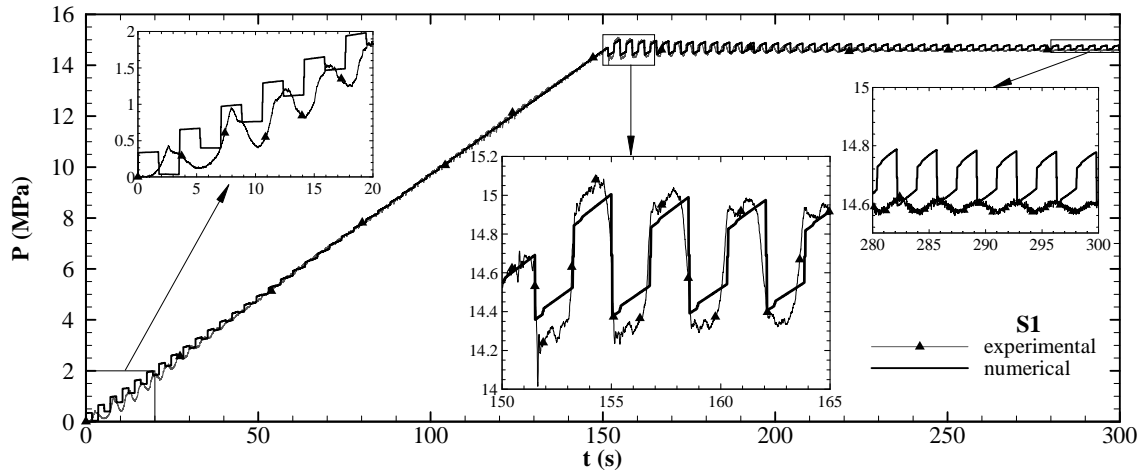
### Model Validation

Three set of experimental data were chosen to be

compared with the current model results. The first test was carried out with water, whereas the other two were performed with two different drilling fluids (fluids A and B).

### Water

For the water case, the pump was turned on with a constant flow rate of 1.122 l/s (0.42 bpm) and was switched off when the pressure at well head was 14.69 MPa (2130 psi). Figs. 3 and 4 depict comparisons of computed and measured values of pressure in the first (located at the drillpipe inlet – S1) and in the third sensor (placed closed to the choke – S3), respectively. The good agreement during the first 150s in Fig. 3 cannot be taken into account, as the flow rate was imposed to make the measured and computed pressures as close as possible. After the pump has been turned off and the flow rate was made equal to zero, the agreement remained reasonable as the measured and computed values oscillate at the same frequency and with similar amplitudes. It is worth noting that the oscillation is due to pressure reflections at the domain boundaries and that the frequency coincides with the wave speed divided by the length of the domain. However, the experimental and numerical values start to diverge soon after the pump shut-off, as the oscillation of the measured pressure dissipates much faster than the computed counterpart. Similarly, the pressure agreement in Fig. 4 is quite reasonable during the pressure rising and soon after the pump shut-off. Despite the similar oscillations, the measured values still dissipate faster than the calculated ones.



**Fig. 3. Time evolution of measured and computed values of pressure at position S1 for a water flow rate of 1.122 l/s.**

