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Onsite Continuous Hydraulics Optimization (OCHO)

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Abstract

Optimization of well hydraulics, while generally understood for several years, has never reached its full potential due to the relative difficulty in the past of implementing the optimization procedures. As today’s well bores reach further and further, largely with equipment that has not changed in decades, it becomes even more important to efficiently utilize available hydraulic energy. Innovative down hole tools currently consume more and more hydraulic energy which leaves less and less for removing cuttings from beneath the drill bit. This energy consumption can be recognized and the maximum hydraulic power or force made available at the bit. A simple, rig-floor method to optimize hydraulics is presented.

Introduction

Hydraulic optimization can mean many things to many different drilling personnel. It can refer to maximizing annular flow rate, maximizing the force with which the fluid strikes the bottom of the hole (impact), or maximizing the power expended through the nozzles in the drill bit (hydraulic power). This development will focus on an attempt to maximize cuttings removal beneath a drill bit. Better cuttings removal will result in elevated bit founder points and promote faster drilling.[1, 2] Equations are derived in the appendix to maximize this cuttings removal based on either hydraulic impact or on hydraulic horsepower.

One of the problems with predicting pressure losses in well bores is the fact that not all of the variables are known. For example: A fluid is flowing at 45 gallons per minute inside of a 2” inside diameter, 100 ft long pipe. What is the pressure loss in the pipe? In this simple example, the fluid description is absent from the problem specifications. Would it make a difference if the fluid were water, alcohol, or honey? Why? Obviously, the viscosity of the fluid would have a big impact on the pressure loss. On a drilling rig, the rheological properties of a drilling fluid are measured at either 120 ^{o}F or sometimes at 150 ^{o}F for an oil-based drilling fluid in a hot hole. Unfortunately, rheological properties cannot be predicted at other temperatures (and pressures) from measurements made at one temperature.

Second, the flow rheology is unknown in a well bore. When fluid is flowing in laminar flow in a pipe, the pressure drop in a pipe is proportional to the flow rate. When fluid is flowing in turbulent flow, the pressure drop in a pipe is proportional to the square of the flow rate. Within a drill string, tool joints disrupt the flow profile between each joint. In a very viscous drilling fluid, very little turbulence may be experienced; in a very low viscosity fluid, a lot of turbulence may be experienced. The induced turbulence would not necessarily continue throughout the pipe joint. So the pressure drop through drill pipe should be proportional to flow rate raised to some exponent between 1 and 2, inclusive. These values cannot be predicted with any certainty before the well is drilled.

Many hydraulic programs assume an exponent (on flowrate) of around 1.82 to 1.85. This range of values is based on measurements made shortly after the Second World War. A better solution would be to measure the effect at the rig site and use the well bore as a rheometer. This “rig-measured exponent” will include all of the unusual features within the circulating system (large diameters, small diameters, changes in viscosity with pressure and temperature, changes in flow regimes, etc.)

This paper describes a proven technique that maximizes either the hydraulic impact force or the hydraulic power of the fluid hitting the bottom of the hole.

Discussion

OCHO (Onsite Continuous Hydraulic Optimization) is an eight-step plan for tailoring the hydraulics program to the well bore as it is being drilled. The eight steps are:

1. Calibrate rig pumps. Measure the rate of liquid level drop in the slugging tank while pumping

down hole through the drill bit. Account for air in the drilling fluid to calculate the volume of liquid moved by the rig pumps.

2. Just before tripping for a new bit, circulate at several pump rates and measure accurately the standpipe pressure at each rate.
3. Calculate and subtract the bit nozzle pressure drops from the measured standpipe pressures (This gives the circulating pressure loss through the system, except for the bit nozzles.)
4. Plot the circulating pressure loss as a function of flow rates on log-log paper.
5. Draw the best straight line through the circulating pressure losses.
6. Measure the slope of the circulating pressure line with a ruler or scale.
7. Calculate the optimum pressure loss through the bit to give either the maximum hydraulic force or the maximum hydraulic power at the bit.
8. Calculate nozzle sizes for the next bit.

Hydraulic impact is a force, F that is the mathematical product of the fluid density ,ρ, flow rate, Q, and velocity, v. With suitable conversion units, the force may be expressed in pounds, kilograms, or newtons of force using the equation:

$$F = \rho \; Qv$$

Hydraulic power, HP, may be calculated by multiplying the force, F, by the velocity of the fluid, v; or

$$HP = F\,v = \rho \, Q \, v^2$$

Both techniques will require calculating the force with which the fluid strikes the bottom of a hole. This force can be related to the pressure drop through the nozzles as shown in the derivation below:

Derivation of Hydraulic Impact Force

Newton's Second Law of Motion, F=ma, can also be expressed as a change in momentum since the acceleration is a rate of change of velocity, or

$$F = \frac{m\,v_2 - mv_1}{t}$$

The fluid moving downward toward the bottom of a hole starts with a velocity v_2 and is stopped by the bottom of the hole from further downward movement, i.e. v_1 =0. The mass flow rate, or m/t, could be expressed as the product of the fluid density, ρ , and the flow rate, Q. The equation then becomes

$$F = \rho \; Qv$$

The pressure loss through a nozzle, ΔP, can be expressed as:

$$\Delta P_{bit} = \frac{\rho \, Q^2}{K_1 \, A^2}$$

Since Q/A is velocity, the equation can be written:

$$\Delta P_{bit} = \frac{\rho \, v^2}{K_1}$$

This equation can be solved for the velocity and that term substituted into the force equation.

$$v = \sqrt{\frac{K_1 \, \Delta P_{bit}}{\rho}}$$

Since force is ρQv, then force must also be

$$F = Q\sqrt{K_1 \; \rho \, \Delta P_{bit}}$$

The density is a constant so it can be combined with the other constants in the equation and gives the expression for force:

$$F = K\,Q\,\left(\Delta P_{bit}\right)^{0.5}$$

This expression is used to develop the mathematical relationships that will maximize the hydraulic impact or power, as shown in the Appendix.

The pressure loss through the system will be related to flow rate raised to an exponent between one and two. This exponent, u, is unique for every well and is characteristic of the well at the time it is determined. Put another way, this characteristic exponent will change over the life of the well and hence must be determined for each bit independently.

Maximizing *Hydraulic Impact*

As shown in the Appendix, two separate regimes exist for maximizing the force with which the fluid strikes the bottom of the hole. The first regime is where the Maximum Standpipe Pressure is limited by the available hydraulic power at the rig. Pumps are powered by connecting them to suitable motors. If small motors drive pumps, very little power will be available for pumping fluid. This regime occurs at the shallower depths where the optimum flow rate is high.

Each drilling rig has a maximum possible standpipe pressure. The limit might be the pressure ratings of the pump liners, or physical piping, or contract limitations, or a bubble in the rotary hose that limits the pressure. This is the second regime. Between these two regimes, the hydraulic impact will be limited by both the hydraulic power and the maximum standpipe pressure.

Optimum Conditions:

Either the impact force or the hydraulic power can be maximized to use as a criterion for optimization. Searching the literature for years has failed to reveal valid correlative investigations that confirm one method is better than the other. Both will be presented here:

MAXIMUM IMPACT FORCE EQUATIONS

If the limiting condition is hydraulic power, the pressure loss through the bit which creates the maximum impact force obtained from the fluid flowing through the bit nozzles, can be calculated from the equation:

$$\Delta P_{\substack{bit\\opt}} = \frac{u+1}{u+2} P_{HP} ,$$

where the term 'u' is the exponent of the flow system as described previously, and P_{HP} is the maximum available pressure as limited by available power.

