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Making a Case for AADE Hydraulics and the Unified Rheological Model

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Abstract

This paper has two objectives. The first is to make a case for an AADE initiative to help improve or create an AADE alternative to the API RP13D recommended practice on drilling fluid rheology and hydraulics. The second objective is to introduce a new Unified rheological model in support of this argument.

Industry practice has deviated from RP13D in recent years, driven primarily by extraordinary demands from today's critical wells. The evolution of new hydraulics technologies, mostly built around newfound computer power, is creating problems with uniformity and widening the gap between theoretical and practical solutions.

The Unified rheological model is so-called because it seeks to "unify" the wide range of industry personnel concerned with hydraulics and rheology. The empirically derived flow equation is expressed in a form easily recognized by field engineers and is sufficiently accurate for most high-end software applications. Attention also is given in this paper to determination of rheological parameters and complexities involved in hydraulics and rheological modeling.

Introduction

Deepwater**,** HTHP and extended-reach projects, particularly when drilled with synthetic-based muds (SBMs), have forced the industry to rethink hydraulics and rheology.[1] Improved techniques, mostly included in new high-end software applications, have helped significantly; however, there have been consequences. Foremost among these are an expanding technology gap between theory and practice, and a growing lack of uniformity for design, analysis, and training.

This paper is divided into two parts. The first part presents an argument for the development of AADE guidelines for drilling fluid rheology and hydraulics. Alternatively, the AADE could work in partnership with the American Petroleum Institute (API) to update RP13D,[2] currently the only standard for this important subject area. In either case, the aim would be to provide guidelines that (a) more closely match current industry practices, (b) are understandable and usable by field and staff personnel, and (c) apply to conventional and critical wells.

The second part introduces a new rheological model in support of this argument. Models are used during planning and drilling operations to evaluate numerous hydrodynamic issues, such as equivalent circulating density (ECD) and hole cleaning. Model parameters also are essential to help formulate, diagnose, and run muds, so synchronization clearly is very important.

The Unified rheological model is a new empirical simplification based on the Herschel-Bulkley flow equation. Herschel-Bulkley has become the model of choice in recent years; but its exact solution, unfortunately, is very complex and mostly restricted to sophisticated computer programs. The "unified" reference demonstrates its resolve to be sufficiently accurate for use in high-end hydraulics software, yet practical enough for field use by front-line drilling and mud engineers.

The rheological model is only one of many controversial issues that would have to be addressed. The intent is to show that opportunities exist even for the most controversial issues. Also discussed are techniques to determine key rheological parameters as well as complexities involved in rheological modeling.

The Case for an AADE Hydraulics Initiative

A successful argument for AADE hydraulics should establish that (a) an improved recommended-practice standard is needed, and (b) the AADE is properly positioned and inclined to undertake such a task. Firstly, RP13D was a significant achievement when first released in 1985. Despite several obligatory revisions, however, it has not kept pace with current industry practice and inadequately addresses today's critical wells. Secondly, the AADE is well suited for this project based on its charter and membership diversity. This clearly would be an opportunity for the AADE, albeit one of great challenge and without guarantee of success.

The issue at hand is not whether the industry benefits from published or *defacto* standards, nor if tools exist to mitigate hydraulics-related problems. Instead, timing is the concern. Updated guidelines are needed now, and technology is advancing at such a rate that soon it may be impossible to meet those needs in a single document with the original intent and scope of RP13D.

Changing Hydraulics Technology

The evolution of hydraulics standards in our company is illustrative of industry changes in recent years. Soon after the merger of Magcobar and IMCO Services in late 1986, the new company adopted RP13D to "unify" two different approaches to drilling fluid hydraulics. This continued until 1995, when a Gulf of Mexico operator initiated a project to measure dynamic pressure and temperature distributions in an offshore well.[3] The concern was the potential for major losses of expensive SBM while drilling, complicated by the inability of API equations to match field data. A month later, a North Sea operator requested an upgrade in our hydraulics programs to incorporate the effects of temperature and pressure on SBM density and rheological properties.

The two operator-driven requests were instrumental in the development of one of the first advanced hydraulics programs.[4] Finally, the same Gulf of Mexico operator "commissioned" development of a system to minimize SBM losses while running casing, a deepwater drilling problem they considered the most challenging of that time. The result was a real-time hydraulics system[5] proven successful for minimizing losses, accurately predicting downhole density and ECD profiles, and filling technology gaps related to annular-pressure-while-drilling measurements.

