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Modeling Real-Time Wellbore Stability within the Theory of Poromechanics

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Abstract
Real-Time wellbore stability mud weight estimates, while drilling, are becoming industry standards. Overcoming wellbore drilling instability in shale formations will always present one of the major challenges to our drilling engineers. What if this shale rock is also fractured? What would be the necessary theories and tools to apply in Real-Time to model these types of problems? What should be the drilling fluid chemistry and salt concentrations if the drilling fluid is water based mud?
The answer to these challenging problems lies in the theory behind the models used in the industry. Poromechanics, or the poroelastic theory, as it is more commonly known, could couple the diffusion processes of the fluid mud pressure, chemistry, temperature, in-situ stresses and the fracture pressure with formation pressure disturbance, in the case of fractured shale formation. In this paper the poromechanics approach and the estimation of the mud weight stability for the vertical or the inclined wellbore are briefly discussed and presented. These solutions, implemented in the software RT-PBORE-3D, are capable of analyzing Real-Time Wellbore Stability in sound and/or fractured shale rock formations.

Introduction
In wellbore drilling, the pore pressure differential between the drilling mud and the drilled formation is governed by the diffusion process which is a time-dependent phenomenon. In addition, the downhole temperature and the chemical reactivity (mud/shale) are also time-dependent variables. As such, tools for Real-Time Wellbore drilling simulation should take into account these physical time-dependent processes, which involve among others stress/fluid pressure and temperature, coupled with the real-time monitoring data obtained periodically from LWD and SWD. The newly developed Real-Time Poromechanics Wellbore Stability simulator, RT-PBORE-3D, with its solid mechanics and poromechanics models is capable of coupling the real-time downhole data with the real-time-dependent diffusive processes, while upgrading the time-dependent mud weight window through the multi-layer drilling operations. The associated theories and their implementation in RT-PBORE-3D are here discussed and examples of real-time drilling scenarios are illustrated.

Biot's theory of poroelasticity[1] represents an accurate and proven prediction theory of coupled time-dependent phenomena induced by stress perturbations in the geomechanical domain. The importance of these coupled processes in field applications from wellbore stability problems and reservoir management to design and analysis of laboratory rock testing procedures has already been well recognized. The extension of Biot's theory of Poromechanics to fractured rock formation within the framework of the dual-porosity approach has also been accoompplished by Wilson and Aifantis[2]

RT-PBORE-3D is designed to simulate the three-dimensional time-dependent wellbore stability in real-time drilling operations for sound and fractured rock formations. The displayed values of mudweight windows for the corresponding drilled layers are calculated and updated as time and the drilling operation progress; thus illustrating the true nature of the phenomena of real-time diffusion, temperature gradient variations, and the evolution of chemical activities between the mud and the rock formation.

In this paper, a brief overview of both approaches to fractured and non-fractured rock formation is illustrated, and a validation of their use is also described. The applicability of RT-PBORE-3D is demonstrated through borehole stability analysis in multi-layer drilling operation with its associated time-dependent mudweight windows updates.

Poromechanics Constitutive Relations
The time-dependent coupled diffusion-deformation effects including the temperature gradients and the chemical activity are best modeled within the general theory of poromechanics. The evolution in time of these dynamic and kinematic variables, within the consistent frame of the poroelastic theory, will allow real-time simulation.

The time-dependent coupling of solid matrix/grain deformation, pore fluid flow and the temperature gradient, for a sound rock formation, is governed by the following constitutive equations:

$$\sigma_{ij} = M_{ijkl}\varepsilon_{kl} - \alpha_{ij}p - \beta_{ij}T \quad\text{................................(1)}$$

$$p = M(\zeta - \alpha_{ij}\varepsilon_{ij} + \beta_m T) \quad\text{................................(2)}$$

Eq. (1) describes how the overall strains of the porous rock rely on the applied total stresses, the pore pressure weighted by Biot's effective stress tensor α_{ij}, and the temperature gradient weighted by the thermic coefficient β_{ij}. Eq. (2), on the other hand, indicates that the fluid pore pressure is not only related to the pore fluid content variation, ζ, but is also affected by the deformation of the rock solid matrix and the change in temperature. It should be noted that under isothermal conditions, the terms $\beta_{ij}T$ and $\beta_m T$ do not contribute.

Fractured rock formation is modeled within the framework of dual-porosity. Specifically, two overlapping and separate continua are considered, one for matrix block and the other for fractures. Fig. 1 provides the dual-porosity realization of naturally fractured rock in this sense. With the introduction of the fractured medium, both the stresses and pore pressures are modified due to the additional coupling of deformation, fluid flow and temperature gradient in the fracture. The constitutive equations for a fractured rock are given as:

$$\sigma_{ij} = M_{ijkl}^{I,II}\left(\begin{array}{c} \varepsilon_{kl} - C_{klmn}^{I}\alpha_{mn}^{I}p^{I} - C_{klmn}^{II}\alpha_{mn}^{II}p^{II} \\ - (C_{klmn}^{I}\beta_{mn}^{I} + C_{klmn}^{II}\beta_{mn}^{II})T \end{array}\right) \text{..(3)}$$

$$p^{I} = M^{I}(\zeta^{I} - M_{ijkl}^{I,II}C_{klmn}^{I}\alpha_{mn}^{I}\varepsilon_{ij} + \beta_m^{I}T) \quad\text{........(4)}$$

$$p^{II} = M^{II}(\zeta^{II} - M_{ijkl}^{I,II}C_{klmn}^{II}\alpha_{mn}^{II}\varepsilon_{ij} + \beta_m^{II}T) \quad\text{.....(5)}$$

In the above equations, the superscripts I and II refer to the rock matrix and the fracture medium respectively, $M_{ijkl}^{I,II}$ is the drained elastic modulus tensor for the whole solid domain, C_{klmn}^{I} and C_{klmn}^{II} are the compliance tensor for the corresponding media. Eq. (3) to (5) are direct extension of Eq. (1) and (2) in the approach to naturally fractured rocks using dual porosity concept. In writing Eq. (3), it is assumed that the changes in applied total stresses in each medium are equal and the overall deformations are obtained by adding the strains of the two media. Therefore, the overall strains of the fractured rock are governed by the applied total stress, the fluid pore pressure in each medium and the temperature gradient. The fluid pore pressures response equations are analogous to that of Eq. (2) except that the strain in each is expressed as a portion of the overall deformation.