However, if the limiting condition is not power but is stand pipe pressure, then the pressure loss through the bit which creates the maximum impact force, obtained from the fluid flowing through the bit nozzles, can be calculated from the equation:

$$\Delta P_{\substack{bit\\opt}} = \frac{u}{u+2} P_{max}$$

On a drilling rig, motors power the mud pumps with a finite amount of power. Generally, the hydraulic power can be obtained by assuming a mechanical efficiency of power transfer of about 85% and a volumetric efficiency of 93% to 95%.

MAXIMUM HYDRAULIC POWER EQUATIONS

For maximum hydraulic power, the limiting feature on a drilling rig will always be the maximum standpipe pressure possible. The optimum pressure loss through the drill bit will be:

$$\Delta P_{\substack{bit\\opt}} = \frac{u}{u+1} P_{max}$$

This will normally result in a lower flow rate in the well than optimizing for hydraulic impact.

Limiting Conditions for Both Optimization Criteria

On a drilling rig a motor, or motors, are dedicated to providing hydraulic power to drive the mud pumps. Thus, the first limiting condition is the hydraulic power. The second limiting condition is the maximum standpipe pressure. Before the well is spudded, these values can be placed on a log-log plot of pressure and flow rate.

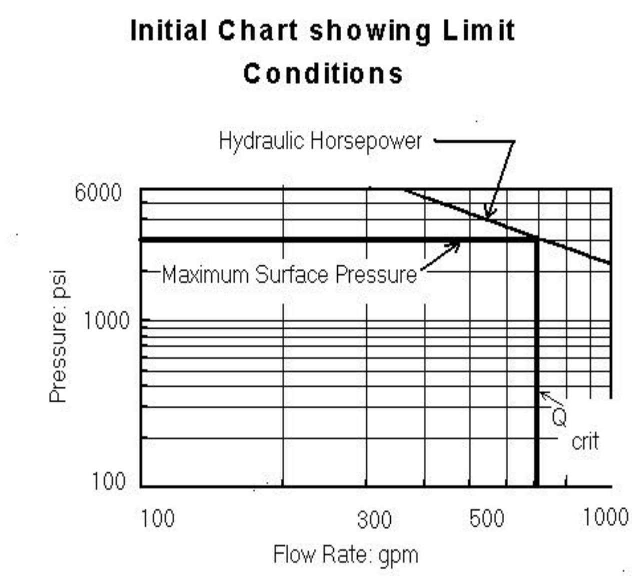

Figure 1
Rig Limiting Conditions

The maximum surface pressure intersects the available hydraulic horsepower line at a flow rate called Q_{crit}, or Q- critical. This represents the flow rate where both the maximum stand pipe pressure and the maximum hydraulic horsepower can be used. The area to the right of Q_{crit} is the region where the limit conditions would be the maximum available hydraulic power; the area to the left of Q_{crit} is the region where the limit condition is the maximum standpipe pressure.

Measuring "u"

The optimum bit pressure drop is related to the constant "u" which is a characteristic of a particular system. It is the slope of pressure loss curve for the entire system (P_{circ}), except for the drill bit, plotted on a log-log graph. The total system pressure losses, which would be equivalent to the standpipe pressure (P_{surf}), may also be calculated as the sum of the bit pressure loss (P_{bit}), and the circulating system pressure loss (P_{circ}). Note that in a physical sense, in the generalized proportionality for pressure drop as a function of flowrate:

$$\Delta P \approx Q^u ,$$

u is the exponent on flowrate.

The pressure loss through the nozzles of a drill bit may be calculated from the equation:

$$\Delta P_{bit} = \frac{(MW)(Q)^2}{12042\,(1.03)^2 (\text{Nozzle Area})^2} .$$

Where:

MW is the mud weight in ppg
Q is the flow rate, in gpm, and
Nozzle area is the total flow area of the nozzles, in square inches.

This equation is slightly different from older and perhaps more familiar equations in that the nozzle coefficient of 0.95 has been replaced by a nozzle coefficient of 1.03 for the new nozzle shapes. This is an attempt to quantify the pressure recovery effect observed from field measurements. This coefficient was also independently validated in controlled laboratory tests.[3,4] Note that in some companies, the 12042 constant and the nozzle coefficient are combined into a single term.

The standpipe pressure is read for at least four flow rates just before tripping the drill bit.

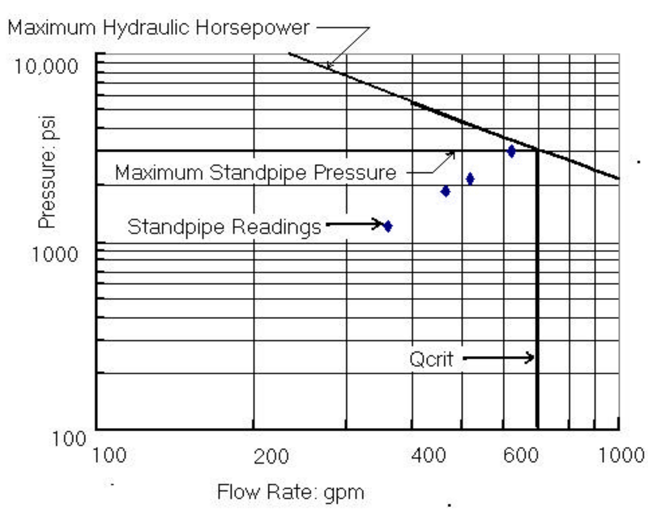

Figure 2
Standpipe Readings Prior to Pulling Drill Bit

At each circulating rate, the pressure loss through the drill bit may be calculated.. The circulating pressure loss, or pressure loss in the system may be calculated by subtracting the calculated bit pressure loss from the measured standpipe pressure.

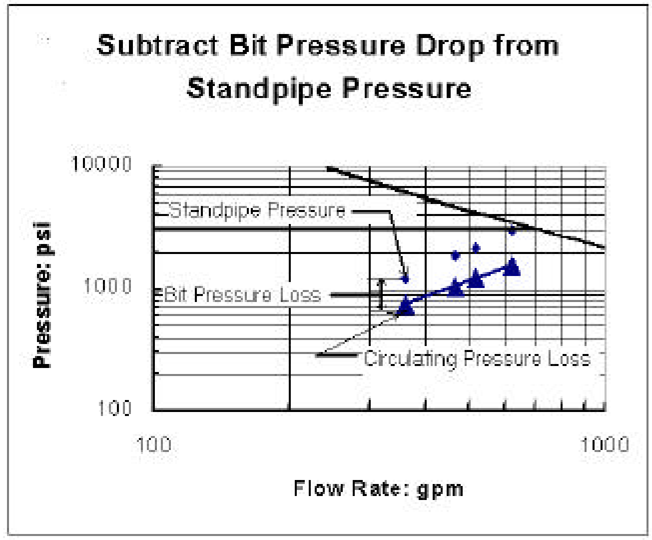

Figure 3
Creating the Circulating Pressure Line

With a ruler, measure the slope of the circulating pressure line. In the example problem, the slope is 1.4. This is the value of u that is used in the optimizing equations