It is noteworthy that both previously mentioned advanced software packages use a flow equation based on the Herschel-Bulkley model for pressure-loss calculations. The model recommended in RP13D is offered as an option in the software, but primarily for comparison purposes.

Additions/Improvements to Recommended Practice

Most of the necessary basic issues already are addressed in RP13D. However, some issues should be added and a few others require updates. A working list of changes to recommended practices can almost single handedly be provided by ultra-deepwater wells. They are characterized by very low temperatures, low fracture gradients, moderate-to-high formation pressures, narrow operating windows, tight casing/hole clearances, and chemically unstable wellbores.[6] These can translate into serious problems inadequately covered by RP13D, including major losses of temperature-sensitive synthetic and oil-based muds, mechanically unstable wellbores, poor hole cleaning, and barite sag.

New issues that should be addressed in an improved hydraulics bulletin along with very brief comments include the following:

1. *Temperature and pressure effects on downhole equivalent static density (ESD)*. This clearly is the most pressing concern,[1,6] especially for extreme-temperature wells (deepwater and HTHP) as illustrated in **Fig. 1**. Generalized mud P-V-T (pressure-volume-temperature) properties will be required, but this information is becoming more widely available in the literature. Perhaps the most challenging aspect will be determining reasonable downhole mud-temperature profiles.
2. *Effects of directional profiles on hole cleaning and barite sag*. Hole inclination creates several critical concerns attributable to gravity forces and pipe eccentricity. RP13D assumes vertical wells for hole cleaning and does not address sag. Nearly all aspects of hole cleaning and sag relationships are highly controversial; however, it is encouraging that suitable curves already are available for hole cleaning.[7] Perhaps these can be extended and the approach applied to sag.
3. *Pressure-loss correlations for downhole tools*. Standpipe pressure calculations would benefit from better non-Newtonian correlations provided by equipment suppliers. Currently, data of any type are somewhat limited on certain tools, and very little information based on real muds is available.
4. *Surge/swab pressures while tripping pipe and running casing*. Traditional equations are adequate for planning, if used with temperature and pressure-adjusted mud properties.[5]
5. *Hydraulics optimization*. Conventional bit-hydraulic optimization would be useful, but an optimization scheme based on pump horsepower, ECD, hole cleaning, and hydraulic power at the bit would be especially valuable.

Many RP13D elements are still valid and in use, but some need to be updated, including the following:

1. *Temperature and pressure effects on downhole rheological properties*. Mud rheology needs adjustment for downhole conditions, especially in deepwater, HTHP, and extended-reach wells drilled with oil or synthetic-based mud. **Fig. 2** shows calculated PV and YP profiles for a deepwater well.[1]
2. *Measurement and treatment of viscometer data taken under temperature and pressure*. Considerable data now are available from Fann 70/75 units, currently the most commonly used oilfield HTHP viscometers. Temperature-effect trends can be established on field viscometers by measuring rheology at three temperatures. Data generated at very low temperature are of particular value for deepwater drilling.
3. *Rheological modeling to include a yield-stress term*. Mud yield stress and low-shear-rate viscosity are now key parameters for hole cleaning, barite sag, and pressure-loss analyses. The Herschel-Bulkley model has become the *defacto* standard.
4. *Transitional and turbulent-flow correlations and pressure losses*. Additional laboratory and analytical work will be required first, as most available techniques do not consistently match field data.

AADE Involvement

The AADE is well within its charter to consider this task on its own or in partnership with the API. For example, the AADE mission[8] is "to provide the forum for the dissemination of practical drilling technology to those employed or interested in the drilling industry," and its vision[8] is "to be a leading forum for the dissemination and interchange of drilling practices and technology."

Also, in the National Letter[8] from the Office of the 2002 AADE President, Paul Hebert encourages members to "make a commitment to participate (by) starting a technical study group in an area of interest to you and others in your chapter relative to our drilling related activities." Finally, the AADE membership, by intent, includes precisely the spectrum of drilling personnel who would benefit from improved guidelines.

Unified Rheological Model

In a controlled environment, like a chemical or manufacturing plant, the rheological model is the focal point of a hydraulics system. In drilling applications, however, fluid complexities, downhole conditions, and uncertainties can be such that the model, while still important, no longer is the defining element. For example, the difference between surface and downhole mud weights in extreme-temperature wells easily can override model-related differences in annular pressure-loss calculations. This situation alone creates an opportunity for empirical solutions like the Unified rheological model that, while not theoretically exact, are still well suited for drilling applications.