In the following sections "we" will illustrate the implementation and the benefits, of these constitutive relations and their associate field equations, in the solution of the aforementioned problems related to industry field applications.

RT-PBORE-3D: Poromechanics Approach to Real-Time Wellbore Stability Applications

Real-time wellbore stability analysis is only possible if the time-dependent processes that are responsible for instability and failures during wellbore drilling are updated periodically from the real-time data collected from the various tools in MWD. Time-dependency in real-time wellbore stability applications has two folds: (1) allows updating of in-situ stresses, pore pressure and rock and drilling mud properties as the drill bit moves ahead; and (2) takes into account the time-dependent coupled phenomena of pore fluid diffusion, temperature gradient and formation stress variation. These coupled diffusion-deformation processes, in wellbore drilling mechanics, are naturally accounted for in the aforementioned constitutive modeling using the theory of poroelasticity[1].

Analytical solutions for inclined boreholes in a general three-dimensional in-situ state of stress, accounting for time-dependent fluid diffusion, thermal and chemical effects, and rock formation material anisotropy, have been published and extensively applied in evaluating potential time-dependent effects on wellbore stability.[3--8] These solutions are implemented in the software, RT-PBORE-3D, developed for real-time drilling application. The solutions can perform analyses of stress/pore-pressure, formation failure, mudweight and mud salinity, temperature effect, etc. design for drilling in shale formations.

RT-PBORE-3D is equipped with a state-of-the-art user-friendly interface to facilitate data input and presentation of the output results. The software allows the real-time updating data from MWD tools and implementation in the time evolution design of wellbore-drilling program for mud-weight pressure windows, mud salinity, and wellbore depth-gradient analysis. These capabilities allow drilling engineers to make the necessary "on-rig" decisions on well control and potential critical time effects in drilling as well as completions, (e.g., time of casing placement and/or cementing, potential blow ups, etc.).

Wellbore Stability Analysis using RT-PBORE-3D

In this section, the use of RT-PBORE-3D is demonstrated in several field examples. The inclined wellbore problem which represents a general class of three-dimension wellbore geometry from vertical to horizontal wells is used hereafter. The Figures 1, 2 and 3 illustrate the corresponding geometries of the problem in both single-porosity and naturally fractured dual-porosity rock formations. Fig. 4 shows a sample input data screen from RT-PBORE-3D. Relevant data such as wellbore orientation, in-situ stress and pore pressure gradients, material properties i.e., the elastic modulus,

Poisson's ratio etc., rock failure parameters and the particular model to be used in the analysis are imported from Excel sheets and/or ASCII files, consistent with outputs from MWD, SWD, PWD, etc. A sample Excel sheet used in RT-PBORE-3D is shown in Fig. 5. To date PBORE-3D[9] (earlier versions of RT-PBORE-3D) has been extensively used in the industry in field operations under a variety of scenarios.[6, 8]

The real-time analysis module in RT-PBORE-3D was applied to a pilot well drilled in the North Sea. The input parameters used for development of the borehole stability model are as given in Table 1. In this case an elastic analysis was carried out for both, the sandstone and mudstone layers encountered and the collapse, fracturing and the stability mudweights were computed as shown in Fig. 6. Additionally, a similar analysis was carried out for a horizontal well drilled in the same field as the pilot well. Table 2 summarizes the mudweights predicted by RT-PBORE-3D and those used in the field.[10]

A poroelastic analysis of under-balanced drilling was carried out for a horizontal well in a highly depleted reservoir in Lake Maracaibo, Venezuela. The input parameters used are as given in Table 3. The collapse, fracturing and the stability mudweights for a given time are as shown in Fig. 7. RT-PBORE-3D predicted a safe mudweight of 2.7 ppg to be used throughout the drilling operation which compares favorably with the actual mudweight of 2.5 ppg used in the field.[11]

RT-PBORE-3D is now designed to accommodate real-time input data from generic Excel™ sheets that contain only relevant data at required depths. The user is provided the freedom to pick the analysis type (elastic or poroelastic), isotropy type (isotropic or transversely isotropic), pore pressure boundary conditions (permeable or impermeable) and the shear failure criteria (see Fig. 8). The data is imported periodically from standard Excel™ files that are organized in a specific format and the mudweight window for each layer is reupdated accordingly. Fig. 9 shows the results for an isotropic, poroelastic, permeable mudweight analysis for 3 depths using the Mohr-Coulomb criteria. RT-PBORE-3D also suggests the use of appropriate casing when the mudweight for collapse exceeds that for fracturing. Results are presented at a fractional distance into the formation, thus assuming strain relaxation on the immediate wellbore wall. Fig. 10 shows a snapshot of the data used, when viewed from within RT-PBORE-3D.

Analytical solutions for fractured formations based on the dual-porosity approach i.e., Eq. (3) to (5) has been developed and are incorporated into RT-PBORE-3D. In Figure 11 the mudweight window for instantaneous drilling of a permeable inclined borehole in a fractured rock formation is shown. For comparison purpose, the mudweight window for the corresponding single-porosity (non-fractured) rock is also plotted. Table 4 provides the data set used to generate the plot. It is obvious that the dual-porosity behavior of the fractured system deviate from the single-porosity as the mudweight window is shifted upward. Thus, the effect of fractures in a fractured rock can not be overlooked.