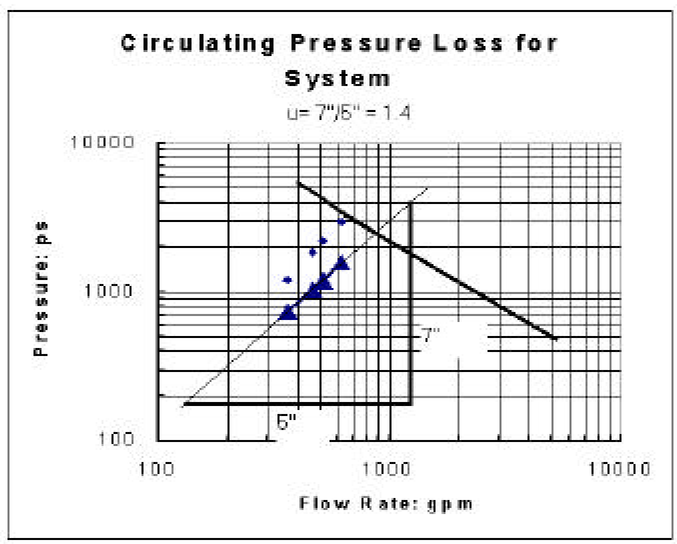

Figure 4
Measuring the slope "u" of
The Circulating Pressure Line

Optimum Circulating Flow Rates:

When the slope, u, has been determined, the optimum pressure loss through the nozzles may be found to either maximize the hydraulic impact or the hydraulic power using the equations from the appendix.

For Maximum Hydraulic Power at the Bit:

For hydraulic power, the optimum pressure loss through the drill bit nozzles would be calculated from the equation:

$$\Delta P_{\underset{opt}{bit}} = \frac{u}{u+1} P_{max} \text{ or}$$

$$\Delta P_{\underset{opt}{bit}} = \frac{1.4}{1.4+1} 3000\text{psi} = 1750 \text{ psi}$$

If the optimum pressure loss through the drill bit nozzles is 1750 psi and the maximum pressure is 3000 psi, the optimum circulating pressure loss would be 3000 psi – 1750psi or 1250 psi. Draw this horizontal line on the pressure-flow rate chart. The proper flow rate to produce this optimum circulating pressure loss will be where that line intersects the circulating pressure line.

The equation for the circulating pressure line would indicate that the circulating pressure is equal to a constant times the flow rate raised to the 1.4 power. The constant can be determined from one of the data points (P=739psi; Q=361gpm or k=0.194). The optimum circulating rate can now be calculated from the equation:

$$1250 \text{ psi} = (0.194)(Q_{opt}^{1.4}),$$

or Q_{opt} = 526 gpm

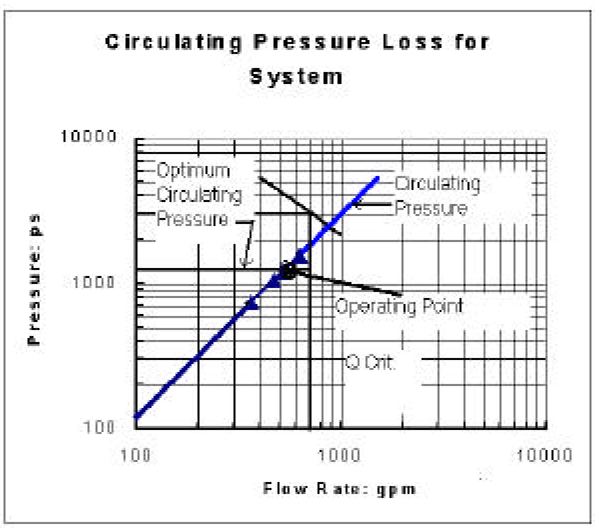

Figure 5
Determining the Operating Point for the Next Bit

In this case the optimum flow rate from the graph would be 520 gpm and the pressure loss through the bit nozzles should be 1250 psi. This will cause the drilling fluid to expend the maximum hydraulic power hit the bottom of the hole. From the equation for pressure loss through the drill bits, the nozzle area may be calculated and proper nozzles dressed into the new drill bit.

$$\text{Area} = \frac{(11.7\text{ppg})(525\text{gpm})^2}{(12042)(1.03)^2(1250\text{psi})} = 0.2019 \text{ in}^2$$

The bit can be dressed with one 11/32” and one 12/32” nozzles or one 9/32” and two 10/32” nozzles.

For Maximum Hydraulic Impact at the Bit:

For hydraulic impact at the bit, the optimum pressure loss through the drill bit nozzles would be calculated from the equations:

$$\Delta P_{\underset{opt}{bit}} = \frac{u}{u+2} P_{max} \text{ , or}$$

$$\Delta P_{\underset{opt}{bit}} = \frac{u+1}{u+2} P_{HP} \text{;}$$

depending upon whether the optimum circulating flow rate is below or above the critical flow rate, Q_{crit} , respectively.

Since u=1.4, and the P_{circ} line crosses the optimum line to the left of, or less than Q_{crit}, these values may be calculated as:

$$\Delta P_{\underset{opt}{bit}} = \frac{1.4}{3.4} P_{max} = 1235 \text{ psi} \text{ , or}$$

$$\Delta P_{\underset{opt}{bit}} = \frac{u+1}{u+2} P_{HP} = 2118 \text{ psi} \text{ ; at } Q_{crit} \text{ .}$$

These lines could be drawn on the initial chart, as shown on the following page. The shaded regions represent the portion of the total available pressure losses that should be devoted to or ‘spent’ across the bit for optimum hydraulics operations.

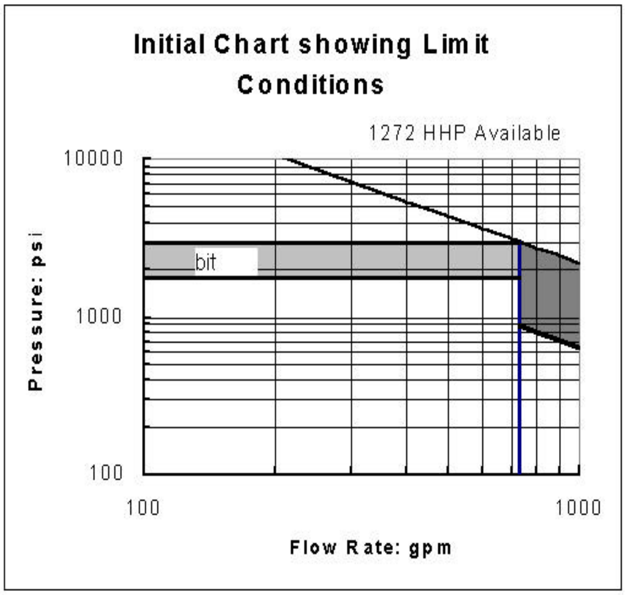

Figure 6
Limit Conditions with Optimum Bit Pressure Loss

The Circulating Pressure Line crosses the Optimum Line at the operating point, as shown below in Figure 7.

