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# An Overview of Annular Displacement Efficiency in Cementing Jobs Using an Efficient Numerical Model

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#### **Abstract**

An important consideration in planning a cementing job is how to effectively remove drilling mud and minimize the cement slurry contamination. Good mud removal ensures good bonding between the cement and the casing or the formation, which leads to zonal isolation and wellbore longevity. Although this subject is highly concerned by the drilling industry, due to the complex nature of fluid dynamics and limitations of both experimental and theoretical research approaches, reliable and timely prediction of displacement efficiency before executing a cementing job remains a challenge. The influences of a variety of parameters and operation conditions are complicated, correlated and largely unknown. Guidelines are needed to assist engineers to select the correct parameters for job planning or evaluate the cement placement after a job.

This paper addresses the concept and calculation of displacement efficiency through a series of numerical simulations performed in a computer simulator developed from an efficient numerical model. It provides an overview of the displacement efficiency affected by various conditions, including fluid density, rheology, casing standoff, flow rate, casing rotation and reciprocation. Snapshots of end-of-job mud/cement concentration in the annulus are captured and displayed in various tables of matrixes, to show the impacts of combinations of the parameters. These case studies provide cement engineers with quick guidelines to avoid risks and optimize the displacement efficiency.

#### Introduction

In drilling and completion practice, the effectiveness of one fluid displacing another in a well is a high concern in both cementing and wellbore cleanup jobs. As an assessment of the effectiveness, displacement efficiency can be analyzed from several considerations such as length of mixed fluids, fluid channeling, mud cakes, fluid volume fraction and fluid contaminations (Dai and Liu, 2017). For cementing jobs, the goal is to place good (or uncontaminated) slurries to the planned depth as referred to Top of Cement (TOC) while minimizing mud channeling and mud cakes, to secure the bond; in the case that a cementing job is poorly done - the defect will be found either in the form of poor cement bonding or communication between zones - a squeeze cementing technique has to be performed to establish zonal

isolation (Farkas et al., 1999; Suman and Ellis, 1977), which is much more expensive than the primary cementing job. Therefore, improving mud displacement efficiency in a cementing job is very important for saving cost.

Annular displacement efficiency can be defined as the volume fraction of displacing fluid in the annulus. This is equivalent to the volume of displaced fluid removed from the annulus divided by the annulus volume:

$$E(t) = \frac{1 - V_n(t)}{V} \tag{1}$$

where  $V_n(t)$  is the native fluid remaining in the annulus, V is the annulus volume and E(t) is the efficiency. As fluid volumes are conserved, at the beginning of displacement, displacement efficiency is proportional to flow rate (Tehrani et al., 1993), as shown in Eqn. 2, where Q is flow rate and  $T_b$  represents breakthrough time.

$$E(t) = \frac{Qt}{V}, t \le T_b \tag{2}$$

After breakthrough, which is the time of first arrival of the interface at the top of the annulus, the efficiency continues to increase but with a reducing rate. The rate of change after breakthrough depends on various parameters. Maintaining a stable and flat interface between fluids will avoid early breakthrough therefore improve displacement efficiency.

When the displacement efficiency is concerned at local depths, it can be defined as an area fraction of displacing fluid,

$$E(x,t) = \frac{1 - A_n(x,t)}{A(x)}$$
 (3)

Where x, t are depth and time, respectively, and  $A_n$  is the area occupied by native fluid (displaced fluid), and A is the total area of cross section. The previously defined volumetric efficiency will be derived by integrating the area efficiency over the entire length of the annulus. For cementing jobs, as a measurement of job quality, volumetric efficiency may be examined at the annulus below TOC only at the end of job,

examined at the annulus below TOC only at the end of job,
$$E(t) = \frac{\int_{TD}^{TOC} E(x, t) A(x) dx}{V}$$
(4)

where V is the volume of the annular section from TOC to TD. It is well known that annular displacement efficiency is greatly affected by casing eccentricity (Siginer and Bakhtiyarov, 1998; Nelson et al, 2006), because eccentricity will introduce an azimuthally non-uniform axial velocity distribution in the cross section, therefore creating non-flat

fluid interface. It's a common practice to install centralizers on casing string to achieve better centralization and there are computer models developed for optimizing this placement (e.g., Liu et al, 2012). On the other hand, however a perfectly centered casing will not give a 100% displacement. This is due to a radially non-uniform velocity profile as fluid attaching to the casing wall and wellbore wall is subject to a no-slip boundary condition. Therefore, native mud tends to remain on the wall although bulk fluid is displaced. High viscosity non-Newtonian fluid such as one with high yield point will be desired because it largely flats out the velocity profile along radial direction. This explains the benefit of selecting fluids of proper rheologies and compatibilities. Density contrast between fluids is also an important factor to consider (Clark et al, 1973), even in the situation of horizontal wells (Feng et al, 2013). Low density fluid displacing high density fluid in an annulus is usually avoided due to interface instability such as Rayleigh-Taylor instability (Rayleigh, 1882; Taylor, 1950). Nowadays density and rheology hierarchy and fluid compatibilities are common considerations for cementing job designs.