Conclusions
The study showed that time-dependency in wellbore stability can be critically examined for real-time support via a poromechanics approach.

Solutions for an inclined wellbore drilled in fractured or intact formations under a three-dimensional in-situ state of stress have been developed as part of the present research. These solutions are based on the poromechanics models, along with its conventional solid mechanics counterpart models. The solutions are all in analytical forms making RT-PBORE-3D efficient software for practicing engineers to simulate "real-time" analyses.

Nomenclature
φ_x = azimuth angle of the bore hole
φ_y = inclination angle of the bore hole
LWD = logging while drilling
M = Biot's modulus
M_{ijkl} = elastic modulus tensor
MWD = measurement while drilling
ppg = pound per gallon
ROP = drilling rate of penetration
rpm = revolutions per minute
S_H = maximum horizontal insitu principal stress gradient
S_h = minimum horizontal insitu principal stress gradient
S_v = vertical insitu principal stress gradient
$S_x, S_y, S_z, S_{xy}, S_{yz}, S_{xz}$ = local borehole stress insitu stress
p_o = virgin pore pressure gradient
TD = total depth
TVD = true vertical depth

References
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2. Wilson, R.K. and Aifantis E.C.: "On the Theory of Consolidation with Double Porosity," *International Journal of Engineering Science*, Vol. 20, pp. 1009-1035, 1982.
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8. Abousleiman, Y. *et al.*: "Time-dependent wellbore-stability predictions", *J. of Petroleum Technology, JPT, Drlling Technology,* February, 2001.
9. Abousleiman, Y. *et al.*: "Tools in Poromechanics with Laboratory and Field Applications", *Proceedings of 2nd Biot Conference on Poromechanics,* France, Grenoble, 2002.
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11. Parra, J.G., Gennaro and D.De: "Wellbore stability simulations for underbalanced drilling operations in highly depleted reservoirs", SPE 65512 presented at the 2000 SPE/Petroleum Society of CIM International Conference on Horizontal Well Technology held in Calgary, Alberta, Canada, 6-8 November, 2000.

Tables

Table 1. Input parameters for real-time wellbore stability analysis (North Sea)

Parameter	Value	Value
Depth Range (ft)	8,776-9,000	9,000-9,500
Rock Type	Sandstone	Mudstone
Overburden Stress Gradient, S_v (psi/ft)	1.0	1.0
Maximum Horizontal Stress, S_H (psi/ft)	0.75	0.75
Minimum Horizontal Stress, S_h (psi/ft)	0.61	0.61
Pore Pressure, P_o (psi/ft)	0.48	0.48
Cohesion (psi)	1,252.3	721
Friction Angle (degrees)	38.7	52.9
Young's Elastic Modulus (psi)	2,250,000	4,000,000
Poisson's Ratio	0.16	0.33

Table 2. Comparison of mudweights predicted by RT-PBORE-3D with actual mudweights used

Well	Mudweights (lb/gal)	
	RT-PBORE-3D	Actual
P2-NE2-Pilot	10.8	10.43
P2-NE2-Horizontal	10.8	10.43

Table 3. Input parameters for poroelastic analysis of under-balanced drilling (Lake Maracaibo, Venezuela)

Parameter	Value
Hole Angle (degrees)	90
Azimuth (degrees)	0
$S_v/S_H/S_h$ gradients (psi/ft)	0.95/0.64/0.52
Pore pressure gradient (psi/ft)	0.14
Cohesion (psi)	2020
Friction angle (degrees)	35.5
Elastic Modulus (psi)	2400000
Poisson's ratio	0.18
Biot's Effective Stress Coefficient	0.91
Undrained Poisson's ratio	0.43
Permeability (md)	100
Fluid viscosity (poise)	0.07

Table 4. Input parameters for dual-porosity mudweight analysis of the inclined wellbore in fractured formation

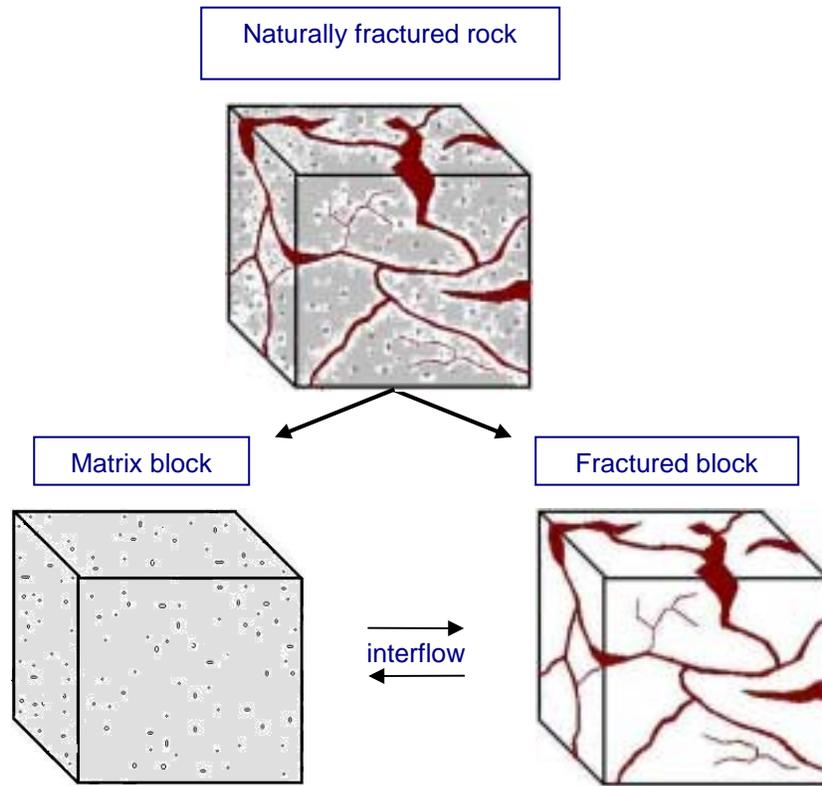| Parameters	Single Porosity	Double Porosity
$S_v/S_H/S_h/ P_o$ (psi/ft)	1.11/1.28/0.88/0.43	
Azimuth (degrees)	70	
Matrix Permeability (md)	0.001	0.001
Fracture Permeability (md)		0.1
Matrix Porosity (fraction)	0.02	0.02
Fracture Porosity (fraction)		0.002
Modulus Young (psi)	268,755	268,755
Poison Ratio (dimensionless)	0.219	0.219
Bulk Grain Modulus (psi)	4,986,397	4,986,397
Fracture Normal Stiffness (psi)		498,639
Fracture Spacing (ft)		1.64
Bulk Fluid Modulus (psi)	35,244	35,244
Fluid Viscosity (cp)	1	1
Time (day)	0.001	0.001