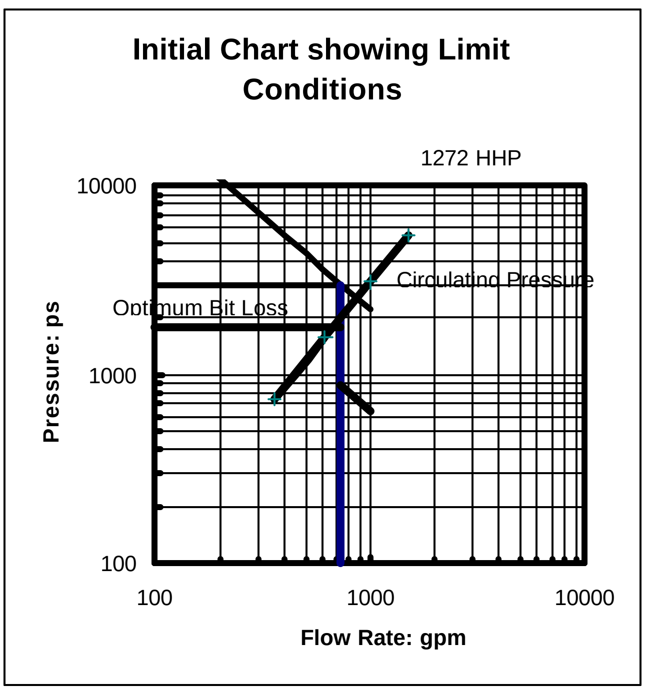

Figure 7
Circulating Pressure Line Crosses the Optimum Pressure Line at 670 gpm

The Equation for the Circulating Pressure Line is

3000psi –1235psi = $1765 \text{ psi} = (0.194)(Q_{opt}^{1.4})$,

Q_{opt} = 673 gpm

For a bit pressure loss of 1235psi at a flow rate of 673 gpm, the nozzle area may be calculated from the equation:

$$\text{Area} = \frac{(11.7\text{ppg})(673\text{gpm})^2}{(12042)(1.03)^2(1235\text{psi})} = 0.3359 \text{ in}^2$$

Three 12/32" nozzles should be installed into the new bit to make the drilling fluid strike the bottom of the hole with the most force possible.

Comments

Optimizing hydraulics on one drilling rig resulted in a surprising savings. Prior to optimization, the drilling rig was using between 1700 and 2300 gallons of diesel fuel per day. The optimization process was initiated around day 20 and resulted in an immediate decrease in fuel usage. (Perfect optimization was not achieved because the field personnel believed the change in hydraulics was too drastic.) Just before reaching casing depth, the procedure was repeated to better identify the correct flow rate and nozzle sizes for maximum hydraulic impact. With the field personnel fully convinced, the correct hydraulics were run on the next bit. For the two days before reaching the casing point, the fuel usage dropped to 920 gallons of diesel. The decrease was unexpected but could be easily rationalized by the fact that the optimization procedure uses energy more efficiently.

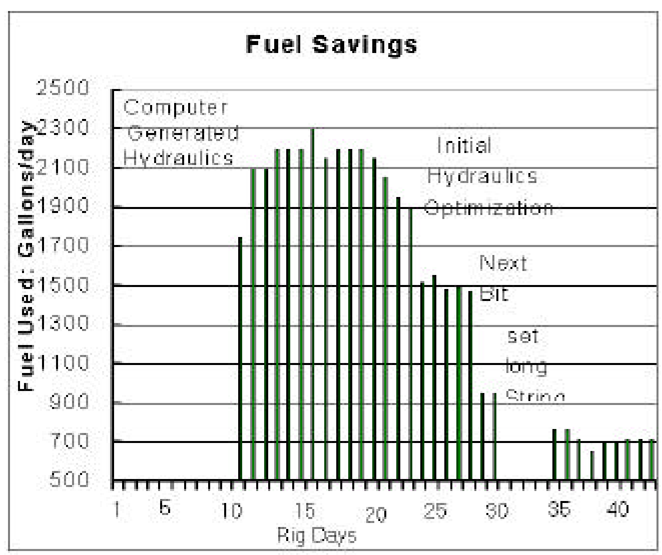

Figure 8
Fuel Usage at a Rig Before and After Hydraulic Optimization.

It seems counter-intuitive that optimizing hydraulics can result in less fuel used for running the pumps. However, field experience has shown that the tendency of field personnel is to run jets a little larger than needed, and then run the pumps a little faster in order to reach a particular standpipe pressure. This practice results in less pressure drop across the bit (as compared to optimized hydraulics), and more pressure drop in the annulus and drill pipe (due to the higher flow rates). Hence, optimizing hydraulics often has the effect of actually decreasing flow rates slightly, and hence the fuel savings.

Correspondingly, if maximum annular velocities are needed in order to clean the hole, then this consideration may take priority over bit hydraulics optimization. Even in this case, however, the above OCHO procedure can be used to optimize the jet nozzles around whatever flow and pressure drop restrictions or limitations exist.

One other feature which is frequently ignored in hydraulic optimization is the effect of hydraulic optimization on drilling rate. The derivations of the equations used do not contain a drilling rate term. (see Appendix) The assumption is made that better cuttings removal from the bottom of the hole will permit faster drilling. If the bit was drilling in a foundered condition before optimization and optimization changed that condition, significant drilling rate improvement will be noticed. If, however, the bit loading was very low and all of the cuttings are being removed from the bottom of the hole, increasing the cuttings removal capabilities will not increase penetration rates.

Conclusions

Optimization of well hydraulics, while generally understood for several years, has never reached its full potential due to the relative difficulty in the past of implementing the optimization procedures. As today's wellbores reach further and further, largely with equipment that has not changed in decades, it becomes even more important to efficiently utilize available hydraulic energy. This becomes even more important when one considers that innovative downhole tools consume more and more of the hydraulic energy seemingly every day. A simple way to optimize rig hydraulics is presented, that can be utilized with or without the use of computers on the rig site, or with rig-gathered data sent to the office.

Appendix

The pressure loss through the drilling fluid circulating (P_{circ}) system can be expressed as $P_{circ} = K\,Q^{u}$, where K is a constant, Q is the flow rate, and u is the exponent. [If the flow is turbulent, u=2.]

The Standpipe Pressure (P_{surf}) can be expressed as the sum of two pressure losses: P_{circ} and P_{bit}.

Derivation of Maximum Hydraulic Impact

The derivation for maximum hydraulic impact depends upon the limiting conditions from the drilling rig. The obvious limiting hydraulic condition will be the amount of power available to drive the mud pumps. The second limiting condition will be the maximum standpipe pressure. Each of these conditions will be discussed.

Maximum Hydraulic Impact for the Power Limited Case:

On a drilling rig, the mud pumps are powered by motors with a finite amount of power. Generally, the hydraulic power can be obtained by assuming a mechanical efficiency of power transfer of about 85% and a volumetric efficiency of 93% to 95%. So the mathematical relationship would be

$$HHP = (E_m)(E_v)(\text{Motor horsepower})$$

This hydraulic horsepower is also the product of the surface (or standpipe) pressure and the flow rate.

$$HHP = \frac{(P_{surf})(Q)}{1714}$$

Where:
HHP is the hydraulic horsepower, HP
P_{surf} is the surface (or standpipe) pressure, psi, and
Q is the flow rate, gpm.

