## Understanding 3D Curves

Beau Bromley
February 26th, 2020
AADE Mid-Continent Symposium

## What is a 3D Curve?

## Three Dimensional Curves

- Building and turning at the same time
- Turning while at an inclination between $0^{\circ}$ and $90^{\circ}$
- Designing a directional plan to be steered between $0^{\circ}$ and $180^{\circ}$ GTF
- Also known as a Build \& Turn or Turnizontal


## Applies for all doglegs except:

1. Only build \& drop (no azimuth changes)
2. Azimuth only changes at $90^{\circ}$ inclination


## 3D Curve Benefits - Why drill them?

- Allows lower inclinations by removing the drop to vertical
- Enables larger displacements with less TVD
- Provides less rod wear during production



## 2D Curve - drop to vertical before KOP

| MD <br> $(\mathrm{ft})$ | INC <br> $\left({ }^{\circ}\right)$ | AZI <br> $\left({ }^{\circ}\right)$ | TVD <br> $(\mathrm{ft})$ | NS <br> $(\mathrm{ft})$ | EW <br> $(\mathrm{ft})$ | VS <br> $(\mathrm{ft})$ | DLS <br> $\left({ }^{\circ} / 100 \mathrm{ft}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2000.00 | 0.00 | 90.00 | 2000.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 4000.00 | 20.00 | 90.00 | 3959.63 | 0.00 | 345.54 | 42.05 | 1.00 |
| 6066.24 | 20.00 | 90.00 | 5901.26 | 0.00 | 1052.23 | 128.06 | 0.00 |
| 7066.24 | 0.00 | 90.00 | 6881.08 | 0.00 | 1225.00 | 149.08 | 2.00 |
| 7161.99 | 0.00 | 0.00 | 6976.82 | 0.00 | 1225.00 | 149.08 | 0.00 |
| 8292.39 | 90.43 | 0.00 | 7693.00 | 721.61 | 1225.00 | 865.32 | 8.00 |
| 17562.05 | 90.43 | 0.00 | 7623.00 | 9991.00 | 1225.00 | 10065.82 | 0.00 |

## 3D Curve - hold inclination through KOP

| MD <br> $(\mathrm{ft})$ | INC <br> $\left({ }^{\circ}\right)$ | AZI <br> $\left({ }^{\circ}\right)$ | TVD <br> $(\mathrm{ft})$ | NS <br> $(\mathrm{ft})$ | EW <br> $(\mathrm{ft})$ | VS <br> $(\mathrm{ft})$ | DLS <br> $\left({ }^{\circ} / 100 \mathrm{ft}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2000.00 | 0.00 | 90.00 | 2000.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3374.55 | 13.75 | 90.00 | 3361.41 | 0.00 | 164.09 | 19.97 | 1.00 |
| 7118.00 | 13.75 | 90.00 | 6997.64 | 0.00 | 1053.57 | 128.22 | 0.00 |
| 8248.25 | 90.43 | 0.00 | 7693.00 | 721.45 | 1225.00 | 865.17 | 8.00 |
| 17518.06 | 90.43 | 0.00 | 7623.00 | 9991.00 | 1225.00 | 10065.82 | 0.00 |

## 3D Curve Benefits - Why drill them?

- Enhances buckling resistance at KOP

$$
F_{\text {crit }}=2 \sqrt{\frac{E I \rho A g \sin \theta}{r}}
$$

- As $\theta$ increases, compressive load at which buckling occurs increases
- Example: 8.75" hole, 5" 19.5\# DP

| Inclination (deg) | $\boldsymbol{F}_{\text {crit }}$ |
| :---: | :---: |
| $1^{\circ}$ | $16,184 \mathrm{lbs}$ |
| $5^{\circ}$ | $36,168 \mathrm{lbs}$ |
| $10^{\circ}$ | $51,052 \mathrm{lbs}$ |
| $15^{\circ}$ | $62,327 \mathrm{lbs}$ |

## 3D Curve Detriments - Why NOT drill them?

1. It may not be needed to get to the desired location
2. It can be difficult to hold an inclination deeper in the well
3. The net forces on the motor change affecting motor yields
4. They are just more difficult to understand


## 3D Curve Case Study

## T Cosner 27-1H

- Curve built on $10^{\circ} / 100 \mathrm{ft}$
- Kickoff Point started at $13^{\circ}$ inclination, $270^{\circ}$ azimuth (west)
- Landing Point was at $89.6^{\circ}$ inclination, $180^{\circ}$ azimuth (south)
- $76.6^{\circ}$ of build, $90^{\circ}$ of turn
- Hardline to the west, can't abandon our turn in the curve