It becomes popular to rotate the casing string while pumping to improve efficiency, which is especially useful for high eccentric scenarios (Bittleston et al., 1991, Moroni et al. 2009). Similarly, reciprocation can also be used and it was studied numerically to show improvement on the efficiency (Bittleston et al, 1994). Today the industry is still lacking of sufficient understanding of the impacts of certain influence factors, even more mysterious are the combinations of them.

Cementing bond logs can provide detailed information about the cement bond, defect and fluid interfaces in post job However early prediction of the displacement analysis. efficiency is most wanted during design stage. Many experimental and mathematical studies were performed with the motivation of understanding the correlation of various contributing factors by varying parameters and flow conditions and establishing empirical or semi-empirical relations which may be used in job planning (Moran et al, 2007; Sairam et al, 2010). Due to the complex nature of the flow physics, numerically calculating the displacement efficiency is very difficult. Many numerical approaches were developed based on advanced CFD techniques to model the dynamic process of displacement (Bittleston et al., 2002; King et al, 2000; Ladva 2001; Enayatpour et al, 2017). Numerical simulators based on regular CFD techniques usually require high mesh and time resolutions to give reliable results which is computational expensive. The method we developed in this study is a quasi-static pseudo-3D approach which greatly speeds up the calculation and makes it possible to run a simulation on a single computer within minutes.

#### Modeling

#### Axial Velocity Profile

Theoretical solution of axial velocity profile is possible for Newtonian fluid assuming single fluid and steady laminar flow. However, it's getting much more complicated for Non-Newtonian fluid such as Herschel-Buckley fluid. Here only H-B fluid is considered as BP and PL models are considered special cases covered in this model. For H-B fluid the shear rate  $\gamma$  is related to shear stress  $\tau$  by

$$\tau = \tau_0 + k \gamma^n \tag{5}$$

Here a parallel-plate approximation is applied, which ignores the curvature of annular section and assumes the flow is between two parallel plates (Nelson et al, 2006). The derivatives in the azimuthal direction are ignored, so the velocity is solved locally at each of the azimuthal point independently. Assuming the channel gap width is  $h(\theta)$ , the pressure gradient along the flow direction with gravity considered is  $P=\frac{dp}{dz}-\rho\,g\,cos(\theta)$ , where  $\theta$  is the inclination angle of the channel. The velocity profile across the annular gap is found as:

 $u(y) = A \left(\frac{T(y)}{k}\right)^{\frac{1}{n}+1} + B \tag{6}$ 

where

$$T(y) = Py - \tau_o \tag{7}$$

$$A = -\frac{k}{P(\frac{1}{n} + 1)} \tag{8}$$

$$B = \frac{k}{P(\frac{1}{n} + 1)} \left( T\left(\frac{h}{2}\right) / k \right)^{\frac{1}{n} + 1} \tag{9}$$

Where y is the radial coordinate with the origin located the gap center, B is the center velocity at y = 0. T(y) was set to 0 when T(y) < 0.

#### Radial and Azimuthal Velocity

Radial velocity is ignored with the assumption that the annular gap is small. The azimuthal velocity however must be considered because it is responsible for traversal fluid transport and fluid redistribution over the cross section. The continuity equation becomes

$$\frac{\partial \mathbf{u}}{\partial \mathbf{z}} + \frac{1}{r} \frac{\partial \mathbf{v}}{\partial \mathbf{\theta}} = 0 \tag{10}$$

where u is the axial velocity and v is the azimuthal velocity. In finite volume method, its integral form is used in the control volume:

$$\int_{\partial V} \boldsymbol{v} \cdot \boldsymbol{n} dS = 0 \tag{11}$$
 Considering azimuthal velocity component is secondary

Considering azimuthal velocity component is secondary compared to axial velocity, this equation is used to solve azimuthal velocity after axial velocity is found in the previous step.

## **Pressure Profile**

In a quasi-static approach, one assumes the pressure is uniformly distributed over a cross section. With this assumption the pressure field becomes one dimensional and  $\frac{dp}{dz}$  is to be determined. Pressure is solved with an iterative procedure at each depth where the bulk flow rate is known. Firstly a  $\frac{dp}{dz}$  value is assumed and using Eqn. (6) to find axial

velocity at each azimuthal position. Integrate the velocities over the cross section to find the overall flow rate. Compare this rate with the target flow rate to update  $\frac{dp}{dz}$ . This procedure is repeated until the given rate is achieved.

