

Time Domain Modeling of Torsional Stick-Slip in Vertical Wells: Comprehensive Evaluation with WellScan Simulator

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Abstract

The study provides a comprehensive evaluation of torsional stick-slip through a detailed numerical simulation using the time domain module of WellScan Simulator. Building on prior studies, the research reviews various techniques and introduces an advanced numerical model to bridge the gap between theory and application. An upscaled drillstring is used, encompassing diverse components and casing strategies, offering a realistic depiction of vertical well drilling dynamics. Input parameters such as torque on the bit (TOB) adjustments are used to analyze drilling dynamics. Results reveal the dynamic response of the system to TOB variations, showcasing stick-slip events and their intricate influence on rotational behavior. Variations in TOB intensify sticking phases, prolonging drilling challenges. The findings address the challenges posed by torsional stick-slip, enhancing the sustainable and efficient extraction of hydrocarbon resources in vertical well drilling.

Introduction

The drilling industry continually faces challenges in optimizing operations for increased efficiency and safety (Adumene and Hope, 2022; Abdalla, 2023; Ejike and Tian, 2022; Ejike and Tian, 2022(b); Saldivar et al., 2014; Li et al., 2021; Edouard et al., 2022; Edouard et al., 2023). Among the issues encountered during vertical well drilling, torsional stick-slip stands out as a critical phenomenon that significantly impacts drilling performance (Liu et al., 2019; Tian et al., 2020). Torsional stick-slip involves the unpredictable and oscillatory motion of the drillstring during rotation, leading to inefficiencies, wear and tear on equipment, and potential safety risks (Ejike et al., 2024; Omojuwa et al., 2011; Ejike, 2021; Alrassas et al., 2024; Tang et al., 2017). The significance of studying torsional stick-slip extends beyond its immediate impact on drilling operations. Economically, the drilling industry is under constant pressure to optimize processes (Ejike, 2019), reduce downtime, and enhance drilling efficiency. In that respect, torsional stick-slip issues should be addressed in order to lower operational costs and delays (Sharma et al., 2023; Srivastava et al., 2022; Srivastava et al., 2023). Furthermore, the safety implications of this phenomenon underscore the need for a comprehensive evaluation, as uncontrolled stick-slip events can pose risks to both personnel and equipment integrity.

Several studies for drillstring vibrations have been performed so far. Germay et al., 2009 derived an analytical representation of the velocity-weakening friction law using a toothed-bit model. This analysis showed the significance of bit-rock interactions in the initiation of stick-slip vibrations. Navarro-Lopez and Cortes, 2007 utilized dynamic sliding mode control, established models for the non-continuous torsional dynamics with dual non-continuous surfaces, managed the rotary speed of the oilwell drill string, and effectively alleviated self-generated stick-slip vibrations. A new method for inspecting the Anti-Stick-Slip Tool at the drill string's terminal end was introduced by Taraghi et al., 2023. This model solves the issues found in previous analyses by including two degrees of freedom in the inactive state and increasing to three degrees upon activation. A modified integral resonant control system with tracking capabilities was developed by Maclean et al., 2022 to reduce stick-slip vibrations and achieve the desired drilling velocity in drill strings. This method outperformed sliding mode control in terms of performance. Krama et al., 2022 presented a robust active controller that uses the fuzzy sliding mode technique to reduce stick-slip vibrations in drill strings. Maintaining the angular velocity of drill components at predetermined values was the goal. Navarro-Lopez and Cortes, 2007 formulated a dynamics model for oilwell drilling, examined the problem of self-induced vibrations in the bottom hole assembly, and presented essential recommendations for selecting drilling parameters to prevent torsional vibrations in the drill string. Richard et al., 2007 suggested that velocity weakening is a system reaction rather than an inherent trait. In their study, Richard and Detournay, 2000 explained the mechanisms underlying both normal and tangential vibrations within stick-slip limit cycles, utilizing a two-degrees-of-freedom model. To reduce limit cycles in drill string operations, Canudas-de-Wit et al., 2008 developed an improved OSKIL mechanism known as D-OSKIL, which adds axial load as an additional control parameter. The closed-loop system's global asymptotic stability was guaranteed by this control method.