Tripped in the curve at $51^{\circ}$ inclination in the curve for build rates


## T Cosner 27-1H Dogleg Severity



## T Cosner 27-1H Dogleg Severity



## T Cosner 27-1H Toolface



## T Cosner 27-1H Toolface



## Review 2D Curves



## Curve Calculations

- Build Rates Needed to Land (linear interpolation)

$$
B R N=\frac{I N C_{L P}-I N C_{P O I}}{M D_{L P}-M D_{P O I}} \times 100
$$

- Gravity Toolface

$$
\begin{aligned}
G T F & =\cos ^{-1}\left(\frac{B R}{D L S}\right) \\
\text { Build Rate } & =\text { DLS } \therefore G T F=0^{\circ}
\end{aligned}
$$

This is constant throughout the curve

- Radius of Curvature

$$
R O C=\frac{18,000}{\pi \times D L S}
$$

## Differences for 3D Curves



## What's different?

- Radius of curvature circle rests in a slice of a sphere
- We use DLS needed to land, not BUR needed to land


## Differences for 3D Curves



## Curve Calculations

- Build Rates Needed to Land
- Turning is harder at higher inclinations
- "Get your turn in first"

$$
B R N=? ? ?
$$

- Gravity Toolface

$$
G T F=\cos ^{-1}\left(\frac{B R}{D L S}\right)
$$

$$
G T F=? ? ?
$$

What toolface should be held in the middle of a 3D Curve?

- Radius of Curvature

$$
R O C=\frac{18,000}{\pi \times D L S}
$$

## Differences for 3D Curves



Primary problem is that 3D curves drill on a rotated plane

- 2D Curves lie flat on two standard axes: TVD \& Vertical Section
- 3D Curves still lie on a flat plane, but it is rotated between our three standard axes
- Converting an arc of a sphere to vertical and horizontal components is difficult to grasp


## Differences for 3D Curves



## Solution

1. Define Points $1 \& 2$ as vectors (1: KOP, 2: Landing Point)

$$
\begin{array}{ll}
\boldsymbol{u}_{\mathbf{1}}=<\sin I_{1} * \cos A_{1}, & \sin I_{1} * \sin A_{1}, \cos I_{1}> \\
\boldsymbol{u}_{2}=<\sin I_{2} * \cos A_{2}, & \sin I_{2} * \sin A_{2}, \cos I_{2}>
\end{array}
$$

2. Define Normal Vector to the 2D Plane

$$
N=u_{1} \times u_{2}
$$

3. Define Radius Vector from Point 1 to the Center of the Sphere $R=N \times u_{1}$
4. Solve for position in terms of $x^{\prime}$ and $y^{\prime}$ in 2D


## Solution

## 5. Project $x^{\prime}$ and $y^{\prime}$ back to EW, NS, TVD cartesian coordinates

- Extend the scalar $x^{\prime}$ value in the direction of the $R$ vector
- Extend the scalar $y^{\prime}$ value in the direction of the $u_{1}$ vector $<\boldsymbol{x}^{\prime}, \boldsymbol{y}^{\prime}>\rightarrow<\boldsymbol{u}_{E W}, \boldsymbol{u}_{\boldsymbol{N S}}, \boldsymbol{u}_{\boldsymbol{T V D}}>$ $<\boldsymbol{x}^{\prime}, \boldsymbol{y}^{\prime}>\rightarrow$

$$
\begin{aligned}
& \Delta \mathrm{EW}=\mathrm{x}=x^{\prime} \cdot \mathbf{R}_{\mathrm{x}}+y^{\prime} \cdot u_{1 \mathrm{x}} \\
& \Delta \mathrm{NS}=\mathrm{y}=x^{\prime} \cdot \mathbf{R}_{y}+y^{\prime} \cdot u_{1 y} \\
& \Delta \mathrm{TVD}=\mathrm{z}=x^{\prime} \cdot \mathbf{R}_{z}+y^{\prime} \cdot u_{1 \mathrm{z}}
\end{aligned}
$$

6. Calculate MD, INC, AZI from EW, NS, TVD
7. Calculate DLS, BUR, TR, GTF from MD, INC, AZI


## Examples - 2D Curve



## Examples - Large Turn and Curve



## Examples - Small Turn and Curve



## Examples - Large turn at some Inclination




## Rules of Thumb

How can I get the $\mathbf{8 0 \%}$ answer without remembering Calculus 3?

- Get all of the planned turn in by $30^{\circ}$ inclination
- Plan curves with more motor yield capacity than needed


## When does this really matter?

- When turning more than $30^{\circ}$ azimuth throughout a curve
- When turning at high inclinations (> $10^{\circ}$ )

How can I see this result without working this solution?

- Export directional plan in 5-25 ft intervals instead of typical 100 ft intervals
- Use the gravity toolface equation:

$$
G T F=\cos ^{-1}\left(\frac{B R}{D L S}\right)
$$

Thank you.


## Incorrect Linear Interpolation on 3D Curves

- Black: Incorrect Interpolation
- Red: Correct Interpolation