It is important to note that drilling fluids are very diverse and can be rheologically complex. For practical reasons, muds are assumed to be time-independent, purely viscous, non-Newtonian fluids. To consider otherwise is beyond the scope of this paper and industry guidelines. Also, discussion is limited to steady state, laminar flow in pipes and concentric annuli (parallel plates).

Practical Constraints

As much as possible, industry guidelines in general should be realistic and practical, and encompass existing technologies and procedures. For a drilling fluid rheological model, it is useful to consider the following practical constraints and targets:

1. Rheological parameters traditionally used to run mud should be preserved and extractable from the model (*e.g.*, PV and YP).
2. Data should be generated on currently available field and laboratory viscometers (*e.g.*, Fann 35A, Fann 70/75, and equivalents).
3. Equations and processes should be programmable in a spreadsheet without macros. This would make computer processing more widely available.

Model Development

RP13D is based on a "dual power law", the lower shear-rate segment for the annulus and the upper segment for inside the drill string. This is a good choice for pressure-loss calculations, because this approach technically is a "generalized correlation" for which explicit laminar-flow solutions are both available and straightforward. However, the Herschel-Bulkley model has re-emerged in the drilling industry and become the model of choice for many applications.[9] Also called the yield-power law, yield-pseudoplastic, and modified power law, it contains a yield-stress term that has become central to evaluating and optimizing hole-cleaning, barite sag, suspension, and other key drilling concerns.

Table 1 (column 2) gives the familiar constitutive equations for four models of interest: Bingham plastic, power law, Herschel-Bulkley, and the proposed Unified model. Similarities among the four are self-evident, and all appear to be fundamentally uncomplicated. However, the wall shear stress (τ_w) is needed to calculate frictional pressure loss, rather than the constitutive shear stress (τ). The "Force-Balance" equations illustrated in **Table 2** and derived in RP13D can be used for this purpose. The constant 1.067 converts shear stress in °Fann to $lb_f/100ft^2$. The Rabinowitsch-Mooney "Flow Equations,"[10] also given in **Table 2**, are required to find τ_w. Their solution requires the function $f(\tau)$, the explicit shear rate defined by the constitutive equation. For example, $f(\tau)$ for Bingham plastic fluids is equal to $(\tau - \tau_y) / k$.

Resulting pipe and annular flow equations are given in **Table 1** (columns 3-4) for each model. The power law equations, the only curves directly solvable for τ_w, are identical to the wall-shear-stress equations used in RP13D. The Bingham plastic solutions are more complex, but they can be solved iteratively or by one of several available specialized procedures. However, the approximations listed in the table are almost universally accepted by the oilfield and certainly are suitable for industry guidelines.

Several iterative techniques are available to solve the daunting Herschel-Bulkley flow equations. A popular procedure[11] uses a generalized correlation technique functionally equivalent to that used in RP13D for the dual power law. While both use the same measured viscometer data, the former assumes true Herschel-Bulkley behavior before processing. The API version relies on the raw data and technically assumes no particular rheological model. A drawback to the API method is the wide shear-rate span between data points (especially 5 to 170 s^{-1}, or 3 to 100 rpm).

Alternatively, the Herschel-Bulkley flow equations can be approximated by the Unified model, as shown in **Table 1**. **Eq. 1** is provided for those who prefer to combine the pipe and annulus equations into a single relationship:

$$\tau_w = \left[\frac{4-a}{3-a}\right]^n \tau_y + k\left[\frac{(3-a)n+1}{(4-a)n}\right]^n \left[\frac{(2+a)(96V)}{2D_{hyd}}\right]^n \quad \ldots(1)$$

The geometry factor a is 0 for pipe flow and 1 for parallel-plate (annulus) flow.

The Unified model as presented here is an enhanced version of a simplified Herschel-Bulkley model introduced to the drilling industry years ago.[12] The correlation has since been significantly improved without major complications by adding the term acting on τ_y. That paper[12] also originated the geometry-factor concept and included a relationship for annular (rather than parallel-plate) flow that has been improperly credited in several publications. Follow-up work[13] verified that the original simplified approach, even with its inherent limitations, yielded better results than using exact Bingham-plastic and power-law solutions, if in fact, the muds were neither.

Rheological Parameters

Rheological parameters for the Unified model are the plastic viscosity PV, yield point YP, and yield stress τ_y. A fourth parameter, the ratio τ_y / YP, is a useful tool to help characterize fluids rheologically, although it is not necessary for solving the model.

