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Mathematical Model for Transient Simulation of Surge & Swab in Wells with Cross-Section Variation

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Abstract

Variations of the well diameter and the use of drill pipe accessories provide changes on the cross section of the annular space between the drill and the borehole. It is worth noting that cross section changes affect the pressure head losses within the well. This work presents a mathematical/numerical model to simulate the transient Surge & Swab problem in wells with cross section variations. The fluid flow yielded by the drill pipe motion is considered to be one-dimensional, isothermal, compressible and unsteady. The model is composed of the mass and momentum conservation equations, together with an equation of state and a constitutive equation for Bingham fluid. The governing equations were discretized by the Finite Volume Method. The well is admitted to be impermeable and the drill pipe end is closed. The results were compared to measured data obtained from an experimental rig with good agreement. It can be anticipated that the changes of cross section area may affect significantly the transient surge and swab pressures.

Introduction

Tripping is a frequent operation during the well construction process. The axial motion of the drillpipe displaces the drilling fluid and pressure changes within the wellbore. A pressure increase is observed when the drillpipe is run into the well since the drilling fluid is compressed against the bottom. If the borehole pressure exceeds the formation pressure, a fracture may be induced, resulting in fluid losses. Conversely, if the drill pipe moves upwards the pressure in the well is reduced in an event called swab. Whenever this swab pressure reaches values lower than those of the rock pore, formation fluid influxes may occur.

A fundamental inquiry to model surge and swab pressures was conducted by Burkhardt (1961). By evaluating pressure of a Bingham fluid flow through his stationary model, he pointed out that the viscous effects are more important than the flow inertia. This study was extended resulting in other stationary models such as of Fontenot and Clark (1974), Bourgoyne et al. (1991) and Bing et al (1995), which considered only the viscous effects.

The first attempt to control and predict kick and gel breaking was presented in the dynamic model due to Lal (1983). He considered the pressure drop in power law fluid flow caused by friction distributed along the well. His results also showed that transient effects are transmitted with the speed of sound in the fluid. A step foward was given by Mitchell (1988), Bing and Kaiji (1996), Kimura (2008) and Almeida (2009) which considered both viscous and inertial effects in the modeling.

Despite the large number of works found in the literature either dealing surge and swab as dynamic or stationary problem, the transient phenomenon in wells with annular cross-sectional variation has never been investigated before. In order to fulfill this gap, the current work presents a dynamic mathematical model for surge and swab in wells with cross-section change. The drilling fluid is treated as Bingham fluid. The governing equations of mass and momentum conservation are discretized by the Finite Volume Method (FVM) (Patankar, 1980) and solved iteratively. A comparison with measured data obtained in an experimental rig is performed. A sensitivity analysis to show the influence of the annular cross section variation in the surge and swab phenomenon is finally presented.

Problem description

Figure 1 depicts a schematic representation of a well with a number of devices connected to the drill pipe. The use of drill bits with different diameters and other accessories attached to the drill pipe can eventually provide variations in the annular space between the wellbore and the drill column. Figure 2, on the other hand, shows the simplified geometry of the wellbore that is considered in the current work. Note that changes of diameters are admitted in both the drill pipe (D_p) and the borehole (D_h). In this scheme, the origin of the coordinate system is placed at the well bottom and the drill pipe moves downwards with a constant velocity (V_p). Still in Fig. 2, notice that the annular space is divided into different cross sections, indexed by the letter *j*.

Since the borehole length is much larger than its diameter, the flow is assumed to be one-dimensional. Additionally, the flow is admitted laminar, isothermal and with constant compressibility. The drilling fluid behaves as a non-Newtonian material which is modeled as Bingham fluid. The bottom of the drill pipe is closed so that the drilling fluid flows only through the annular space. The borehole is assumed to be vertical, entirely rigid with non-permeable walls.

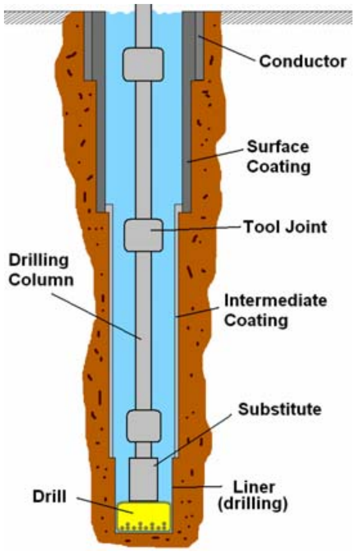

Figure 1. Schematic drawing of the borehole and drillpipe with devices attached.