For the hydraulic case where the limit condition is the available hydraulic power on the drilling rig, the limit condition could be expressed as:

$HHP = (P_{surf})(Q) = \text{constant} = C$

Since the standpipe (or surface) pressure consists of the sum of two components, P_{surf} can be written as:

$P_{surf} = P_{circ} + P_{bit}$

This can also be written:

$$\frac{HHP}{Q} = P_{circ} + P_{bit}$$

Solving this equation for P_{bit} and expressing P_{circ} in terms of Q and u:

$$P_{bit} = \frac{HHP}{Q} - K\,Q^{u}$$

The expression derived which related the hydraulic impact (force) to the pressure drop through the bit nozzles was:

$$F = KQ[\Delta P_{bit}]^{0.5} .$$

This force may now be calculated in terms of flow rate from the calculation for the pressure drop through the bit nozzles:

$$F = K\,Q\left[\frac{HHP}{Q} - K\,Q^{u}\right]^{0.5}$$ or rearranging terms:

$$F = K\left[Q(HHP) - K\,Q^{u+2}\right]^{0.5}$$

This is the expression for the force of the fluid striking the bottom of the hole. To find the maximum value, differentiate with respect to flow rate and set the differential equal to zero.

$$\frac{\partial F}{\partial Q} = \frac{K\left(HHP - K(u+2)Q^{u+1}\right)}{\left(Q(HHP) - K\,Q^{u+2}\right)^{0.5}} = 0$$

For this to be true, the numerator must be equal to zero or

$$HHP = K\,(u+2)\left(Q_{opt}\right)^{u+1}$$

Since HHP is the product of the standpipe pressure and the flow rate,
this could be written as

$$\left[\left(P_{\substack{surf\\opt}}\right)\left(Q_{opt}\right)\right] = K\,(u+2)\left(Q_{opt}\right)^{u+1}$$

Solving for the optimum surface (or standpipe) pressure, results in:

$$P_{\substack{surf\\opt}} = k\,(u+2)\left(Q_{opt}\right)^{u}$$

The pressure loss through the circulating system was

$P_{circ} = K\,Q^{u}$

The Optimum surface (or standpipe) pressure would therefore be

$$P_{\substack{surf\\opt}} = (u+2)\left(P_{\substack{circ\\opt}}\right)$$

The optimum pressure drop through the bit nozzles would be the difference between the optimum surface pressure and the optimum circulating pressure, or;

$$P_{\substack{bit\\opt}} = \left(1 - \frac{1}{u+2}\right) P_{\substack{circ\\opt}}$$

This can also be written:

$$P_{\substack{bit\\opt}} = \left(\frac{u+1}{u+2}\right) P_{\substack{circ\\opt}}$$

If the (u+1/u+2) fraction of the standpipe pressure is applied across the jet nozzles, the hydraulic impact will be the maximum value possible for the Hydraulic Power Limited Case.

Surface Pressure Limit.

As wells get deeper, the limits on surface pressure prevents utilization of all available hydraulic power. The surface pressure becomes the limiting condition. The pressure drop through the bit nozzles would be the difference in pressure between the maximum standpipe pressure, P_{max}, and the circulating pressure drop, P_{circ}..

$$P_{bit} = P_{max} - P_{circ}$$

The circulating pressure loss, P_{circ}, is proportional to the flow rate, Q, raised to an exponent, u, or:

$$P_{circ} = K\,Q^{u}$$

The hydraulic impact force, F, derived earlier, is related to the pressure drop across the bit nozzles, or;

$$F = KQ[P_{bit}]^{0.5} .$$

Again, to maximize the force with respect to flow rate, the force equation (expressed in terms of flow rate) must be differentiated and the differential set equal to zero.

$$F = KQ\left[P_{max} - P_{circ}\right]^{0.5}$$

$$F = KQ\left[P_{max} - K'\,Q^{u}\right]^{0.5}$$

$$F = K\left[Q^{2}\,P_{max} - K'\,Q^{u+2}\right]^{0.5}$$

$$\frac{\partial F}{\partial Q}=\frac{K\left(2QP_{max}-K'(u+2)Q^{u+1}\right)}{\left(Q^2P_{max}-K'Q^{u+2}\right)^{0.5}}=0$$

For this to be true, the numerator must be equal to zero; or

$$2QP_{max}=K'(u+2)Q^{u+1}$$

$$P_{max}=\left(\frac{u+2}{2}\right)K'Q^u$$

$$P_{max}=\left(\frac{u+2}{2}\right)\left(P_{\substack{circ\\opt}}\right)$$

Since $P_{bit}=P_{max}-P_{circ}$

$$P_{\substack{bit\\opt}}=P_{max}-\left(\frac{2}{u+2}\right)P_{max}$$

or

$$P_{\substack{bit\\opt}}=\left(\frac{u}{u+2}\right)P_{max}$$

Derivation of Equation for Maximum Hydraulic Horsepower at Bit

To find the maximum hydraulic power available at a drill bit for any flow rate, the expression for hydraulic power must be differentiated with respect to flow rate and the derivative set equal to zero.

Hydraulic horsepower at the bit has been expressed by the equation:

$$HHP_{bit}=K''P\,Q=K'(P_{max}-P_{circ})\,Q$$

The circulating pressure loss is proportional to the flow rate raised to the exponent u power. Substituting this into the HHP equation results in"

$$HHP_{bit}=K''(P_{max}-K\,Q^u)\,Q$$
$$=K''(\,QP_{max}-K\,Q^{u+1})$$

Differentiating this equation with respect to the flow rate, Q;

$$\frac{\partial(HHP)}{\partial Q}=K''\left[P_{max}-K(u+1)Q^u\right]=0$$

Since K" is not zero, the term in the bracket must be zero, or, for optimum conditions:

$$P_{max}=(u+1)\left(K\,Q_{opt}{}^u\right)=(u+1)\left(P_{\substack{circ\\opt}}\right)$$

The optimum pressure loss through the bit would be the difference between the maximum standpipe pressure (P_{max}) and the optimum circulating pressure loss (P_{circ}), or

$$P_{\substack{bit\\opt}}=P_{max}-P_{\substack{circ\\opt}}=\left[1-\frac{1}{u+1}\right]P_{max}=\left[\frac{u}{u+1}\right]P_{max}$$

Nomenclature

ΔP_{bit} – Pressure drop across the bit
$\Delta P_{bit\,opt}$ – Optimum pressure drop across the bit
E_m – Mechanical Efficiency
E_v – Volumetric Efficiency
F - force
Gpm – gallons per minute
HP – horsepower or hydraulic power
HHP – Hydraulic Horsepower
M - mass
MW – Mud weight
P_{bit} - Pressure drop across the bit
P_{circ} – Pressure drop through rig circulating system exclusive of the pressure drop through the bit
Psi – pounds per square inch
P_{max} – Maximum surface pressure available on a rig
Ppg – pounds per gallon
$P_{surf\,opt}$ – Standpipe pressure at the surface under optimum conditions
Q - Flowrate
Q_{crit} – Flowrate where the two operating limits of maximum pressure and maximum available hydraulic power intersect
Q_{opt} – Optimum flowrate for optimized hydraulics across the bit
ρ - rho - density
t - time
u – exponent of flowrate, slope of best fit line on logarithmic paper
v - velocity

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4. Warren, Tommy M., "Evaluation of Jet-Bit Pressure Losses", SPE Drilling Engineering, December, 1989 pp335-340.

NB: Most Figures Shown in altered aspect ratio to conserve space. When measuring the exponent "u" manually, the logarithmic axes must be to the same scale.