Torsional vibration within a small-scale drillstring has been the subject of investigation by Srivastava and Teodoriu, 2019. Their study provides a comprehensive review of existing downscaled laboratory test rigs, aiming to propose a

standardized approach to experimental design. This involves addressing critical aspects such as the determination of an appropriate downscaling factor, methods for rock-bit interaction, and the selection of suitable components and materials. Building upon this foundation, subsequent work delved into exploring the impact of proper sampling rates on diagnosing stick-slip vibrations, as undertaken by Sharma et al., 2020. Their research sought to unravel how the choice of sampling frequency influences the accuracy and effectiveness of detecting stick-slip vibrations. In a more recent investigation, Srivastava et al., 2022 delved into the intricate details of factors influencing the performance of various popular classifiers when dealing with drilling data. Their study considered variables such as data sampling frequency, data labeling technique, feature extraction technique, and class imbalance. The researchers not only provided insights into the implications of these factors but also offered recommendations specific to vibration classification, while shedding light on the potential pitfalls associated with certain techniques in the context of drilling data. Interestingly, the stick-slip index was found to vary across different sampling frequencies, despite the vibration tests remaining entirely identical. This observation led to the inference that the sampling frequency of measurements significantly impacts the accuracy and efficacy of downhole vibration detection, as highlighted by Srivastava et al., 2023.

As drilling depths and complexities increase, there is a growing need for precise and reliable numerical models to predict and manage torsional stick-slip. Previous studies frequently fall short in capturing the nuances of this phenomenon, leading to a demand for advanced and comprehensive modeling approaches. This paper endeavors to bridge the gap between theoretical understanding and practical applications by evaluating torsional stick-slip vibrations in an upscaled drillstring. The goal is to advance drilling technologies and contribute to the sustainable and efficient extraction of hydrocarbon resources through a thorough evaluation using the WellScan Simulator.

Numerical Simulation

Drillstring Details

The study utilizes the wellscan simulator as a tool to create a detailed model of an upscaled drillstring, with the primary aim of investigating torsional stick slip phenomena in vertical wells. The upscaled model uses the laboratory small scale setup presented by Srivastava and Teodoriu, 2019 with a total length of 15 meters. The geometrical scaling factor is 1:30 (Srivastava, 2022). We then obtain an upscaled well of 450 meters. Previous simulations of the downscaled model have shown a good correlation between the simulator results and the small scale experimental setup (Ejike et. al., 2024). Hence this paper will show the upscaling potential of the simulator. The simulated drillstring consisted of various components comprehensively presented in Table 1. The bottom-up approach of these components plays a pivotal role in shaping the overall dynamics and behavior of the drillstring, particularly concerning the

phenomenon of torsional stick slip under investigation. Furthermore, the choice of materials for the heavy weight drill pipe (HWDP) and drill pipe (DP) components is specified as S135, providing crucial information about the grade of these elements within the drillstring.

Trajectory

The study begins its assessment with an initial measured depth (MD) of 0 meters, marking the starting point for the investigation. The final measured depth, concluding the survey, is reached at 450 meters (figure 1). This intentional range in measured depth captures a significant vertical span, allowing for a comprehensive analysis of the subsurface conditions under analysis. Table 2 presents a detailed breakdown of the survey data, offering information crucial for the understanding of the wellbore. This includes comprehensive details such as the MD, inclination, azimuth, and true vertical depth (TVD). The inclination values show the wellbore's deviation from the vertical plane, providing insights into the trajectory of the drilled path. The azimuth values denote the horizontal orientation of the wellbore, indicating its alignment concerning the magnetic or true north. Additionally, the TVD provides a critical measure, representing the vertical distance from the wellbore's origin to a specific point in the subsurface. This depth metric is vital for accurate spatial representation and analysis, contributing significantly to the overall characterization of the wellbore's geometry.

Table 1: Bottom hole tools used in the simulation (bottom up), polycrystalline diamond compact (PDC), near bit stabilizer (NBC), short drill collar (SDC), variable gauge stabilizer (VGS), drill collar (DC), string stabilizer (SS), measurement while drilling (MWD), heavy weight drill pipe (HWDP), and drill pipe (DP)

Type	Name	Length (m)	OD (inch)	ID (inch)	Gauge (inch)	Total length (m)	Contact (m)	Mass (kg)	Total mass (kg)	Linear mass (kg/m)
PDC	-	0.45	-	-	5 5/8	0.45	-	28.16	28.16	62.59
NBS	-	1.6	4 1/2	2 1/8	5 5/8	2.05	1.25	100.14	128.3	62.59
SDC	-	1.75	4 1/2	2 1/8	-	3.8	-	109.53	237.83	62.59
VGS	-	3	4 1/2	2 1/8	VARIABLE	6.8	5.3	187.76	425.58	62.59
SDC	-	3.7	4 1/2	2 1/8	-	10.5	-	231.57	657.15	62.59
DC	-	9	4 1/2	2 1/8	-	19.5	-	563.27	1220.42	62.59
SS	-	1.6	4 1/2	2 1/8	5 5/8	21.1	20.3	100.14	1320.56	62.59
MWD	-	10	4 1/2	2 1/8	-	31.1	-	625.86	1946.42	62.59
Generic	Sensor Package	-	-	-	-	-	24.1	-	-	-
DC x 6	-	54	4 1/2	2 1/8	-	85.1	-	3379.64	5326.06	62.59
HWDP x 10	-	95	3 1/2	2 1/16	-	180.1	-	3170.08	8496.14	33.37
DP x 28	-	266	3 1/2	2 5/8	-	446.1	-	6416.2	14912.3	24.12