Governing equations

By applying the above hypothesis, the mass and momentum conservation equations can be respectively written as,

$$\frac{\partial(\rho_j \overline{V_j})}{\partial z} + \frac{\partial \rho_j}{\partial t} = 0 \tag{1}$$

$$\frac{\partial(\rho_j \overline{V_j})}{\partial t} + \frac{\partial(\rho_j \overline{V_j}\,\overline{V_j})}{\partial z} = -\frac{\partial P_j}{\partial z} + \\ + \frac{\pi}{A_{tj}}(D_{hj}\tau_{hj} - D_{pj}\tau_{pj}) - \rho_j g \tag{2}$$

where ρ, $\overline{V}$ and P are, respectively, the average values of fluid density, fluid velocity and pressure across the sectional area. g is the gravity acceleration. A_t is the cross-section area of the annular space, and τ_h and τ_p are the shear stress at the external and internal walls of the annular space, respectively. t is the time and z is the axial position.

The pressure is evaluated according to the definition of compressibility (Anderson, 1990) as follow:

$$P_j = P_{atm} + \frac{1}{\alpha}\ln\left(\frac{\rho_j}{\rho_{atm}}\right) \tag{3}$$

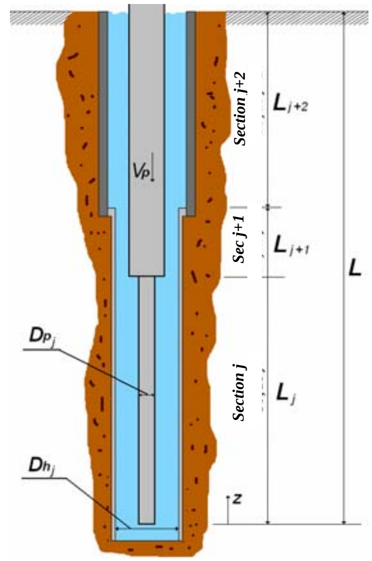

Figure 2. Representation of drill column and annular space with cross section variations.

where α is the fluid compressibility and the subscript *atm* indicates the atmospheric condition. Anderson (1990) also stated an expression to evaluate the fluid sound speed, C, defined as a function of compressibility:

$$C = \frac{1}{\sqrt{\rho_{atm}\alpha}} \tag{4}$$

Constitutive Equations

The shear stress term on the right hand side of the momentum conservation equation is modeled assuming a linear change of the shear stress across the cross-sectional area, similarly to what takes place in fully developed flows. Based on the friction factor correlation for Bingham fluids that was proposed by Melrose et al. (1958), an expression for the shear stress at the walls is defined:

$$\frac{\pi}{A_{tj}}\left(D_{hj}\tau_{hj} - D_{pj}\tau_{pj}\right) = -\varepsilon_{2j}\frac{\mu_j^2}{\rho_j(D_{hj} - D_{pj})^3} \tag{5}$$

where μ is the fluid plastic viscosity, ε_2 is a parameter that is a function of Reynolds and Hedstrom numbers:

$$\varepsilon_{2j} = \frac{2He_j\zeta_j}{3} + 16\,\mathrm{Re}_{efj}\,\zeta_j + \\ -\frac{4}{3}\zeta_j\sqrt{He_j^2 + 16He_j\,\mathrm{Re}_{efj} + 64\,\mathrm{Re}_{efj}^2}\cos\left(\frac{\theta_j}{3} + 120^{\circ}\right) \tag{6}$$

where the angular direction, θ_j , is:

$$\theta_j = \arccos\left[\frac{-\zeta^3\Gamma_j}{\sqrt{\zeta^6\left(He_j + 8\,\mathrm{Re}_{efj}\right)^6}}\right] \tag{7}$$