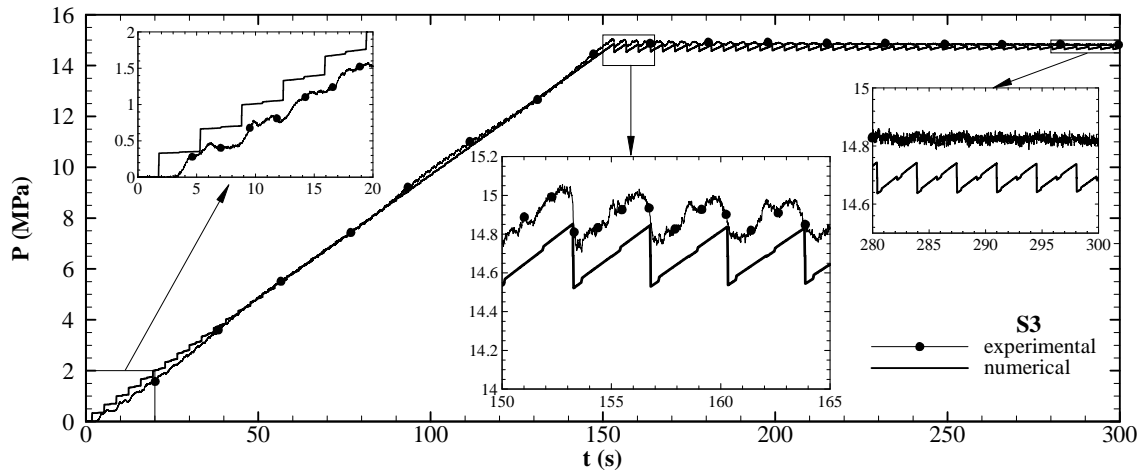


Fig. 4. Time evolution of measured and computed values of pressure at position S3 for a water flow rate of 1.122 l/s.

### Drilling Fluids

Similar tests were conducted for two drilling fluids, fluids A and B. For fluid A, the flow rate was set to the constant value of 1.462 l/s (0.55bpm) until the pressure at wellhead reached 13.82 MPa (2004 psi) and then the pump was switched off. Figs. 5a, 5b and 5c show the time evolution of measured and computed values of pressure for sensors S1, S2 and S3. Similar to the previous case, measured and computed values agree quite well. However, both measured and computed values do not oscillate as much as the pressures for the water case and this can be attributed to the fluid yield stress. As soon as the pump is turned off, the pressure starts to reflect back and forth at the boundaries and the pressure gradient begins to reduce within the domain. Whenever the magnitude of the pressure gradient is not high enough to exceed the yield stress, the pressure does not reflect anymore and then stabilizes. This issue was further discussed in a previous paper (Oliveira et al., 2012).

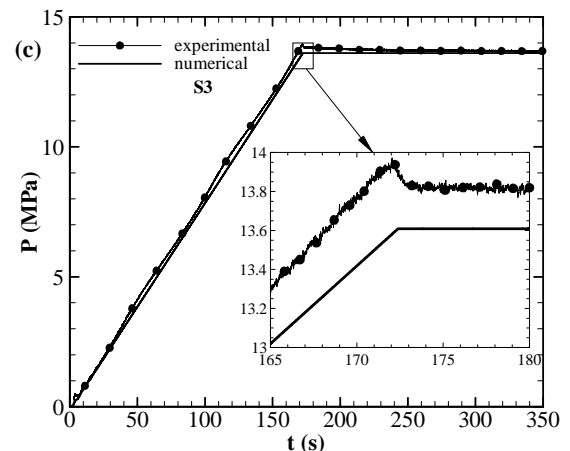
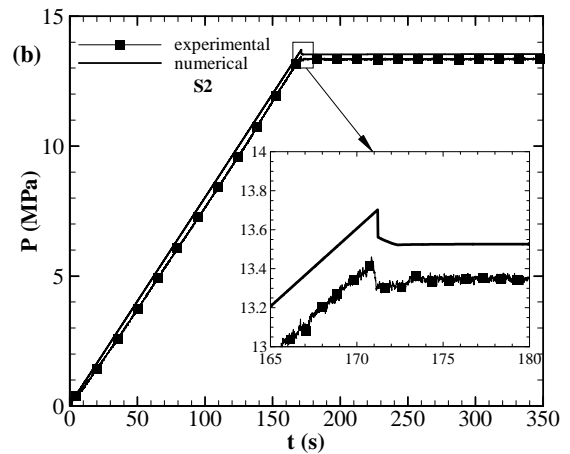
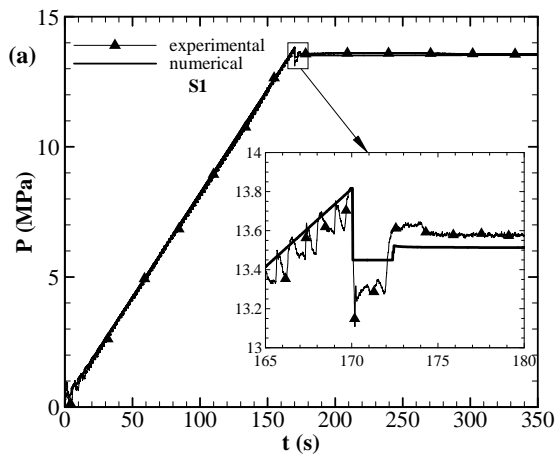


Fig. 5. Time evolution of measured and computed values of pressure for fluid A at (a) S1, (b) S2 and (c) S3 positions.