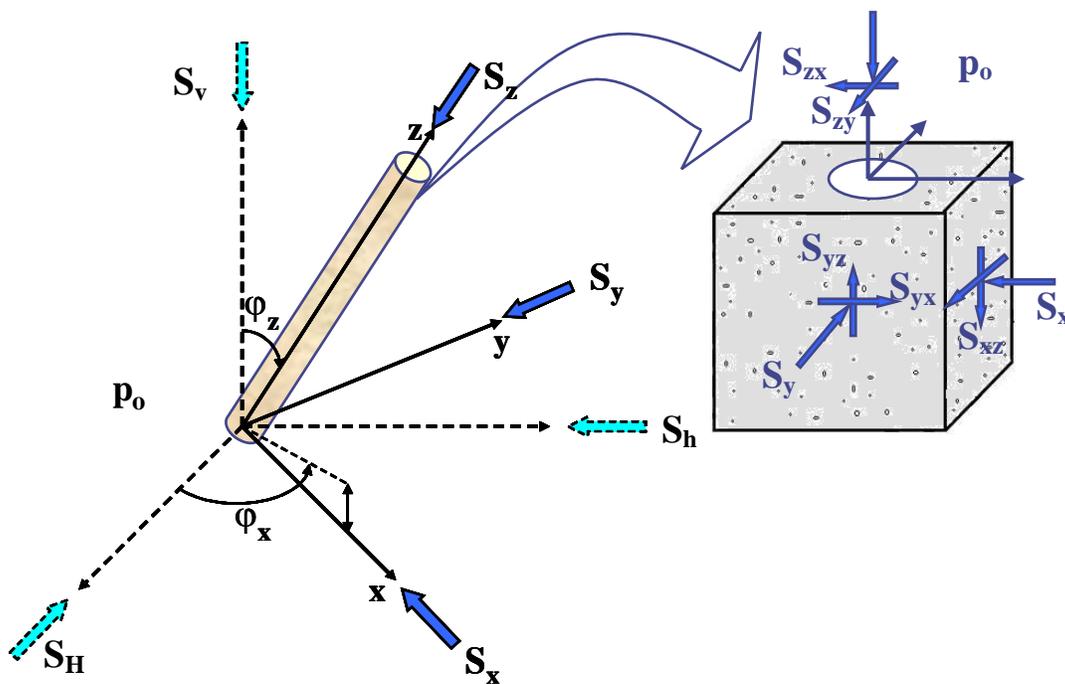Figure 1: Visualization of a double porosity naturally fractured rock formation