where $\Gamma_j = 24He_j^2\,\mathrm{Re}_{efj} - He_j^3 + 192He_j\,\mathrm{Re}_{efj}^2 + 512\,\mathrm{Re}_{efj}^3$ and He_j is the Hedstrom number, which is defined as:

$$He_j = \frac{\rho_j \tau_o (D_{hj} - D_{pj})^2}{\mu_j^2} \tag{8}$$

In the above equations, τ_o is the Bingham fluid yield stress and ζ_j is a geometric parameter given by the following correlation:

$$\zeta_j = \frac{(1-\beta_j)^2}{(1+\beta_j^2) - \dfrac{(1-\beta_j^2)}{\ln\left(\dfrac{1}{\beta_j}\right)}} \tag{9}$$

where β_j is the diameter ratio (D_{pj}/D_{hj}).

An effective velocity (V_{ef}), which accounts for the influence of the fluid average velocity ($\overline{V}$) and the drill pipe velocity (V_p), is defined as $V_{efj} = \overline{V_j} + \kappa_j V_{pj}$, where κ_j a geometric parameter given by:

$$\kappa_j = -\left[\frac{1-\beta_j^2 + 2\beta_j^2 \ln\beta_j}{2(1-\beta_j^2)\ln\beta_j}\right] \tag{10}$$

Thus, an effective Reynolds number, Re_{efj}, can be evaluated as:

$$\mathrm{Re}_{efj} = \frac{\rho_j V_{efj}(D_{hj} - D_{pj})}{\mu} \tag{11}$$

The local pressure loss that takes place at the cross-section variations is evaluated as follows:

$$\Delta P_L = K_L \frac{\rho \overline{V}^2}{2} \tag{12}$$

being K_L is the pressure loss coefficient that is calculated by:

$$K_L = \left(1 - J\frac{A_j}{A_{j+1}}\right)^2 \tag{13}$$

where J is the local pressure drop coefficient which is equal to 1 for expansions and is computed by the following expression (Assy, 2004) for contractions:

$$J = -1.52\left(A_1/A_2\right)^3 + 2.19\left(A_1/A_2\right)^2 + \\ -1.22\left(A_1/A_2\right) - 0.135 + 1.685\left(A_1/A_2\right)^{-1} \tag{14}$$

Boundary Conditions

Initial and boundary conditions are shown in Figure 3. As initial condition, the fluid is assumed to be at rest within the wellbore:

$$\overline{V}(z,0) = 0 \tag{15}$$

Because of the fluid weight, the initial fluid density and pressure increase with the well depth. Based on the hydrostatic change of pressure and on the definition of compressibility, the following expressions are derived for the initial density and pressure, respectively:

$$\rho(z,0) = \frac{1}{\dfrac{1}{\rho_{atm}} - \alpha g\left(L-z\right)} \tag{16}$$

$$P(z,0) = P_{atm} - \frac{1}{\alpha}\ln\left[1 - \rho_{atm}\alpha g(L-z)\right] \tag{17}$$

As the drill column is assumed to be closed, the rate in which the fluid displaced by the column is balanced by flow rate on the annular space. Therefore, the fluid velocity at the well bottom can be calculated from such balance as:

$$\overline{V}(0,t) = \frac{A_p}{A_t} V_p(0,t) \tag{19}$$

At the well head, the derivative of the product between density and velocity is assumed to zero in the axial direction:

$$\left.\frac{\partial\left(\rho_{atm}\overline{V}\right)}{\partial z}\right|_{z=L} = 0 \tag{20}$$

The governing equations (1) to (3) are discretized and solved by means of the control volume method of Patankar (1980) with a staggered grid; the fluid velocity is computed at the centre of the control volumes whereas, the pressure and density at their boundaries. The grid, as displayed in Figure 3, is uniform with a constant Δz value.

Results

In order to validate the model, a comparison between the model results and measured data obtained from an experimental rig is firstly performed. A case study based on real dimensions of a well is thus defined to evaluate the effect of cross section variation on the values of surge pressure. Finally, the influence of the speed of the column and of the rheological properties of the drilling fluid is analyzed.