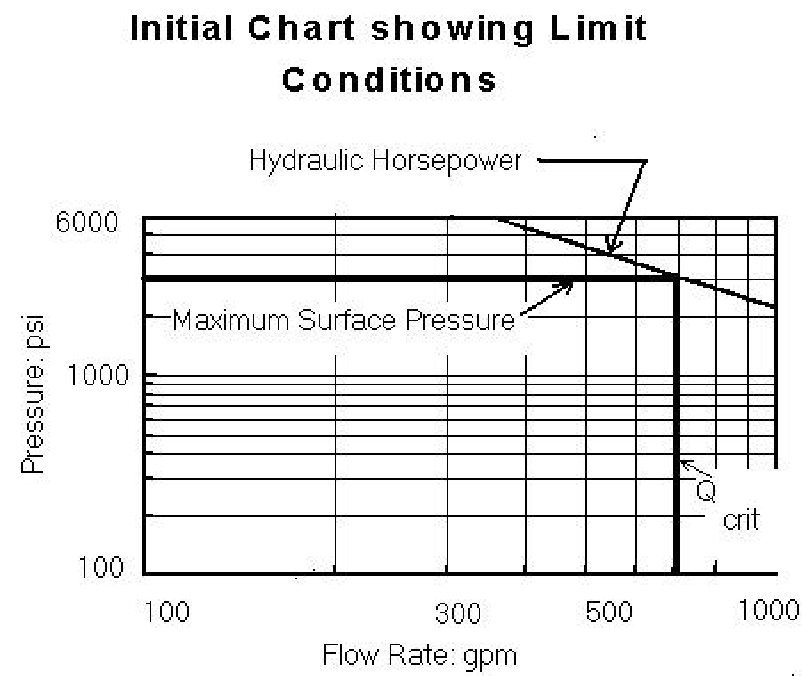

Figure 1 - Rig Limiting Conditions

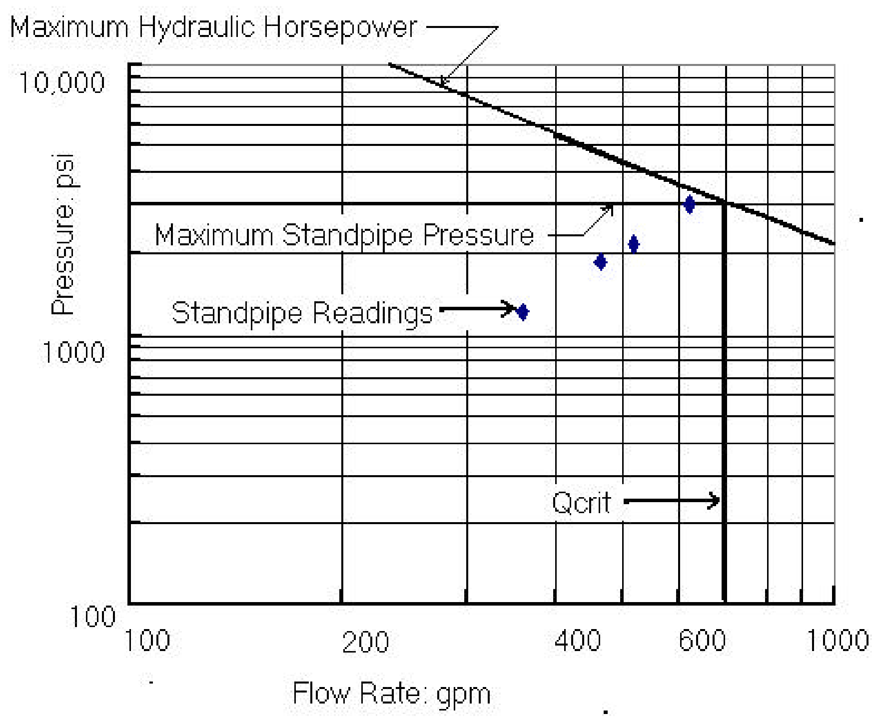

Figure 2 - Standpipe Readings Prior to Pulling Drill Bit

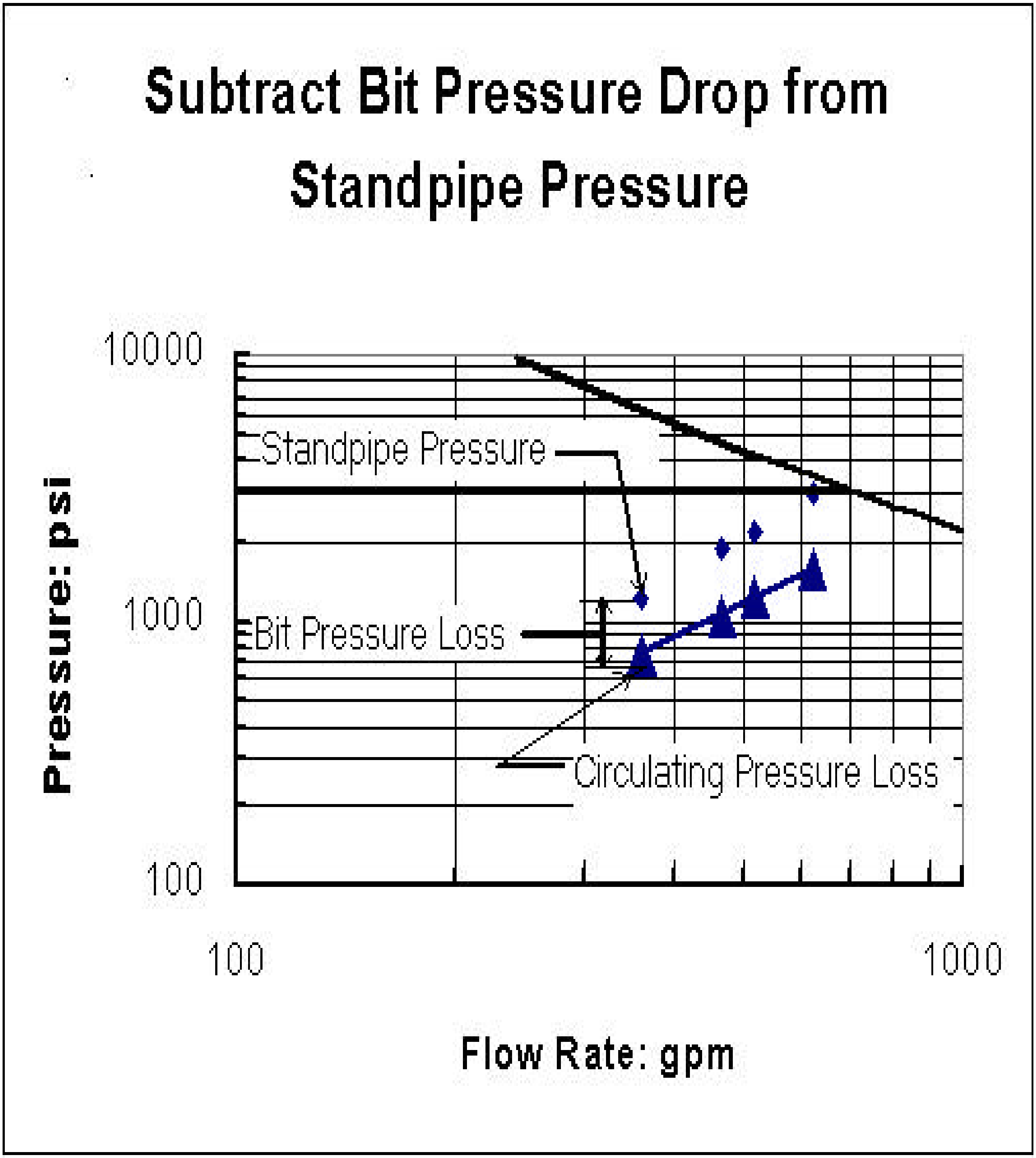