Figure 2: Geometry of the inclined wellbore problem in single-porosity rock
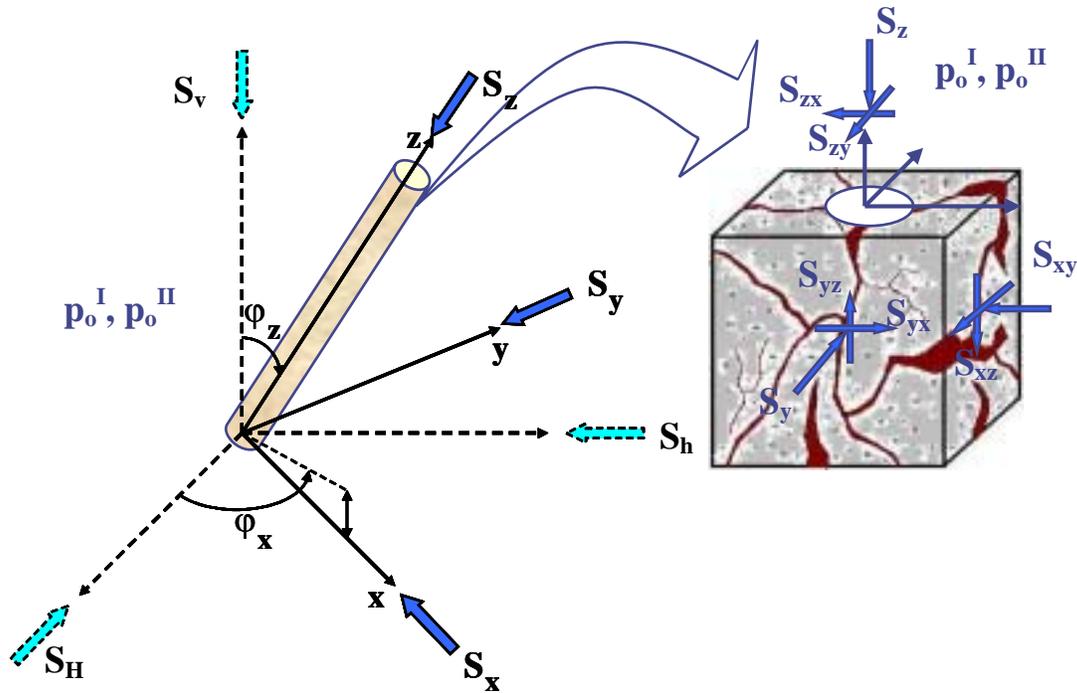
Figure 3: Geometry of the inclined wellbore problem in dual-porosity fractured rock
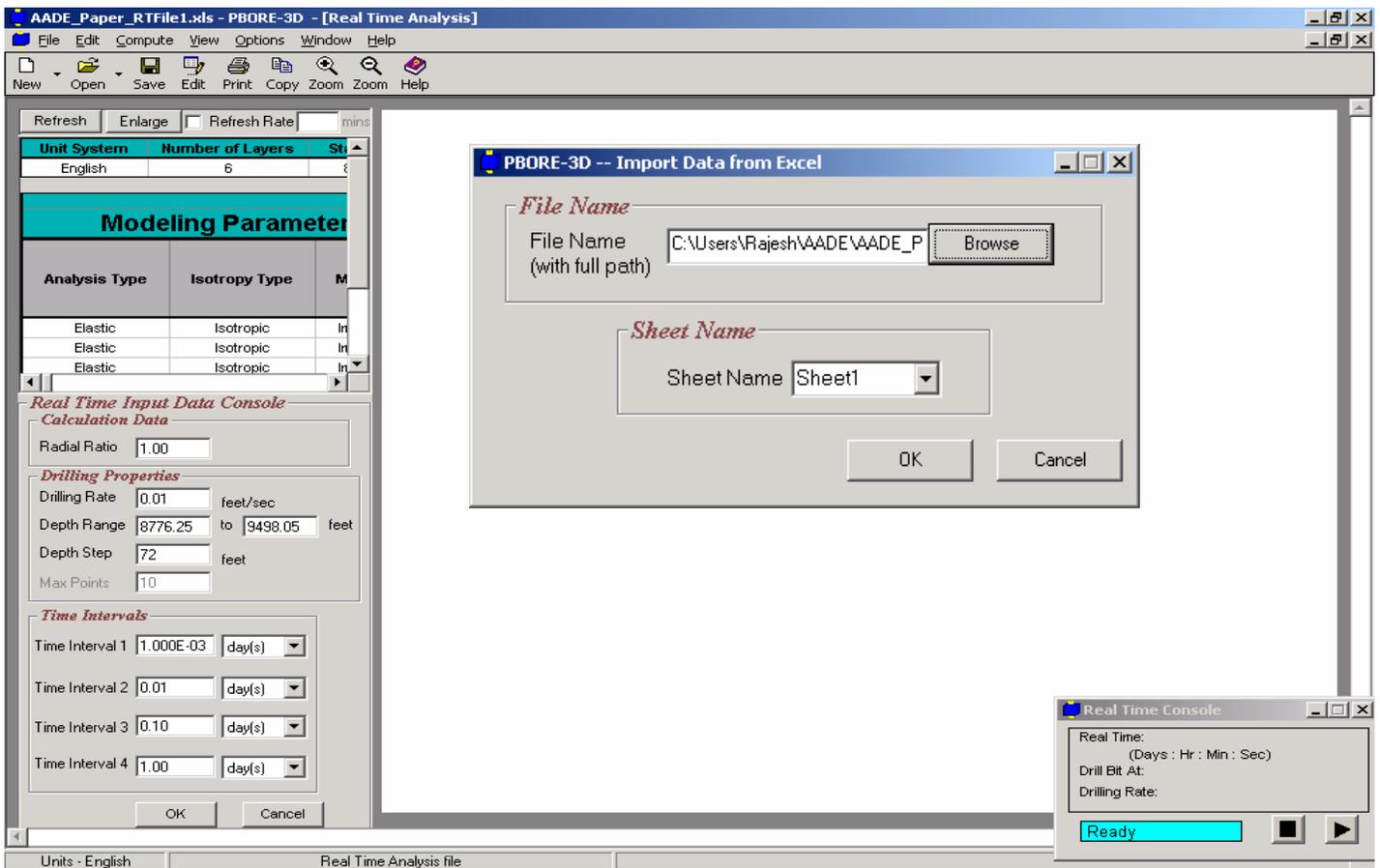

Figure 4: The RT-PBORE-3D Input data screen

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	A	B	C	D	E	F	G	H	I	J	K
1	Unit System	Number of Layers	Start Depth	Hole Radius	Drilling Rate	Time Int. 1	Unit	Time Int. 2	Unit	Time Int. 3	Uni
2	English	6	8776.25	0.33	1.00E-02	1.00E-03	days	1.00E-02	days	1.00E-01	days
3											
4	**Modeling Parameters**			**Wellbore Geometry**							
5											
6	Analysis Type	Isotropy Type	Model Type	Thickness	Hole Angle	Azimuth	Sv	SHMax	SHMin	Pore Pressure	Failur Criter
7	Elastic	Isotropic	Impermable	65.62	55	90	1	0.75	0.61	0.48	Drucker-F
8	Elastic	Isotropic	Impermable	98.43	55	90	1	0.75	0.61	0.48	Drucker-F
9	Elastic	Isotropic	Impermable	65.62	55	90	1	0.75	0.61	0.48	Drucker-F
10	Elastic	Isotropic	Impermable	229.66	55	90	1	0.75	0.61	0.48	Drucker-F
11	Elastic	Isotropic	Impermable	32.81	55	90	1	0.75	0.61	0.48	Drucker-F
12	Elastic	Isotropic	Impermable	229.66	55	90	1	0.75	0.61	0.48	Drucker-F
13											
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Sheet1 / Sheet2 / Sheet3 /