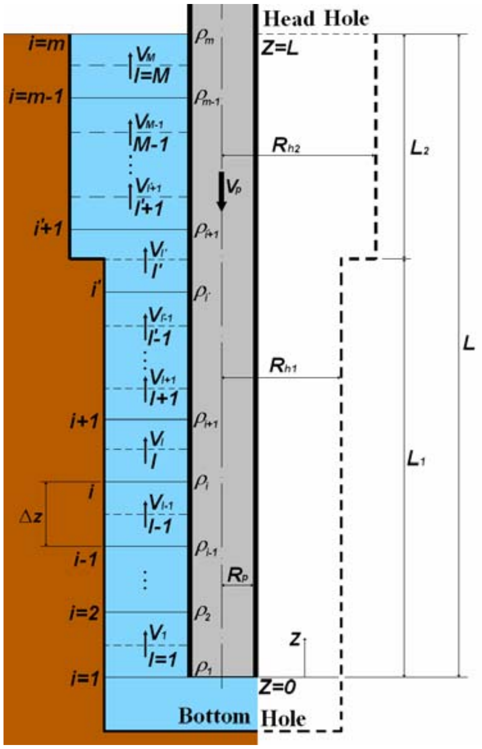

Figure 3. Numerical grid.

Comparison with experimental data

The experiments were conducted by PETROBRAS in an experimental drilling rig which was equipped with a data acquisition system and optical fiber sensors (acquisition frequency of 2 Hz) to measure pressure at the borehole. The borehole geometry and the rheological properties for the drilling fluid are presented in Table 1.

The speed of the drill column was measured by observing the rate of displacement of a marked point in the drillpipe. The rheological data was obtained by using a FANN 35 viscometer and the results were fit to the Bingham model. Since the fluid compressibility was not measured, its value was estimated by matching the measured values of hydrostatic pressure with the calculated counterparts. Inasmuch as the borehole and the drill column presented nearly constant cross section, the section of the annular region was admitted constant.

Figure 4 shows a comparison of measured and computed values of pressure at the bottom of the well with a drill column speed of -0.225 m/s, to simulate a swab situation. Note that the curves are similar so that the measured and computed pressures decrease abruptly from the hydrostatic to a minimum value, followed by oscillations that reduce with time. In case of the numerical curve, the oscillations due to the pressure wave reflections at the ends of the wellbore are dumped and the steady state is reached after 23s. In contrast, small oscillations for the experimental results never vanish due to the difficulty of maintaining the column at constant speed. It is noteworthy that the maximum difference observed between the experimental and numerical values was only 0.55%.

Table 1 - Rheological and geometric data of the experimental rig.

Parameter	**Value [units]**
Lenght, L	1192 [m]
Borehole diameter, D_h	0.16848 [m]
Column diameter, D_p	0.1143 [m]
Fluid density, ρ	1833.3 [kg/m^3]
Fluid plastic viscosity, μ	0.0738 [Pa.s]
Fluid yield stress, τ_o	9.02 [Pa]

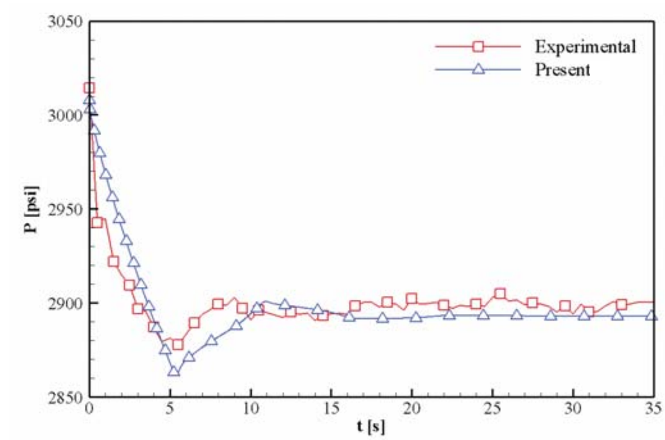

Figure 4. Comparison between numerical and experimental pressure values for a swab test.

The pressure at the well bottom is now shown in Figure 5 for a downward drill column speed of 0.618m/s. For the first 8s of experiment, a good agreement between measured and computed values was again observed, with the difference between the two datasets not larger than 1.02%. After that, the numerical data oscillates to reach the steady state in about 28s and the experimental results continue to fluctuate. Such behavior of the measured values is possibly due to the extreme difficulty to maintain the downward movement of the drilling column at a constant speed. Notably, the calculated pressure values were more conservative than the measured ones in both cases, since they are larger than their experimental counterpart.