Although the measured and computed curves have similar responses the computed values stabilize slightly first. Besides, the pressure peak observed in the measured value is not seen in the computed counterpart in sensor S3. Differently from the Newtonian fluid whose pressure is uniform along the whole

domain after the equilibrium (if the hydrostatic pressure is subtracted), the Bingham fluid pressure stabilizes in different values at each domain position. One can see that the stable value at the outlet (sensor S3) is larger than the inlet one (sensor S1), for both measured and computed curves (see Oliveira et al., 2012 for further explanations).

The fluid B results are now discussed. The flow rate was set to the constant value of 0.952 l/s (0.359bpm) and the pump was turned off when the well head pressure reached 7.6 MPa (1102 psi). Comparisons of the measured and computed values are shown in Fig. 6. Not only the measured and computed

values are very similar again, but also they do not oscillate, as noted in fluid A results. The S1 and S2 pressures reduce suddenly after the peak and then stabilize. Despite the deviation of the measured and computed pressures during the pump-up period, their values at S3 position respond very similarly, as they both reach a maximum and then become constant. The complete absence of oscillation in this case is due to a higher yield stress of this drilling fluid which dissipates the pressure wave faster in comparison to the previous case.

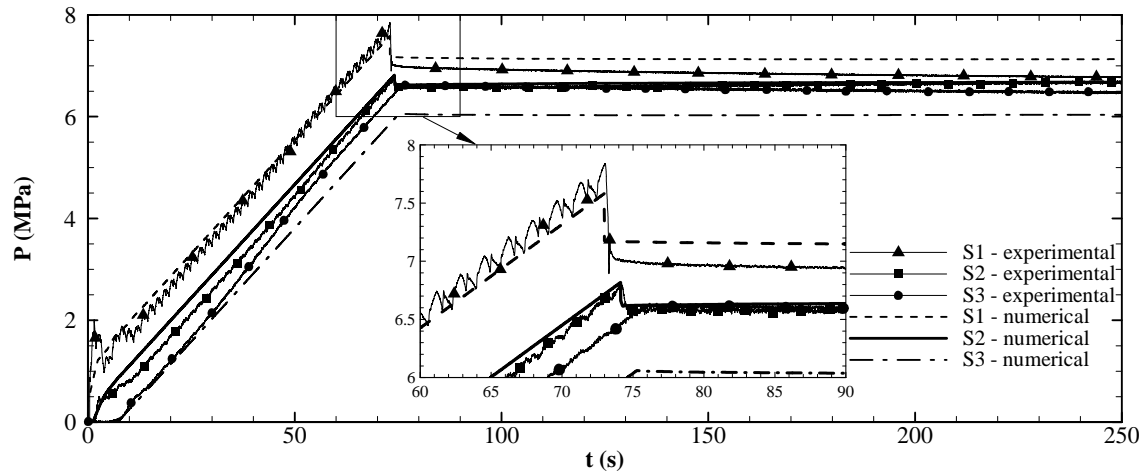


Fig. 6. Time evolution of measured and computed values of pressure for fluid B at positions S1, S2 and S3.

## Conclusions

The current work presents a mathematical model to predict pressure transmission when a drilling fluid is pumped through a closed well. The model is based on the mass and momentum balance equations applied to a compressible, transient and one-dimensional fluid motion. The drilling fluid is modeled as a Bingham fluid and the viscous effect is taken into account by employing the friction factor approach. The governing equations are solved by using the method of characteristics and the results are compared to measured data obtained for water and two drilling fluids. The tests were performed in an experimental rig in which the fluid was pumped into a closed well until the pressure reached a pre-defined value and then the pump was turned off for pressure stabilization. Good agreement between measured and computed values was found. For the water case, not only the measured and computed pressures increase at the same rate but also they oscillate at the same frequency soon after the pump shut-off. After that, the measured and computed values start to diverge as the measured oscillations dissipate faster than the computed counterparts. For Bingham fluids, nevertheless, the comparison is even better as the measured and computed values do not oscillate much. Besides, it was also found that both measured and computed oscillations diminish with the increase of the fluid yield stress. As indicated by Oliveira et al. (2012), the computed results show that the fluid yield stress is

the parameter that damps the oscillations. As soon as the pressure gradient is not high enough to overcome the fluid yield stress, the oscillation stops and the pressure stabilizes within the domain. In fact, the final pressure distribution is not uniform as noted in Newtonian fluids and a pressure gradient is observed.

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