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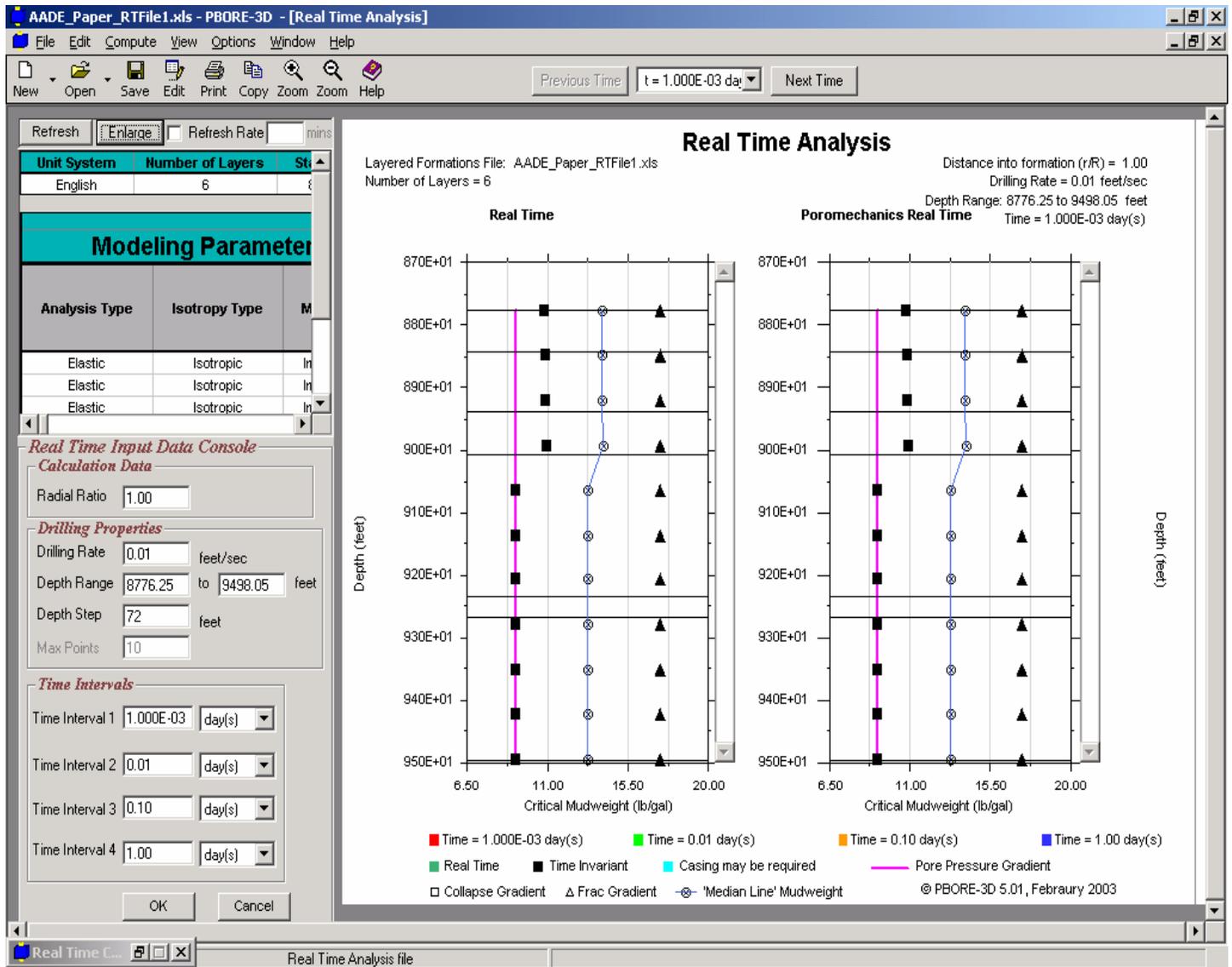Figure 5: A view of the Excel input data sheet for Real-Time Analysis