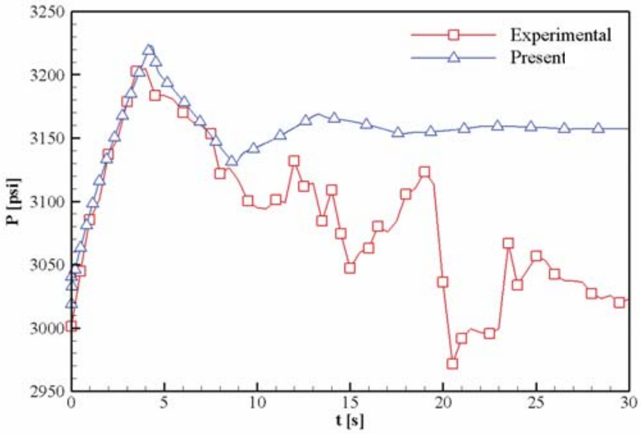

Figure 5. Comparison of numerical and experimental pressure values for a surge test.

Case study: A Borehole with real dimensions The influence of cross-sectional area variations on the swab and surge pressures is now investigated. The geometry chosen to represent the borehole is shown in Figure 6 and the drilling fluid properties are the same considered in the previous case, as depicted in Table 1. Given that the levels of pressure differences calculated for surge and swab tests are very similar, only results of surge are considered in the following analysis.

Figure 7 presents a comparison between the surge pressures obtained for the wellbore shown in Figure 6 and the results of a wellbore with an equivalent uniform cross section. The equivalent cross-section well is defined to provide the same steady state surge pressures as that of the well with cross section change. In addition, the effect of the drill column speed (V_p) on the pressure was observed. As expected, the results show not only that the pressure peaks are larger than the steady-state pressures but also that the differences between the peak and the steady state values increase with the speed of the column. Note that these peaks are caused by reflection of the pressure wave at the head of the wellbore.

Another remarkable aspect is that the wellbore with cross-sectional variation presents higher pressure peaks than those found for the well with uniform cross section. The largest difference, 7.5%, between the pressure peaks is observed for a drill column moving at 1m/s.

Considering the rheological properties of the drilling fluid plays a significant role in the surge and swab pressure, their effects are now evaluated for the drill column of Figure 6 moving at 0.2m/s. Table 2 shows the properties of three typical drilling fluids considered in the simulations. Figure 8 shows the surge pressure changes for the fluids listed in Table 2. As expected, the higher the apparent viscosity of the fluid (higher plastic viscosity and yield stress), the greater the pressure in the steady state regime. On the other hand, the fluid with the higher viscosity (fluid 3) dissipates more rapidly the pressure wave energy, and therefore, the time to reach the steady state is smaller.

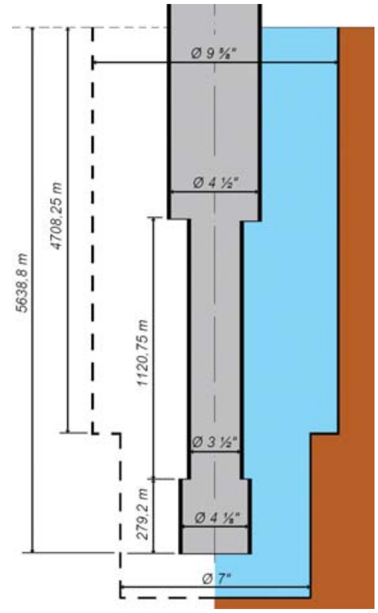

Figure 6. Schematic representation of a wellbore with real dimensions.