Equation parameters n and k were not selected, because they have not met with success as practical mud indicators, and their connotations are complicated by the presence of a yield-stress term. For example, fluids with no yield stress are "more non-Newtonian" at lower values of n, but the same trend may not apply for fluids with a yield stress. As such, focus on the n value could create unnecessary training problems. Therefore, it is convenient to use n and k as equation parameters only and define them in terms of the three rheological parameters as shown in **Eqs. 2-3**:

$$n = 3.322\log_{10}\left(\frac{2PV + YP - \tau_y}{PV + YP - \tau_y}\right) \quad \ldots(2)$$

and

$$k = \frac{PV + YP - \tau_y}{511^n} \quad \ldots(3)$$

It is especially meaningful to track τ_y / YP in wells where rheology profiles are available (like those shown in **Fig. 2**). Some fluids may exhibit more plastic behavior in one part of the well and more pseudoplastic behavior in another. This is important for hole-cleaning and barite-sag considerations. As the ratio τ_y / YP approaches 1 ($\tau_y \rightarrow YP$), fluids take on Bingham-plastic behavior. For τ_y / YP approaching 0 ($\tau_y \rightarrow 0$), they behave more like pseudoplastic (power-law) fluids.

Yield-Stress Determination

Several controversial measurement and curve-fitting techniques currently are in use, but it remains a great challenge to determine the yield stress (τ_y) for a given fluid. Generally, direct measurement is preferred, because τ_y is a material property of the fluid and independent of the rheological model. Also, τ_y can be recorded at the same time rheological data are taken without computer processing or extensive calculations.

A classic definition of yield stress is "that stress below which no flow can be observed under conditions of experimentation".[14] This hints at the difficulties that can be encountered, even with a wide choice of measurement tools.[15] Unfortunately, conventional drilling industry Couette viscometers are not ideally suited for taking this measurement. For fluids with a yield stress, any readings taken at low shear rates are suspect because of the presence of a plug flow region in the viscometer gap.[16] Nevertheless, the following options are available for measuring reasonable, usable values for τ_y:

1. Fann R_3
2. Fann R_6
3. Low-shear yield point ($LSYP = 2R_3 - R_6$)
4. "Zero" gel strength (no time delay)
5. Initial gel strength (10-sec delay)
6. 10-min gel strength (10-min delay)

Standardization on any one of the six options would be acceptable in updated rheology and hydraulics guidelines. However, discussion of their individual merits is appropriate here. The first three are based on stabilized readings; the last three on gel-strength-type measurements. It could be argued that $LSYP$ is the best choice from the first group and the initial gel strength is the best from the second group. Regardless, τ_y should not exceed the YP.

$LSYP$ implies a value at zero shear rate, involves both R_6 and R_3, and should best match results from a curve fit. However, $LSYP$ would under predict the expected "true" yield-stress value by the greatest amount. R_6 and R_3 perhaps are better approximations, but their use could be confusing since, by definition, τ_y is the shear stress at zero shear rate.

An important advantage of gel-strength measurements is that they also can be taken on 2-speed viscometers. The "zero" gel concept[12] has been used successfully for a long time and fundamentally would be a good choice. However, the other two already are reported and considerable data already exist. The 10-sec gel is preferred because the 10-min gel could give unreasonably high yield-stress values.

Curve fits determine τ_y by extrapolation based on an assumed model. This is not a disadvantage, since the Herschel-Bulkley form is central to the Unified model. By definition, a logarithmic plot of τ - τ_y vs γ is a straight line (slope of n and intercept of k). The best fit for τ_y must be determined iteratively, and all methods require computer processing. Standard[9] and weighted-average[17] techniques are acceptable, but the latter is preferred because it favors the better-quality data at high shear rates.

If curve fitting is chosen as the means to find τ_y, a slightly different method should be considered. This method forces the curve fit through R_{600}, R_{300}, and iteratively through one other point, rather than use a least-square technique with all six data points. This preserves values for PV and YP. The additional point can be R_3, R_6, the average of R_3 and R_6 (at 4.5 rpm), or even R_{100}. The R_3 and R_6 average gives results almost identical to the weighted-average curve fit. R_{100} could be the most interesting, because the shear rate is high enough to remove the plug zone for fluids where $YP / PV > 5$.[16]

Unified and Herschel-Bulkley Comparisons

Assuming that drilling fluids generally fit the Herschel-Bulkley model, validation of the Unified model is best achieved by direct comparison between the two. Comparisons are made using annular pressure loss and parallel-plate flow curves (τ_w vs γ) over a broad range of data.