## Interface Tracking

In a cement job a number of fluids are pumped into a wellbore in sequence. These fluids mix upon contact and form complex interfaces. To keep track of the interfaces between different immiscible phases in the flow, a modeling technique named Volume of Fluid methods is applied, in which volume fraction equation is solved for each fluid phase (Hirt et al, 1979). As all fluids are assumed incompressible, the differential form of the volume fraction equation for each phase based on mass conservation is written as

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s = 0 \tag{12}$$

Where v is the velocity vector,  $f_s$  is the volume fraction of the sth phase. The following constraint is applied in any control volume and any time,

$$\sum_{s}^{n} f_{s}(i,j,k) = 1, \quad 0 \le f_{s} \le 1$$
 To solve the VOF equation in finite volume method, the

integrated form is used,

$$\frac{\partial \int f_s dV}{\partial t} + \int_{\partial V} f_s \, \mathbf{v} \cdot \mathbf{n} dS \, = 0 \tag{14}$$

This equation is discretized in control volume. An explicit Euler scheme is used for time marching. Special treatment is required to suppress numerical diffusion with advection in axial direction to maintain implicit fluid interfaces.

## Mesh

Three dimensional structured mesh is created on which the velocity and VOF function are solved. Resolutions of each dimension (MD,  $\theta$  and r) can be specified separately. Azimuthal and radial mesh points are distributed by equal At depths where annular geometry thickness or angles. changes, it is assumed the flow always maintains parallel to the wall, i.e., radial component of the velocity is ignored. Velocities are defined at cell faces and volume fractions are defined at the cell center. Fig. 1 shows a typical mesh structure that is generated inside the simulator.

## **Model Validation**

This numerical model has been validated against experimental and numerical models in other works that have been published (Dai and Liu, 2017; Wang and Dai, 2018). Here we present additional case studies for validation of this model.

The first validation includes two actual cementing job designs simulated using current model and compared with a leading simulator. The first case is to cement a 9 5/8 in liner in an offshore well of 9394 ft (TVD=3291 ft). TOC is planned at 7400 ft. Limited number of centralizers are used and standoff is poor over the entire length. As seen in Fig. 2, Both

numerical model and the simulator shows significant mud channel at the narrow side. In the second case, a surface casing of 13-5/8 inch is to be cemented at shoe depth of 5443 ft (TVD=2821 ft). 13.5 ppg lead cement and 15.8 ppg tail cement are pumped to 1830 ft (TOC) in annulus. 67% standoff is reached below previous casing shoe and the casing was poorly centered above it. Two simulations give similar mud maps and show little mud below around 4000 ft and severe risk near the top.

In the second validation we compared the simulation results of modeling the displacement in a 3 m long annular section at a Reynolds number of 176 between the current model and the leading CFD software ANSYS. The annulus has an inner diameter of 40mm, and an outer diameter of 50mm, with an eccentricity of 0.5. Both fluids are 15.5 ppg in density and modeled as yield power-law model:  $\tau 0 = 1.22 \text{ Pa}$ , k = 0.197 Pa.sn, n = 0.505. Fig. 3 shows the effect of rotation speeds on the displacement efficiency computed from both the model and ANSYS. Both simulators predict similar values and trends, and reveal pipe rotation at relatively low rotation speed (20 rpm) leads to a slightly better efficiency in eccentric annuli compared to cases with higher rotation speeds.

## Case Study

In this study we consider a simple scenario of cementing a 9.625" casing string (ID=8.535") in a 12.25" open hole and run several groups of simulations using the current model to obtain an overview of annular displacement efficiency under varying parameters and operation conditions. The hole is vertical and has a measured depth of 10000 ft. The first two sets of simulations will study the effect of fluid density and rheology, with properties of native and displacing fluids shown in Table 1. In each case of them one annulus volume of displacing fluid was pumped into the annulus using the base operation conditions shown in Table 2. The other three sets of simulations consider 3-fluid system (mud, spacer and cement with properties also shown in Table 1) to study the effects of standoff, casing rotation and reciprocation and flow rate. There is a 60 bbl spacer pumped before one annulus volume of cement is pumped into the annulus. All simulations are run on the mesh of 100 divisions in MD, 30 divisions in radial and 30 in azimuthal directions.