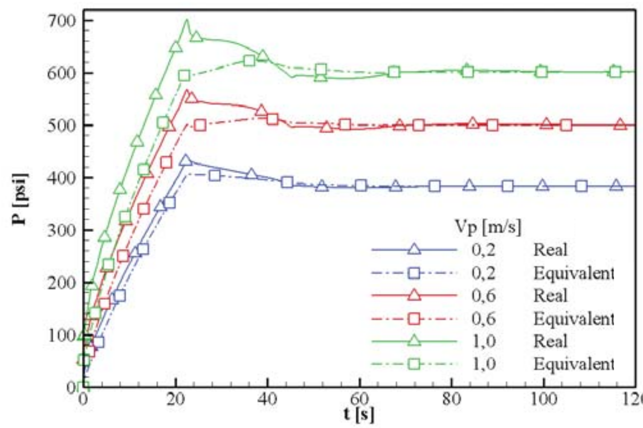

Figure 7. Comparison of surge pressures for wells with uniform and variable cross sections and drill pipes with three different speeds.

Table 2 - Properties of three typical drilling fluids.

Parameter	Fluid 1 [units]	Fluid 2 [units]	Fluid 3 [units]
Density, ρ [lbm/gal]	9.6	9.6	16.1
Fluid plastic viscosity, μ [cP]	23.8	31.6	67.7
Fluid yield stress, τ_o [Pa]	2.17	6.19	10.88

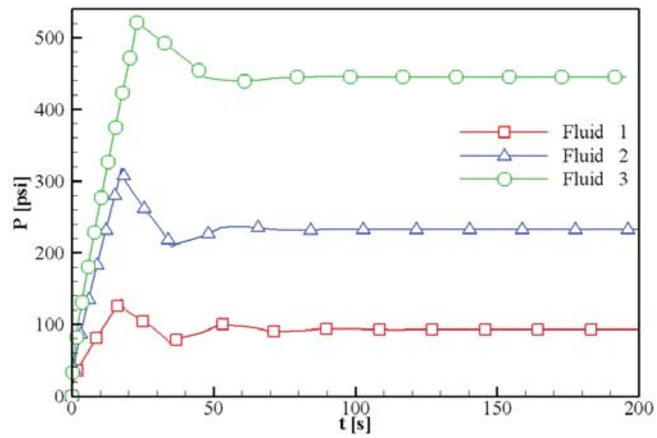

Figure 8. Comparison of surge pressures for a well with uniform section and three different drilling fluids.

Conclusions

In this paper a mathematical/numerical model, capable of predicting transient surge & swab pressures in wellbores with varying cross section, is presented and investigated. Cross-section variations can occur either in the annular region as well as in the drill string. Model validation was accomplished by comparing the numerical results with the values measured in an experimental rig. Despite the difficulty in controlling the process, especially the speed of the drill string, good agreement between the measured and the calculated pressure values was observed. Noteworthy key findings include:

- The maximum relative difference between the measured values and those calculated is less than 1%;
- The difference between the pressure peaks observed in a well with varying cross section and a well with uniform cross section increases with the column speed, reaching 7.5% for a speed of 1m/s;
- The increase of the fluid apparent viscosity not only elevates the surge pressure (or reduce the swab pressure) but also attenuates pressure oscillations, causing the steady state to be reached more rapidly;

Finally, it is important to remark that the presence of cross section variations in the borehole or in the drill column must not be neglected in the modeling, especially in the case of a transient analysis. Wells with varying cross section may present pressure peaks more pronounced than those with uniform section.

Acknowledgments

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Nomenclature

A_t = annular region area
C = speed of sound
D_p = column outer diameter
D_h = borehole diameter [m]
g = gravity acceleration [m/s^2]
He = Hedstrom number
J = distinguishing factor for contraction - expansion
K_L= local pressure drop coefficient
L = borehole length [m]
P = Pressure [Pa]
Re = Reynolds number
t = time [s]
$\overline{V}$ = flow average velocity [m/s]
V_p = drill column velocity [m/s]
z = axial direction [m]

Greek
α = isothermal fluid compressibility [Pa^{-1}]
β = diameter ratio [m]
κ = correction coefficient of the effective velocity
ε_2 = function of Reynolds and Hedstrom numbers
θ = angular direction [m]
μ = fluid plastic viscosity [Pa.s]
ρ = fluid density [kg/m^3]
τ_o = Bingham fluid yield stress [Pa]
τ = fluid shear stress [Pa]
ζ = Correction factor for the annular region

Subscripts
atm = atmospheric condition
ef = effective
h= related to the well
I = velocity control volume
i = pressure and density control volume
j = borehole section
p = drill column
R = reference

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