Table 2: Survey

MD (m)	Inclination (deg)	Azimuth (deg)	TVD (m)
0	0	0	0
100	2	0	99.98
300	2	0	299.86
450	1	0	449.8

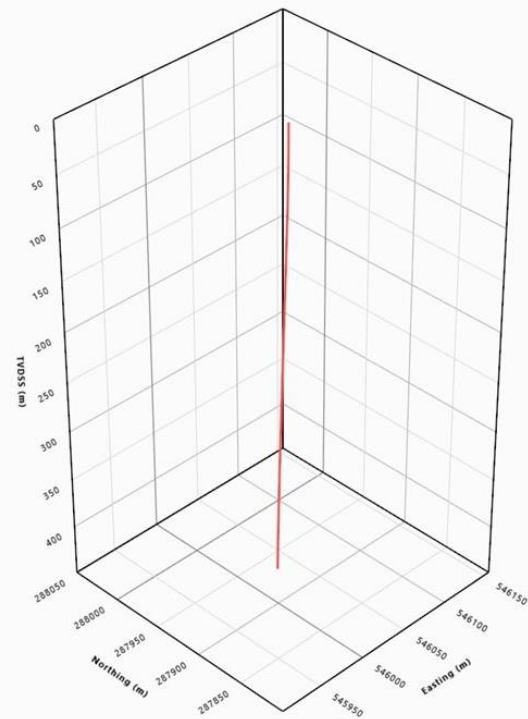


Figure 1: Trajectory

Casing Program

To ensure the structural integrity and stability of the wellbore, an upscaled casing program is implemented. Although this is a shallow well, derived from our upscaling technique, we decided to include a conductor, surface, and an intermediate casing string. The conductor and surface casings are not used in the simulation and do not play any role. However in order to have a complete wellbore schematic and given the utmost importance of conductor and surface casing in the well construction we decided to show them. The conductor casing seamlessly spans from the wellbore's origin to a depth of 50 meters, providing crucial support and preventing potential

instability in the upper sections of the wellbore. Following the installation of the surface casing to a depth of 150 meters, the intermediate casing string is then run to a depth of 300 meters. The schematic of the casing used in this study is presented in Figure 2. The casing fortifies the wellbore by addressing challenges encountered in the downhole and ensures the integrity of the well. Table 3 presents data on the casing used in the simulation, including information on material type and grades. The inclusion of these specifications is vital for comprehending the mechanical properties and the resistance of the casing components against downhole pressures, corrosive environments, and other operational considerations.

Input Parameters

In the investigation of drilling dynamics, the time domain module is used to monitor the rotational behavior of the drilling bit closely. This focused analysis involves subjecting the drillstring to varying rotation speeds at the top, as illustrated in Figure 3, over a duration of 30 seconds. Concurrently, the torque on the bit (TOB) is adjusted according to the specifications outlined in Table 4, with the drilling operation being conducted at a bit depth of 450 meters. A set of carefully chosen input parameters was utilized to capture the intricacies of these dynamic conditions accurately. A bit rock law is formulated, featuring a dynamic/static TOB ratio of 100%, and a critical rotation speed was set at 10 RPM. These parameters play a pivotal role in simulating the mechanical interactions between the bit and the subsurface formations during drilling operations. Moreover, to account for frictional influences within the wellbore, a casing friction factor of 0.2 and an open hole friction factor of 0.3 are integrated into the simulation. These factors contribute to the overall drilling dynamics, affecting the torque and rotation behavior of the bit as it engages at the specified depth.