Figure 3 - Creating the Circulating Pressure Line

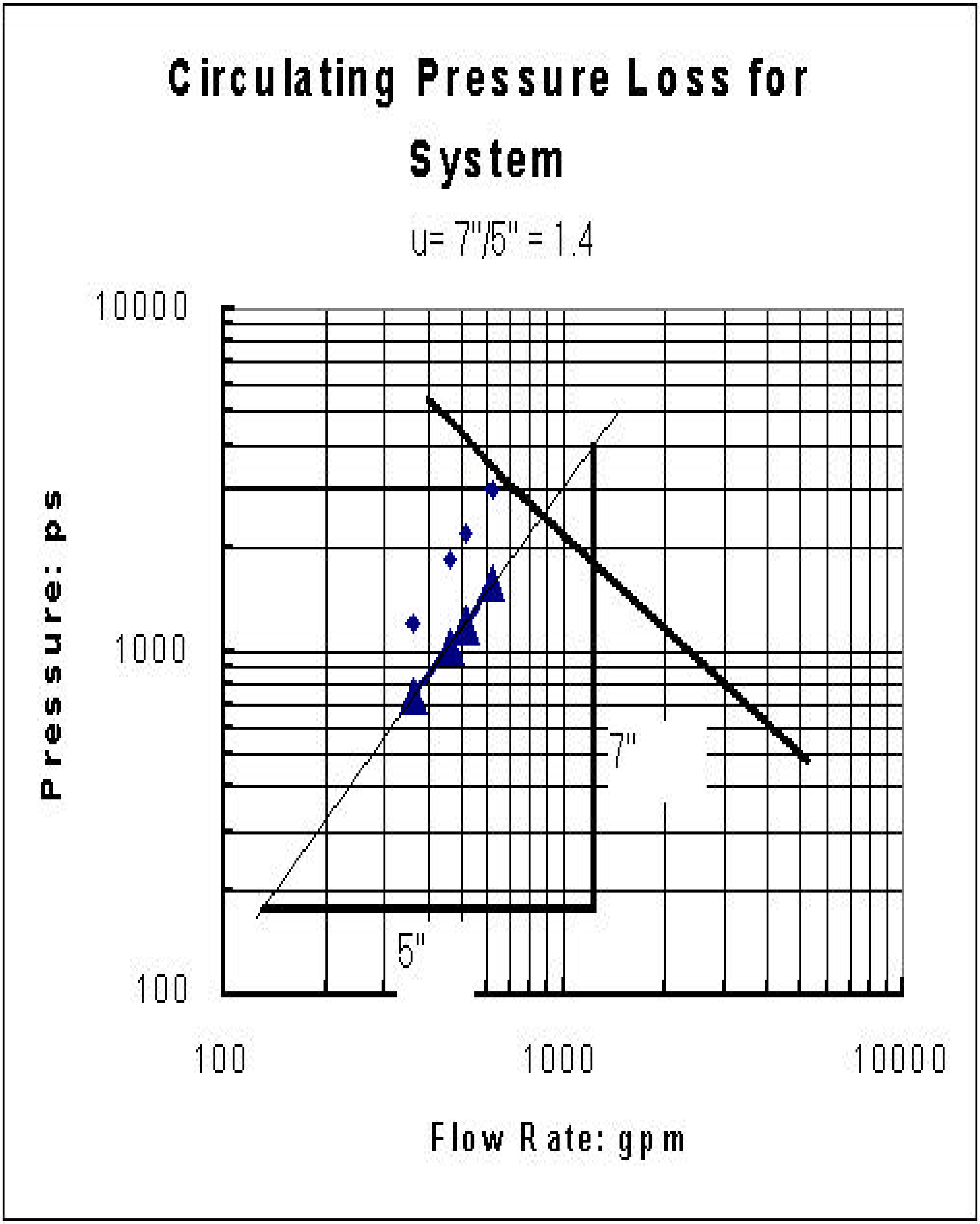

Figure 4 - Measuring the slope "u" of The Circulating Pressure Line

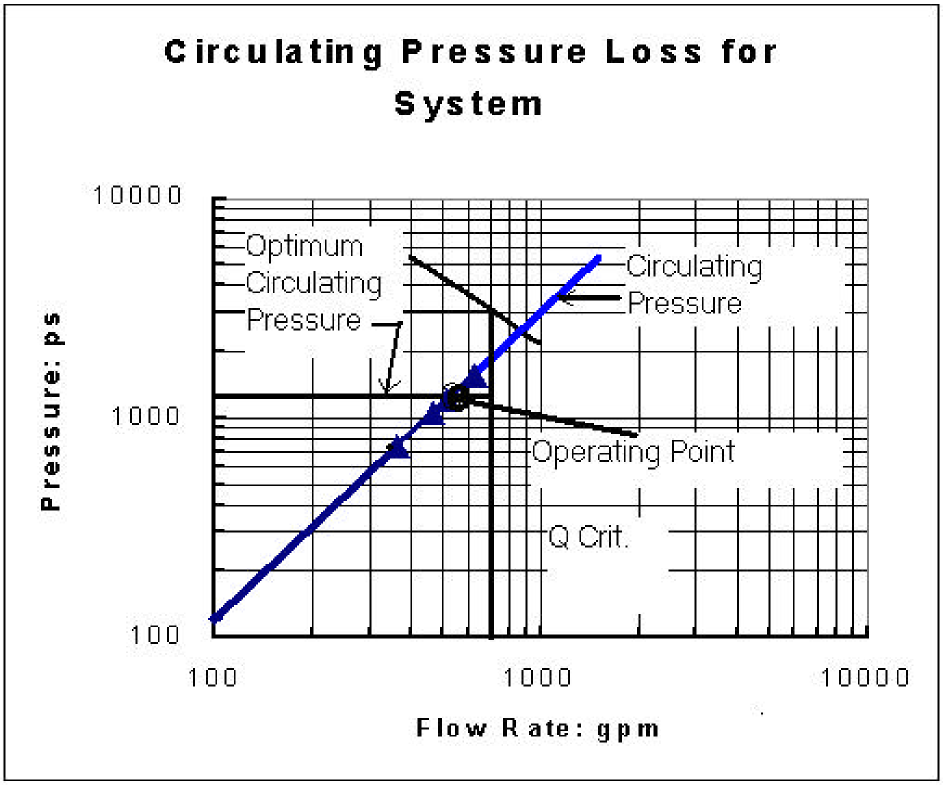

Figure 5 - Determining the Operating Point for the Next Bit.