Table 3 and 4 show the results of 2D distribution maps of displacing (green) and native fluids (brown); Table 5~7 show the maps of cement (grey), spacer (green) and mud (brown). In each map vertical axis is depth and horizontal axis is azimuth with the narrow side in the middle.

### Effect of Density

As discussed in previous section density contrast has a great impact on the displacement efficiency (Clark et al, 1973; Feng et al, 2013); this is also revealed in the simulation as shown in Table 3. When a low density fluid is used to displace a much heavier fluid, even with well centralized casing, there is an apparent mud strip remained in annulus, indicating that the light fluid was escaping through one side of the annular flow path as the ideal uniform velocity profile is easily broken

due to instability. It is also observed a positive density contrast with huge difference such as using a fluid of 18 ppg to displace another fluid of 9 ppg is not the most efficient, as some mud branches appear near the top.

#### Effect of YP

A broadly followed rule in cementing industry is to pump fluids in a sequence of increased viscosity to achieve good displacement efficiency. In this study only YP of the Bingham Plastic fluid is considered. As seen in Table 4, worse displacement is seen either in the form of mud strip (20 lbf/100ft² to displace 40 lbf/100ft²; or 40 lbf/100ft² to displace 60 lbf/100ft²) or mud texture (such as 0 lbf/100ft² to displace 20,40 and 60 lbf/100ft²). It's found among all these cases the one using 60 lbf/100ft² to displace 60 lbf/100ft² gives best result.

## Effect of Casing Rotation

Rotating the pipe while pumping is most beneficial in nonideal circumstances such as when casing string is poorly centralized. We examine the effect of changing rotating speed from 0 to 30 rpm versus standoff from 0 to 100% in Table 5. As expected applying a rotation of 30 rpm on a 0% standoff pipe markedly improves efficiency by entirely removing the mud channeling. Rotating a 100% centralized pipe to 30 rpm does not show much improvement compared to no rotation. Furthermore, higher rotating speed can possibly reduce efficiency under certain circumstances, for example 30 rpm at 70% standoff appears less efficient than lower rotating speeds. While the pipe rotates, the less viscous fluid at the narrow side will be brought to the wide side, causing an overall reduction of viscosity at the wide side, consequently lower shear force near the wall and more chance to have a mud layer stick to the wall at the wide side. This is one contrary mechanism while rotation is widely accepted as an efficiency enhancement operation.

#### Effect of Reciprocation

In this study the pipe is modeled to reciprocate with a frequency of 0.5, 1 and 2 strokes per minutes. The stroke length is 30 ft. The mean axial moving speed of the pipe will be 0.5, 1 and 2 ft/s. As shown in Table 6, for all the standoffs from 0 to 100%, a moderate improvement of efficiency is found as the reciprocation frequency increases, which is consistent with the study by Bittleston et. al (1991) who found overall efficiency is improved, although the down-stroke alone has an opposite contribution to reduce the efficiency.

#### Effect of Flow Rate

Effect of flow rate on displacement efficiency is known depending on flow pattern. Both a lower rate of laminar flow and a high rate of turbulent flow may be desired to enhance displacement effectiveness. As seen in Table 7, by varying flow rates from 4 to 12 bpm, the tops of good cement appear to be slightly raised to a lower depth, indicating efficiencies are improved, for all different standoffs from 0 to 100%,

especially at 100%. As the flow in the studied scenarios mostly falls into the turbulent regime, this conclusion agrees with the previous studies (Nelson et al, 2006).

#### **Tables**

Table 1. Fluid properties used in the simulations

<u>,                                      </u>	Density	Dhaalagu	PV	YP
	(ppg)	Rheology	(cP)	(lbf/100ft <sup>2</sup> )
mud	9	BP	15	0
spacer	10	BP	15	0
cement	12	BP	15	0
native	9,12,15,18	BP	15	0,20,40,60
displacing	9,12,15,18	BP	15	0,20,40,60

Table 2. Operation conditions

	Standoff (%)	Rotation (rpm)	Reciprocation (spm)	Flow rate (bpm)
Base	100	0	0	4
Varying	0,40,70,100	0,1,10,30	0,0.5,1,2	4,6,8,12

Table 3. Fluid distribution maps showing the effect of densities of both native and displacing fluids.

