Understanding 3D Curves

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AADE Mid-Continent Symposium
What is a 3D Curve?

**Three Dimensional Curves**
- Building and turning at the same time
- Turning while at an inclination between 0° and 90°
- Designing a directional plan to be steered between 0° and 180° GTF
- Also known as a Build & Turn or Turnizontal

**Applies for all doglegs except:**
1. Only build & drop (no azimuth changes)
2. Azimuth only changes at 90° inclination
3D Curve Benefits – Why drill them?

- Allows lower inclinations by removing the drop to vertical
- Enables larger displacements with less TVD
- Provides less rod wear during production

### 2D Curve – drop to vertical before KOP

<table>
<thead>
<tr>
<th>MD (ft)</th>
<th>INC (°)</th>
<th>AZI (°)</th>
<th>TVD (ft)</th>
<th>NS (ft)</th>
<th>EW (ft)</th>
<th>VS (ft)</th>
<th>DLS (°/100ft)</th>
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</tbody>
</table>

### 3D Curve – hold inclination through KOP

<table>
<thead>
<tr>
<th>MD (ft)</th>
<th>INC (°)</th>
<th>AZI (°)</th>
<th>TVD (ft)</th>
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<th>EW (ft)</th>
<th>VS (ft)</th>
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<td>1225.00</td>
<td>10065.82</td>
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</table>
3D Curve Benefits – Why drill them?

- Enhances buckling resistance at KOP

\[ F_{\text{crit}} = 2 \sqrt{\frac{EIpAg \sin \theta}{r}} \]

- As \( \theta \) increases, compressive load at which buckling occurs increases
- Example: 8.75” hole, 5” 19.5# DP

<table>
<thead>
<tr>
<th>Inclination (deg)</th>
<th>( F_{\text{crit}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1°</td>
<td>16,184 lbs</td>
</tr>
<tr>
<td>5°</td>
<td>36,168 lbs</td>
</tr>
<tr>
<td>10°</td>
<td>51,052 lbs</td>
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<tr>
<td>15°</td>
<td>62,327 lbs</td>
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</tbody>
</table>

Equation from SPE-11167-PA
3D Curve Detriments – Why NOT drill them?

1. It may not be needed to get to the desired location
2. It can be difficult to hold an inclination deeper in the well
3. The net forces on the motor change affecting motor yields
4. They are just more difficult to understand

![Diagram showing 3D vs 2D curves and their implications](image-url)
3D Curve Case Study

T Cosner 27-1H
• Curve built on 10°/100 ft
• Kickoff Point started at 13° inclination, 270° azimuth (west)
• Landing Point was at 89.6° inclination, 180° azimuth (south)
• 76.6° of build, 90° of turn
• Hardline to the west, can’t abandon our turn in the curve

Tripped in the curve at 51° inclination in the curve for build rates
T Cosner 27-1H Dogleg Severity

Measured Depth vs Dogleg Severity (deg/100 ft)

- **Kick Off Point**
- **Landing Point**
- **Tripped for Build Rates**

Graph showing measured depth along the x-axis and dogleg severity along the y-axis. The graph includes two lines: one for "Survey Station DLS" and another for "DLS Needed to Land."
T Cosner 27-1H Dogleg Severity

Why are the DLS needed to land increasing if we’re achieving higher DLS than needed?
DLS needed to land was increasing because we were not sliding in the best direction.
T Cosner 27-1H Toolface

How do we know what toolface to hold in the middle of a 3D curve?
Review 2D Curves

**Curve Calculations**

- **Build Rates Needed to Land** (linear interpolation)
  \[
  BRN = \frac{INC_{LP} - INC_{POI}}{MD_{LP} - MD_{POI}} \times 100
  \]

- **Gravity Toolface**
  \[
  GTF = \cos^{-1}\left(\frac{BR}{DLS}\right)
  \]

  Build Rate = DLS  \(\therefore\) GTF = 0°

  This is constant throughout the curve

- **Radius of Curvature**
  \[
  ROC = \frac{18,000}{\pi \times DLS}
  \]
Differences for 3D Curves

What’s different?