Fig. 3 shows the annular pressure loss (laminar flow only) for a 12.25-in hole with 5.5-in drill pipe, $PV = 10$, $YP = 10$, and $\tau_y = 5$. The RP13D dual power law also is included. Overall, apparent differences in the curves are not significant – all three would be suitable for use in field situations. At low flow rates, however, it can be noted that the Unified model is slightly higher, but becomes asymptotic to the exact Herschel-Bulkley solution at higher flow rates. The API model is interesting because, as expected, calculated pressure losses are lower for equivalent shear rates below 5 s^{-1} and higher between 5 and 170 s^{-1} (expressed as flow rates) as seen on the graph.

Figs. 4-7 are the annular flow curves for a wide range of PV and YP pairs plotted on rectangular coordinates. **Fig. 8** is a single example for pipe flow plotted on a logarithmic scale. Each graph compares the Unified model (solid lines) to the exact Herschel-Bulkley model (markers only) for $\tau_y / YP = 0$, 0.5, and 1.0.

For each case, the solutions are identical for $\tau_y / YP = 0$ and approach the error consistent with the Bingham plastic approximation as $\tau_y / YP \rightarrow 1.0$. Curves for $\tau_y / YP = 0.5$ are all quite close.

PV and YP pairs were chosen to highlight comparisons rather than realism. The largest observable error for the case of inverted rheology (**Fig. 5**) is acceptable if the traditional Bingham plastic solution also is acceptable. At high PV/YP ratios (**Fig. 7**), all the curves are virtually indistinguishable. In between, the Unified model fits well with the exact solutions.

Additional Complexities

There are certain complexities related to the rheological model that are worthy of mention, but beyond the scope of this paper. These issues should be evaluated individually and collectively on merits and practicality, along the lines of the approach taken with the Unified model. These complexities include:

1. *Shear-rate correction for viscometers*. Correction is needed because viscometer data are used to find pressure loss in tubes. Closed solutions are available for Bingham plastics[16] and pseudoplastics,[18] but not for Herschel-Bulkley fluids. Furthermore, available numerical methods are inadequate at low shear rates for yield-stress fluids.
2. *Hydraulic diameter for annulus*. The traditional equation used for D_{hyd} is more due to simplicity than theoretical accuracy.[19] Methods[11,12] are available to adjust for annular shear rate, but their significance should be evaluated first.
3. *Eccentricity effects on pressure loss*. Explicit analytical methods currently do not exist and numerical solutions are very complex; however, available empirical curves[20] seem to work well.
4. *Drill-pipe rotation*. Empirical relationships will need to be established for various geometries and conditions.

Conclusions

1. An argument is made for the AADE to work with the API to improve or create an AADE alternative to the API RP13D recommended practice on drilling fluid rheology and hydraulics.
2. Realization of uniform practices will be challenging, due to the complexity of the subject (technical and practical), lack of defining relationships in key areas, existing proliferation of different technologies, entrenched philosophies, and industry politics.
3. Hydraulics guidelines should address complex issues and critical wells, but still be usable on conventional wells by the wide range of staff and field engineers concerned with rheology and hydraulics.
4. The proposed initiative can be seeded by the Unified rheological model, a new empirical flow equation based on the Herschel-Bulkley model.
5. The three parameters of the Unified model, PV, YP, and τ_y are also key parameters required to formulate, diagnose, and treat drilling fluids. A fourth parameter, the ratio τ_y / YP, is a useful tool to help characterize fluids rheologically.
6. A measured yield-stress value using one of six suitable options is generally preferred over curve-fit extrapolation.

Nomenclature

ΔP = *pressure loss, psi*
γ = *shear rate, s^{-1}*
τ = *shear stress, $lb_f/100\ ft^2$*
τ_w = *wall shear stress, ° Fann (≈ $lb_f/100\ ft^2$)*
τ_y = *yield stress, ° Fann (≈ $lb_f/100\ ft^2$)*
a = *geometry factor (0=pipe, 1=annulus)*
D = *pipe inside diameter, in.*
D_h = *pipe inside diameter, in.*
D_{hyd} = *hydraulic diameter (D or D_h - D_p), in.*
D_p = *pipe outside diameter, in.*
k = *laminar consistency factor, $lb_f{\cdot}s^n/100\ ft^2$*
L = *length, ft*
$LSYP$= *low shear yield point, ° Fann (≈ $lb_f/100\ ft^2$)*
n = *laminar flow behavior index*
PV = *Bingham plastic viscosity, cP*
R_3 = *Fann reading at 3 rpm, ° Fann (≈ $lb_f/100\ ft^2$)*
R_6 = *Fann reading at 6 rpm, ° Fann (≈ $lb_f/100\ ft^2$)*
R_{100} = *Fann reading at 100 rpm, ° Fann (≈ $lb_f/100\ ft^2$)*
R_{300} = *Fann reading at 300 rpm, ° Fann (≈ $lb_f/100\ ft^2$)*
R_{600} = *Fann reading at 600 rpm, ° Fann (≈ $lb_f/100\ ft^2$)*
V = *velocity, ft/s*
YP = *Bingham yield point, ° Fann (≈ $lb_f/100\ ft^2$)*