Figure 6: Collapse, fracturing and median-Line mudweights from real-time analysis (North Sea)
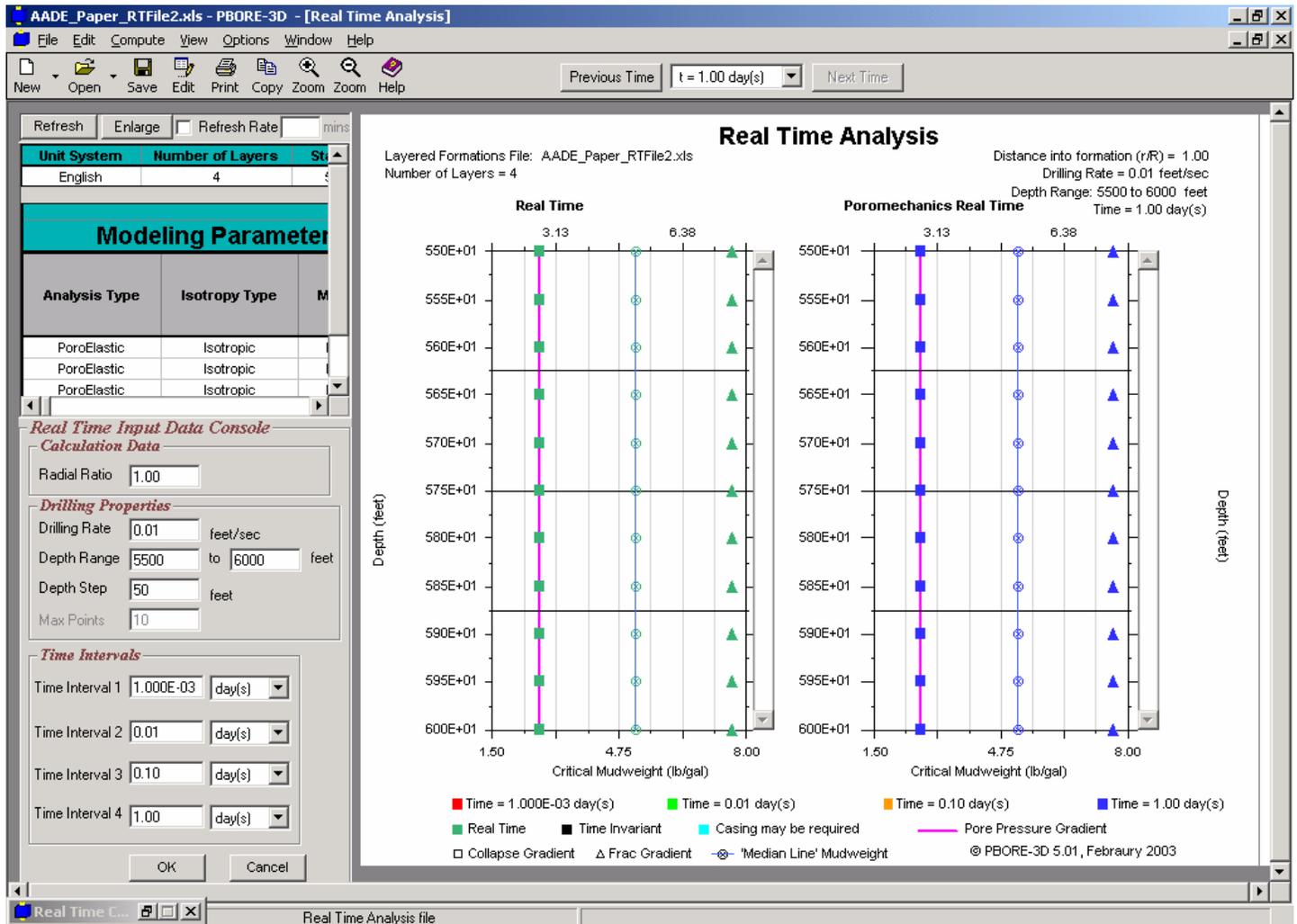
Figure 7:Collapse, fracturing and median-Line mudweights from real-time analysis (Lake Maracaibo, Venezuela)

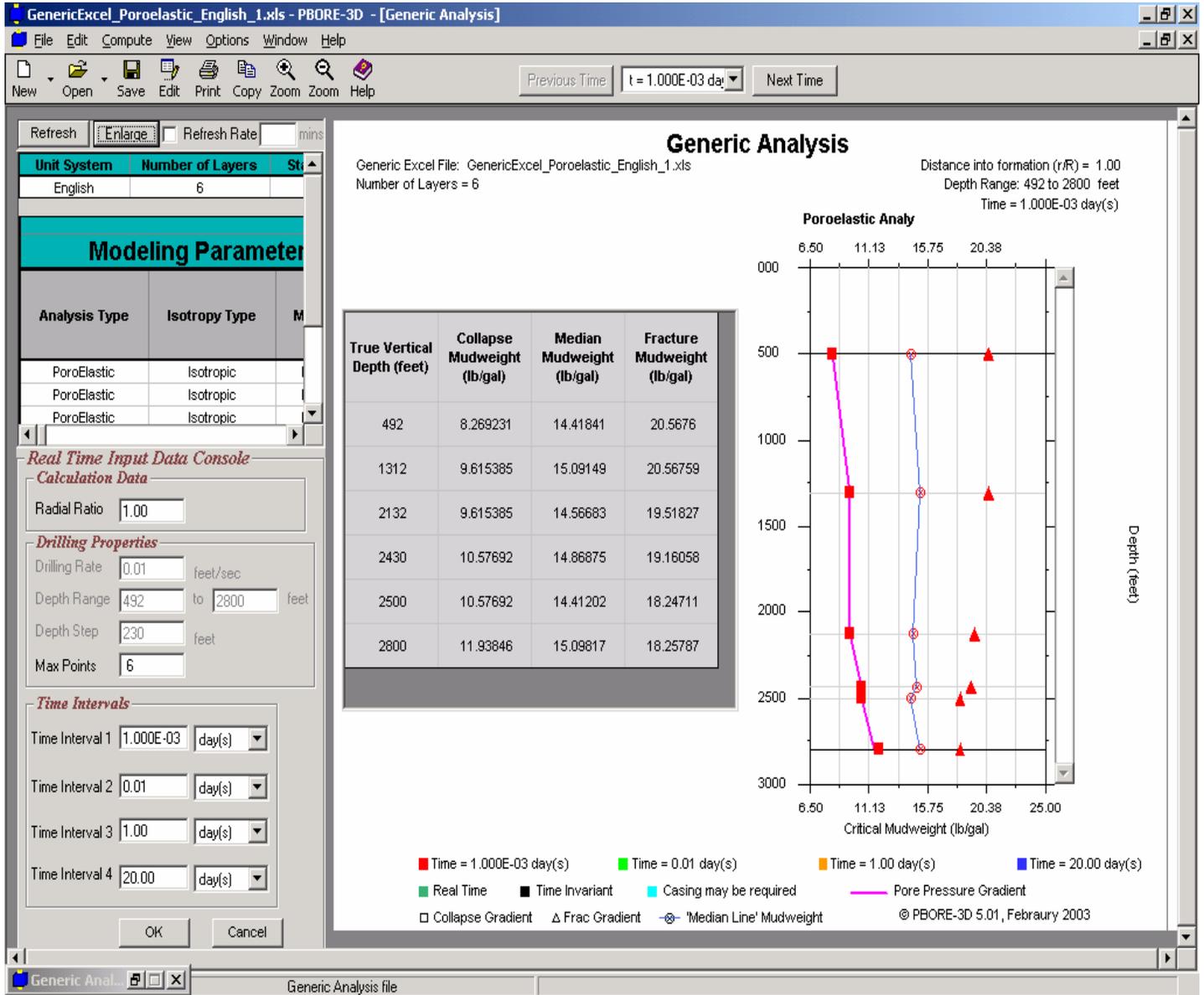Figure 8: Input screen for the generic Excel file option in RT-PBORE-3D

Figure 9: Results from the Generic Excel File option using a permeable, poroelastic analysis with the Mohr-Coulomb failure criteria.