Native Displacing	9 ppg	12 ppg	15 ppg	18 ppg
Displacing	118	118	118	118
9 ppg				
12 ppg				
15 ppg	A CONTRACTOR OF THE PARTY OF TH			
18 ppg	A STATE OF THE PARTY OF THE PAR	Contract (		

Table 4. Fluid distribution maps showing effect of YP of both native and displacing fluids

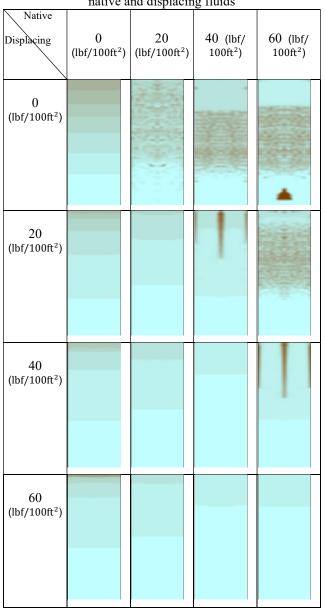


Table 5. Fluid distribution maps showing effect of standoff and casing rotation

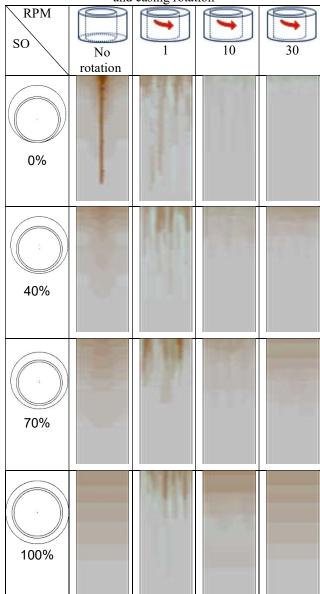


Table 6. Fluid distribution maps showing the effects of standoff and reciprocation

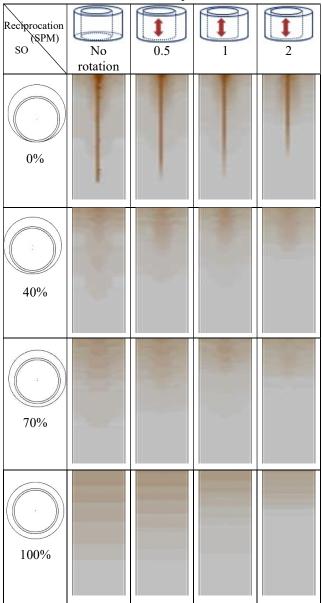


Table 7. Fluid distribution maps showing the effects of standoff and flow rate

Standon and now rate				
Flow Rate	4 bpm	6 bpm	8 bpm	12 bpm
0%				-
40%				
70%				
100%				

### **Figures**

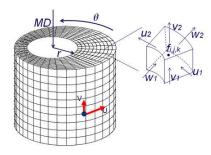


Fig. 1. An illustration of annular mesh and a control volume used in the current modeling technique. Three velocity components are shown.

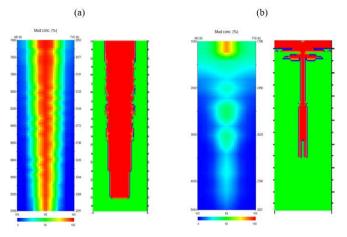


Fig. 2. Mud risk in the annulus after cement jobs comparison of the simulation results from the current model ((a)left; (b)left) and a leading computer simulator ((a)right; (b) right). (a) 9 5/8 in liner job; (b) 13 5/8 in casing. In the simulator, red color represents high mud risk and green represents no risk; in the numerical model, the mud existence is displayed in percentage from 0 to 100 (i.e., from blue to red)

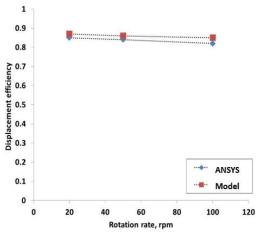


Fig. 3. Displacement efficiency shown as a function of pipe rotation in eccentric annuli after one annulus volume pumped.

#### Conclusions

In this study we present an efficient numerical model based on quasi-static solution of flow and volume of fluid method to simulate annular fluid displacement. Using this model, we run a large set of simulations considering annular displacement efficiency of cementing jobs to study the effects of different parameters and operation conditions including fluid density, rheology, casing rotation, reciprocation and flow rate against different eccentricities. The following conclusions can be made from this paper:

- We described a numerical method in details including mathematics, physics and numerical techniques. The model introduces some simplifications and has good computational efficiency, compared to a typical CFD program.
- Validation studies are conducted and the reliability of the model is tested; the model generally shows good agreement with other simulators.
- 3. Through groups of simulations we clarified the roles of several important factors which contribute to cementing job quality and we provided an overview of the displacement efficiency. The results may be used as a guideline for planning cementing jobs or other jobs involving fluid displacement.

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# **Nomenclature**

TD = Total depth
TOC = Top of cement
SO = Standoff
SPM = Stroke per minute
YP = Yield Point

CFD =Computational fluid dynamics

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