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Model	Constitutive	Pipe	Annulus (Parallel Plates)
Bingham Plastic	$\tau = \tau_y + k\gamma$	$\frac{96V}{D} = \frac{\tau_w}{k}\left[1 - \frac{4}{3}\left(\frac{\tau_y}{\tau_w}\right) + \frac{1}{3}\left(\frac{\tau_y}{\tau_w}\right)^4\right]$ $\tau_w \approx \frac{4}{3}\tau_y + k\left(\frac{96V}{D}\right)$	$\frac{144V}{D} = \frac{\tau_w}{k}\left[1 - \frac{3}{2}\left(\frac{\tau_y}{\tau_w}\right) + \frac{1}{2}\left(\frac{\tau_y}{\tau_w}\right)^3\right]$ $\tau_w \approx \frac{3}{2}\tau_y + k\left(\frac{144V}{D_h - D_p}\right)$
Power Law	$\tau = k\gamma^n$	$\tau_w = k\left(\frac{3n+1}{4n}\right)^n\left(\frac{96V}{D}\right)^n$	$\tau_w = k\left(\frac{2n+1}{3n}\right)^n\left(\frac{144V}{D_h - D_p}\right)^n$
Herschel-Bulkley	$\tau = \tau_y + k\gamma^n$	$\frac{96V}{D} = \left(\frac{\tau_w}{k}\right)^{\frac{1}{n}}\left(1 - \frac{\tau_y}{\tau_w}\right)^{\frac{n+1}{n}}\left[\frac{4n}{3n+1} + \frac{8n^2\left(\frac{\tau_y}{\tau_w}\right)\left[(n+1) + n\left(\frac{\tau_y}{\tau_w}\right)\right]}{(n+1)(2n+1)(3n+1)}\right]$	$\frac{144V}{D_h - D_p} = \left(\frac{\tau_w}{k}\right)^{\frac{1}{n}}\left(1 - \frac{\tau_y}{\tau_w}\right)^{\frac{n+1}{n}}\left[\frac{3n}{2n+1} + \frac{3n^2\left(\frac{\tau_y}{\tau_w}\right)}{(n+1)(2n+1)}\right]$
Unified	$\tau = \tau_y + k\gamma^n$	$\tau_w = \left(\frac{4}{3}\right)^n\tau_y + k\left(\frac{3n+1}{4n}\right)^n\left(\frac{96V}{D}\right)^n$	$\tau_w = \left(\frac{3}{2}\right)^n\tau_y + k\left(\frac{2n+1}{3n}\right)^n\left(\frac{144V}{D_h - D_p}\right)^n$

Table 1 - Constitutive equations and flow curves for four rheological models, including the approximate Bingham plastic solution. The Unified model flow curves are approximations of the exact Herschel-Bulkley model.

Flow Conduit	Pipe	Annulus (Parallel Plates)
Force Balance	$\Delta P = \frac{1.067\tau_w L}{300D}$	$\Delta P = \frac{1.067\tau_w L}{300(D_h - D_p)}$
Flow Equation	$\frac{96V}{D} = \frac{4}{\tau_w^3}\int_{\tau_y}^{\tau_w}\tau^2 f(\tau)d\tau$	$\frac{144V}{D_h - D_p} = \frac{3}{\tau_w^2}\int_{\tau_y}^{\tau_w}\tau\ f(\tau)d\tau$

Table 2 - Force-balance equations for pipe and parallel plates. The constant 1.067 converts °Fann to $lb_f/100ft^2$. Flow equations are the Rabinowitsch-Mooney relations used to determine wall shear stress from constitutive equations.