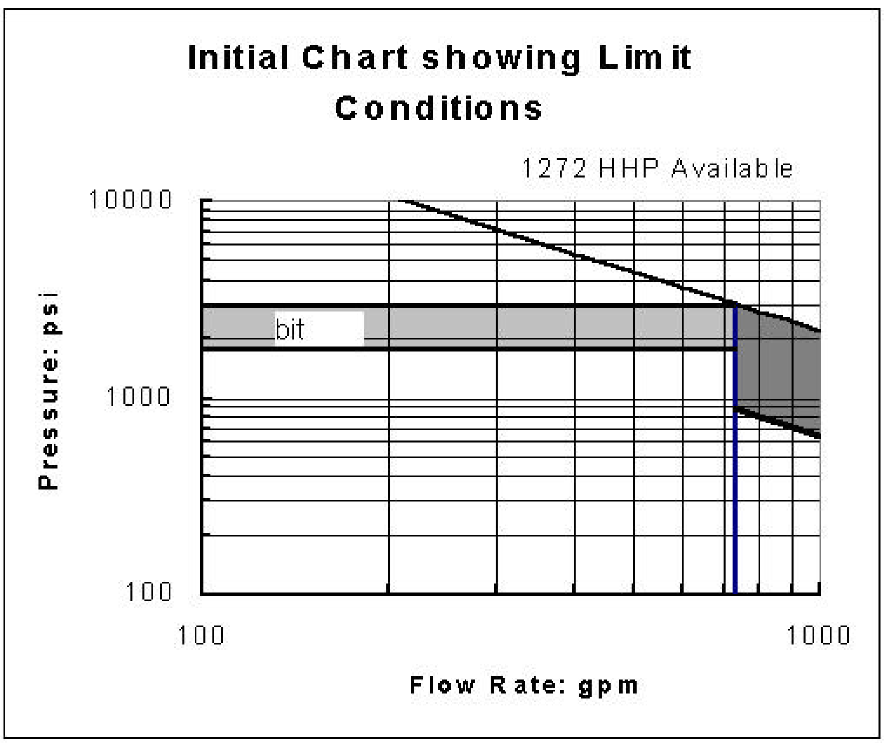

Figure 6 - Limit Conditions with Optimum Bit Pressure Loss

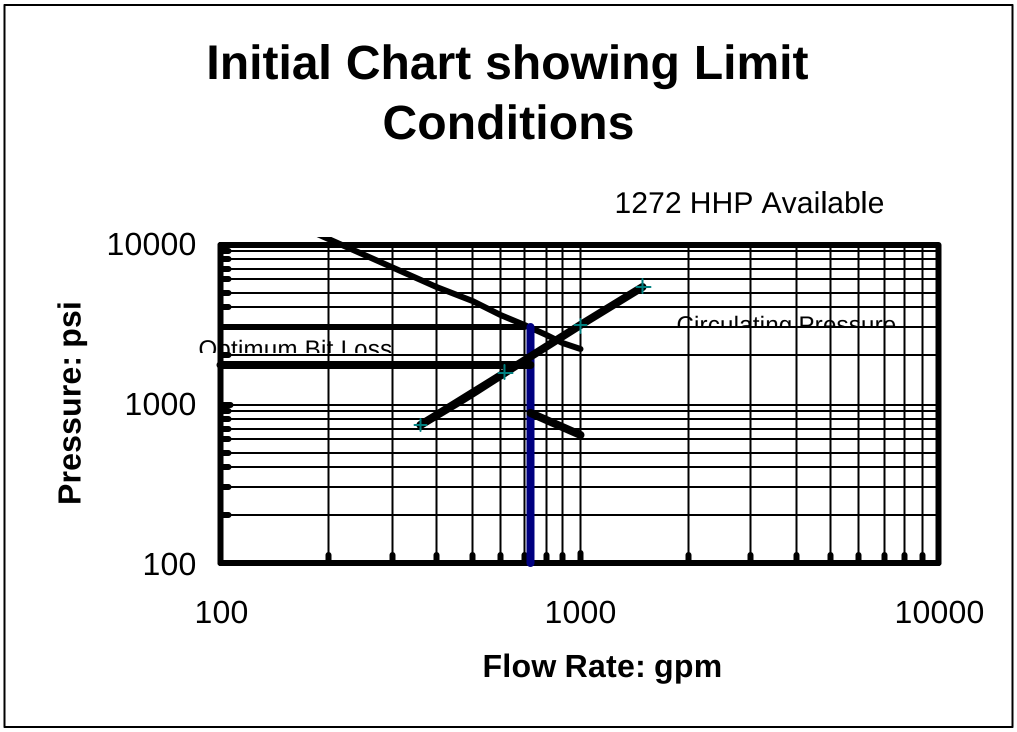

Figure 7 - Circulating Pressure Line Crosses the Optimum Pressure Line at 670 gpm

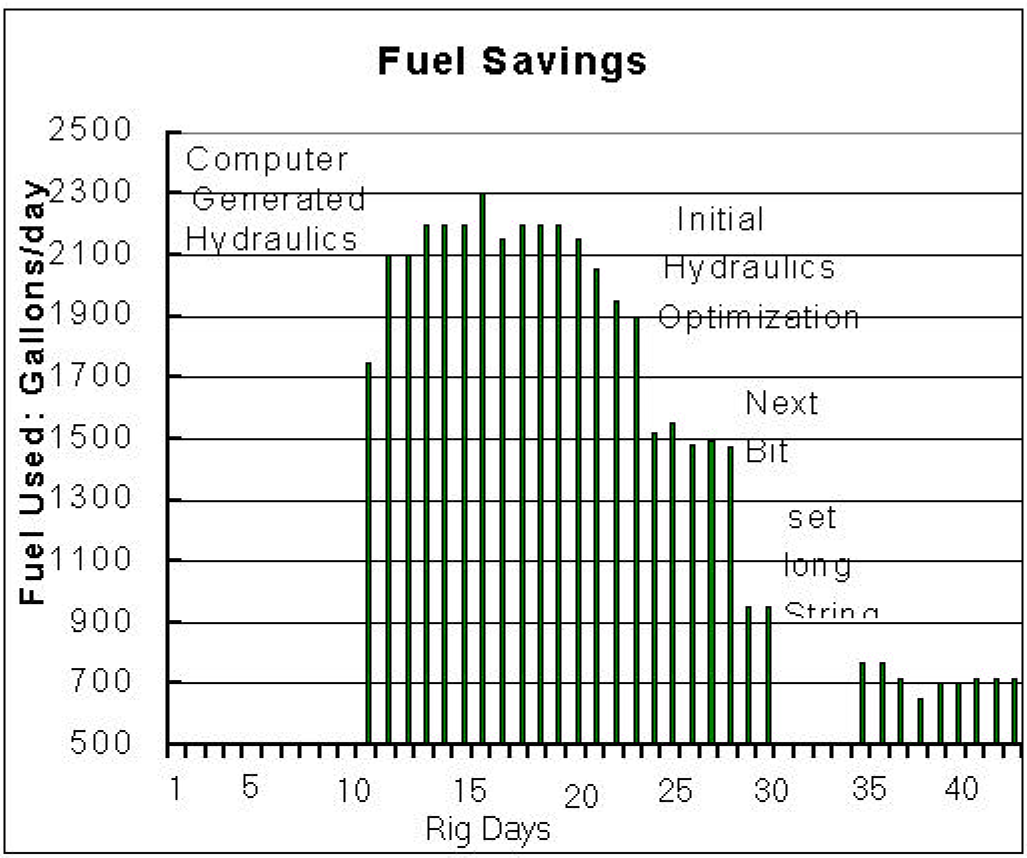

Figure 8 - Fuel Usage at a Rig Before and After Hydraulic Optimization.