Unit System	Number of Layers	Start Depth	Hole Radius	Drilling Rate	Time Int. 1	Unit	Time Int. 2	U
English	6	492.00	0.10	0.01	1.000E-03	days	0.01	

Modeling Parameters			Wellbore Geomet					
Analysis Type	Isotropy Type	Model Type	True Vertical Depth	Hole Angle	Azimuth	Sv Gradient	SHMax Grad	
PoroElastic	Isotropic	Permeable	492.00	50.00	30.00	1.28	1.11	
PoroElastic	Isotropic	Permeable	1312.00	55.00	30.00	1.28	1.11	
PoroElastic	Isotropic	Permeable	2132.00	60.00	30.00	1.28	1.11	
PoroElastic	Isotropic	Permeable	2430.00	65.00	30.00	1.28	1.11	
PoroElastic	Isotropic	Permeable	2500.00	65.00	30.00	1.28	1.11	
PoroElastic	Isotropic	Permeable	2800.00	65.00	30.00	1.28	1.11	

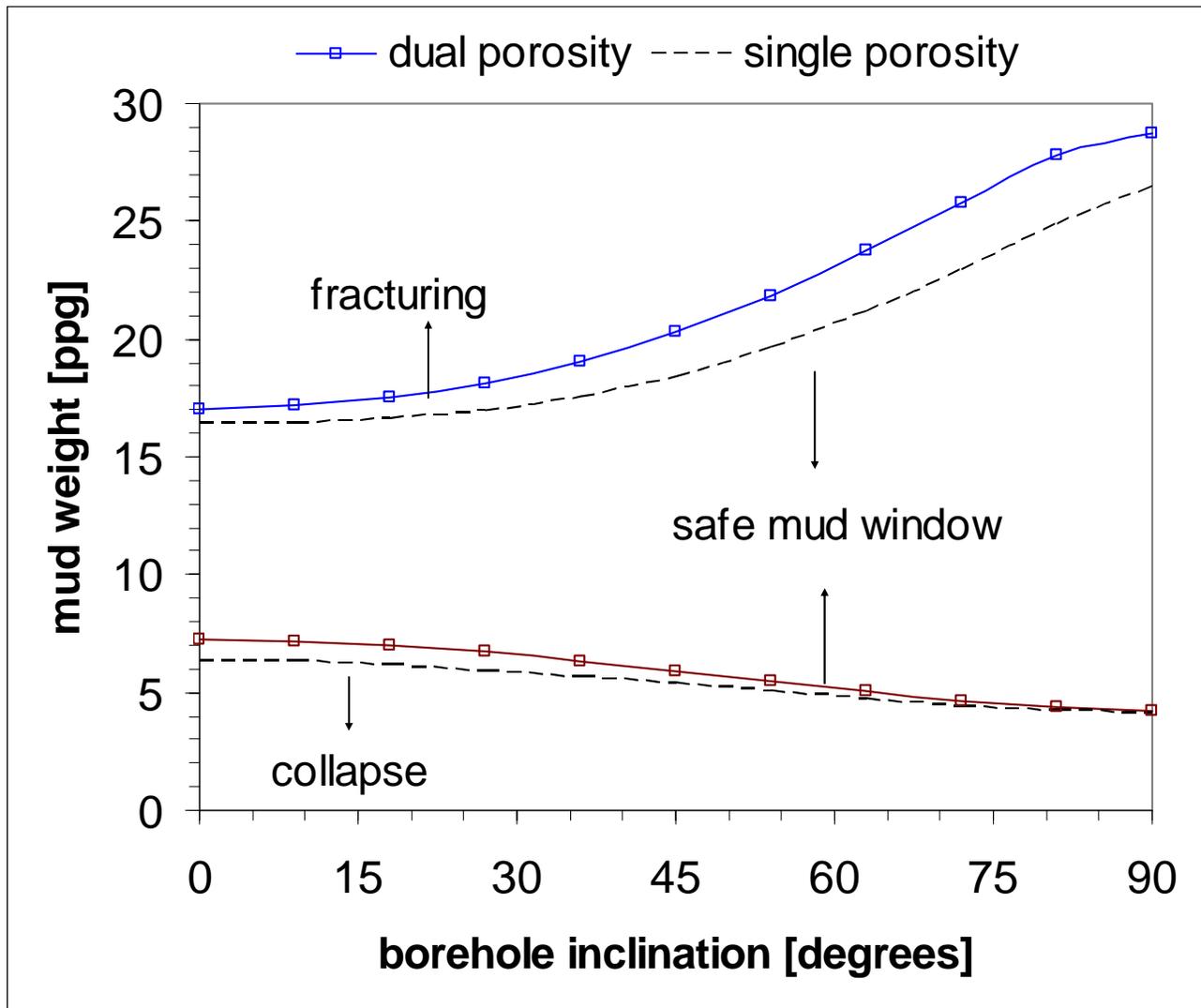Figure 10: Snapshot of data used to generate results in Figure 9 (when viewed from within RT-PBORE-3D).

Figure 11: Mudweight window for an inclined wellbore in naturally fractured rock formation after 0.001 day