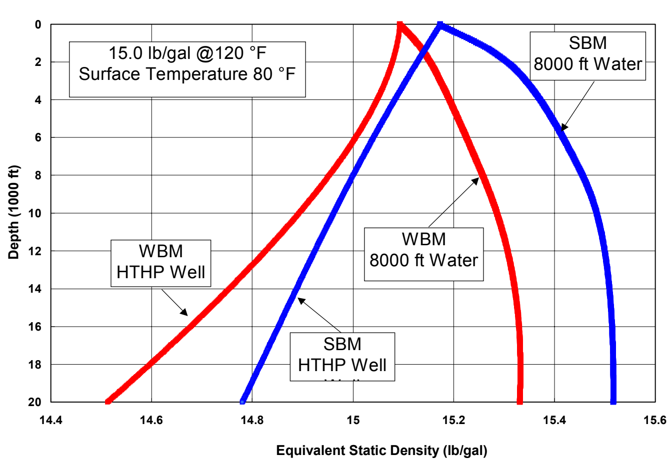

Fig. 1 - ESD comparisons for 15-lb/gal water and synthetic-based muds in 20,000-ft deepwater and HTHP wells.[1]

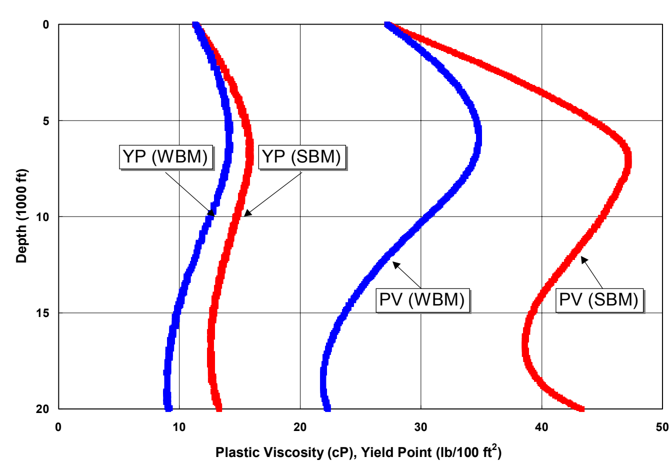

Fig. 2 - Comparison of downhole *PV* and *YP* profiles for water and synthetic-based muds in a deepwater well.[1]

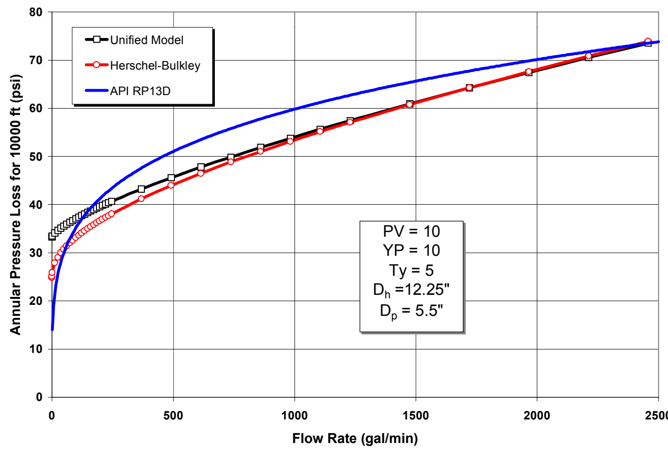

Fig. 3 - Annular pressure-loss curves for three rheological models.

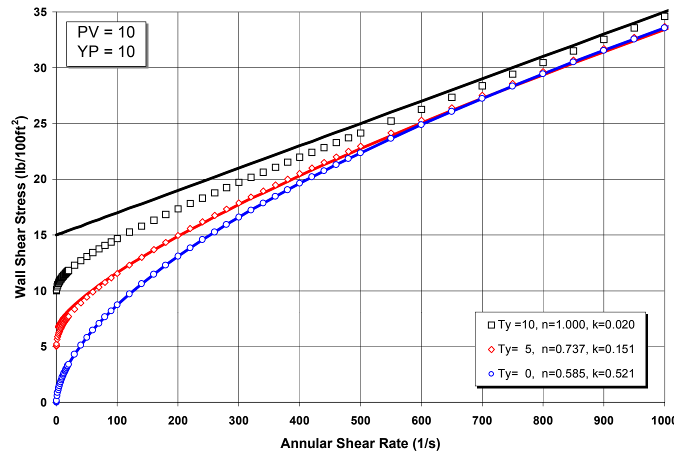

Fig. 4 - Annular flow curves, PV = 10, YP = 10, markers = exact solutions, solid lines = Unified model.