• Radius of curvature circle rests in a slice of a sphere
• We use DLS needed to land, not BUR needed to land
Differences for 3D Curves

Curve Calculations
- Build Rates Needed to Land
  - Turning is harder at higher inclinations
  - “Get your turn in first”

\[ BRN = ?? \]

- Gravity Toolface

\[ GTF = \cos^{-1} \left( \frac{BR}{DLS} \right) \]

\[ GTF = ?? \]

What toolface should be held in the middle of a 3D Curve?

- Radius of Curvature

\[ ROC = \frac{18,000}{\pi \times DLS} \]
Differences for 3D Curves

Primary problem is that 3D curves drill on a rotated plane

- 2D Curves lie flat on two standard axes: TVD & Vertical Section
- 3D Curves still lie on a flat plane, but it is rotated between our three standard axes
- Converting an arc of a sphere to vertical and horizontal components is difficult to grasp
Differences for 3D Curves

Every 3D curve can be redrawn as a 2D curve
1. Define Points 1 & 2 as vectors (1: KOP, 2: Landing Point)
   \[ \mathbf{u}_1 = \langle \sin l_1 \cos A_1, \sin l_1 \sin A_1, \cos l_1 \rangle \]
   \[ \mathbf{u}_2 = \langle \sin l_2 \cos A_2, \sin l_2 \sin A_2, \cos l_2 \rangle \]

2. Define Normal Vector to the 2D Plane
   \[ \mathbf{N} = \mathbf{u}_1 \times \mathbf{u}_2 \]

3. Define Radius Vector from Point 1 to the Center of the Sphere
   \[ \mathbf{R} = \mathbf{N} \times \mathbf{u}_1 \]

4. Solve for position in terms of \( x' \) and \( y' \) in 2D

\[ \langle x', y' \rangle \]
Solution

5. **Project $x'$ and $y'$ back to EW, NS, TVD cartesian coordinates**
   - Extend the scalar $x'$ value in the direction of the $R$ vector
   - Extend the scalar $y'$ value in the direction of the $u_1$ vector
   
   \[
   \begin{align*}
   < x', y' > & \rightarrow < u_{EW}, u_{NS}, u_{TVD} > \\
   < x', y' > & \rightarrow \\
   \Delta EW &= x = x' \cdot R_x + y' \cdot u_{1x} \\
   \Delta NS &= y = x' \cdot R_y + y' \cdot u_{1y} \\
   \Delta TVD &= z = x' \cdot R_z + y' \cdot u_{1z}
   \end{align*}
   \]

6. **Calculate MD, INC, AZI from EW, NS, TVD**
7. **Calculate DLS, BUR, TR, GTF from MD, INC, AZI**
Examples – 2D Curve

90° of Build, 0° of Turn
INC: 0° to 90°
AZI: 0° to 0°
Examples – Large Turn and Curve

75° of Build, 90° of Turn
INC: 15° to 90°
AZI: 0° to 90°
Examples – Small Turn and Curve

75° of Build, 30° of Turn
INC: 15° to 90°
AZI: 60° to 90°
Examples – Large turn at some Inclination

0° of Build, 70° of Turn
INC: 20° to 20°
AZI: 90° to 160°
Rules of Thumb

How can I get the 80% answer without remembering Calculus 3?
• Get all of the planned turn in by 30° inclination
• Plan curves with more motor yield capacity than needed

When does this really matter?
• When turning more than 30° azimuth throughout a curve
• When turning at high inclinations (> 10°)

How can I see this result without working this solution?
• Export directional plan in 5-25 ft intervals instead of typical 100 ft intervals
• Use the gravity toolface equation:

\[ GTF = \cos^{-1}\left(\frac{BR}{DLS}\right) \]
Thank you.
Incorrect Linear Interpolation on 3D Curves

- Black: Incorrect Interpolation
- Red: Correct Interpolation