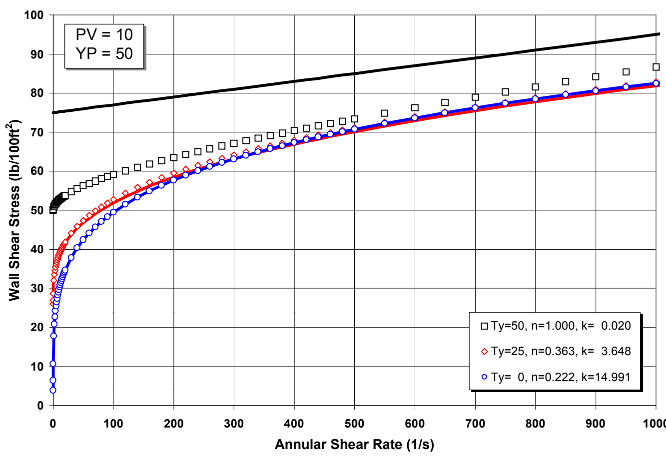

Fig. 5 - Annular flow curves, PV = 10, YP = 50, markers = exact solutions, solid lines = Unified model.

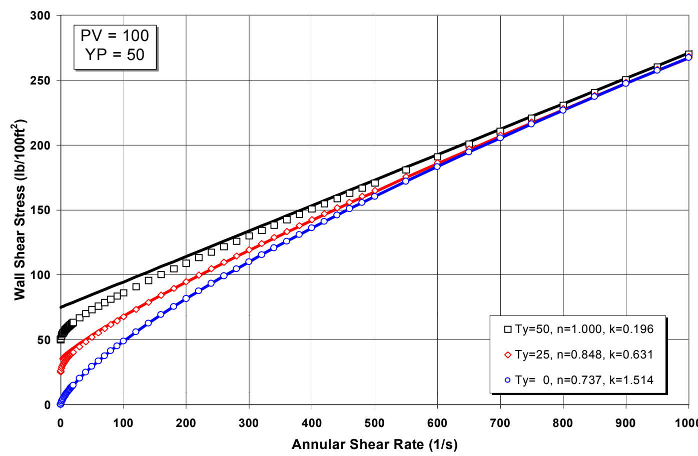

Fig. 6 - Annular flow curves, PV = 100, YP = 50, markers = exact solutions, solid lines = Unified model.

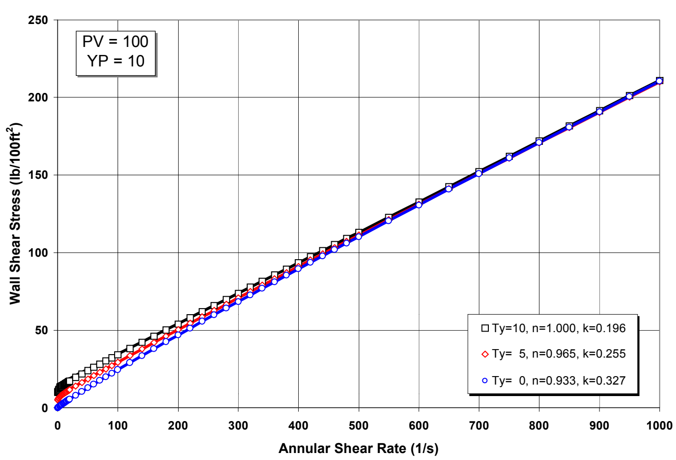

Fig. 7 - Annular flow curves, PV = 100, YP = 10, markers = exact solutions, solid lines = Unified model.

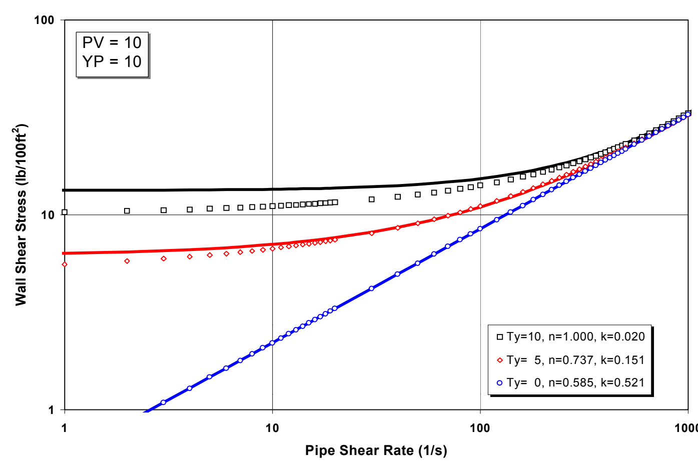

Fig. 8 - Logarithmic pipe flow curves, PV = 10, YP = 10, markers = exact solutions, solid lines = Unified model.