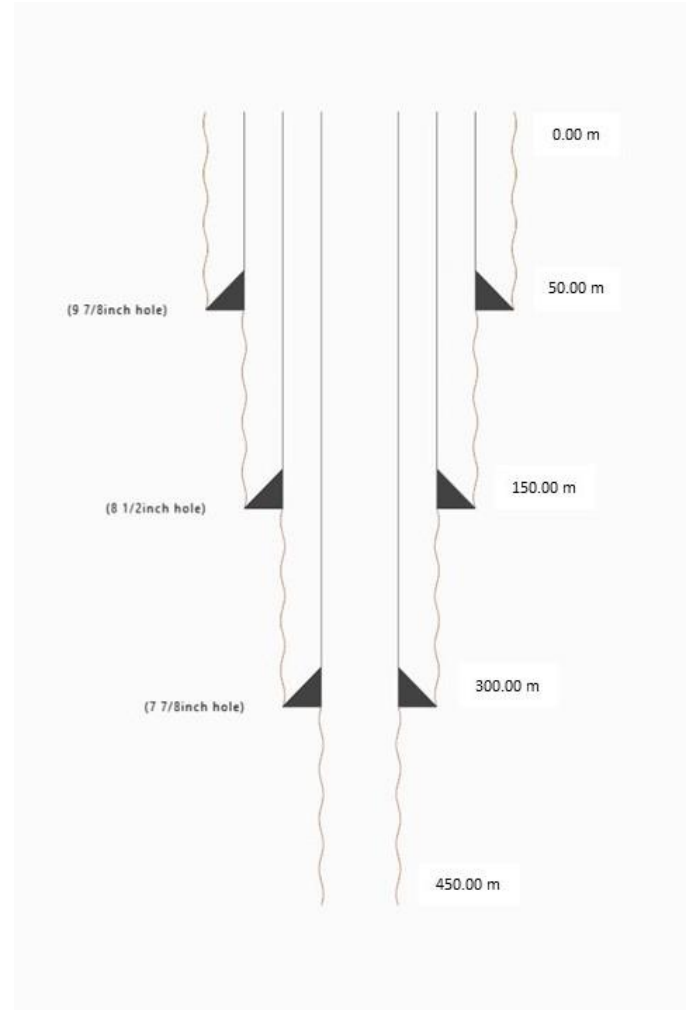


Figure 2: Casing program

Table 3: Casing Section

Hole size (inch)	Top MD (m)	Final MD (m)	Material	Grade
9 7/8	0	50	Steel	P110
8 1/2	0	150	Steel	P110
7 7/8	0	300	Steel	P110

Table 4: Varying TOB

	TOB (Kgf.m)
1	500
2	800
3	1100
4	1400

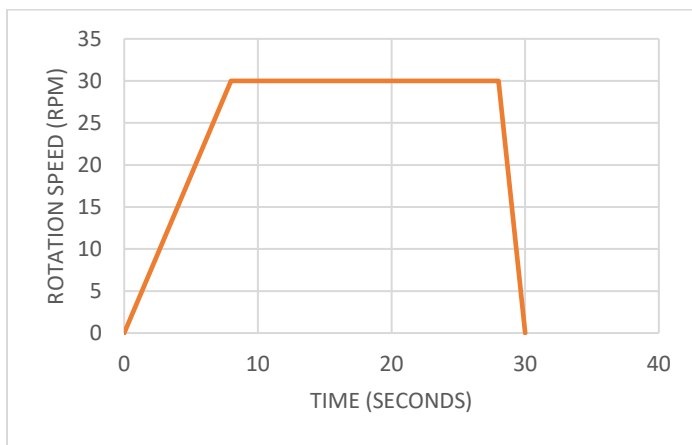


Figure 3: Top rotation speed

Results

In Figure 4, a comprehensive analysis of the rotation speeds at the bit, and the top is presented, providing insights into the system's behavior under a TOB value of 500 kgf.m, while keeping other simulated parameters constant. The blue spikes depict the rotation speed at the bit, whereas the orange line represents the top rotation speed. The figure shows a notable difference in rotation speeds between the two components. At the initiation of the run, a distinct occurrence is observed as the rotation speed at the bit experiences a slip, reaching almost double the rotation speed recorded at the top. This initial stick event persists for a duration extending from 2 to 5.2 seconds, as clearly highlighted in the graph. Subsequently, there is a significant change in the system dynamics. Beyond the 5.2 second mark, the rotation speed at the bit undergoes a significant slip to 35 RPM, while the rotation speed at the top for the same time interval remains at 21 seconds. This prolonged slipping phase is maintained for a considerable duration before a convergence point is reached. After a series of slips, a transition becomes evident in the latter part of the run. The rotation speed at the bit gradually aligns itself with that at the top, indicating a synchronization in their rotational behavior. This convergence implies an equilibrium in the drilling dynamics, signifying a stabilization of the system after the earlier slipping episodes. The observed differences and subsequent adjustments in rotation speeds provide valuable insights into the transient behavior of the drilling process under the specified TOB conditions.

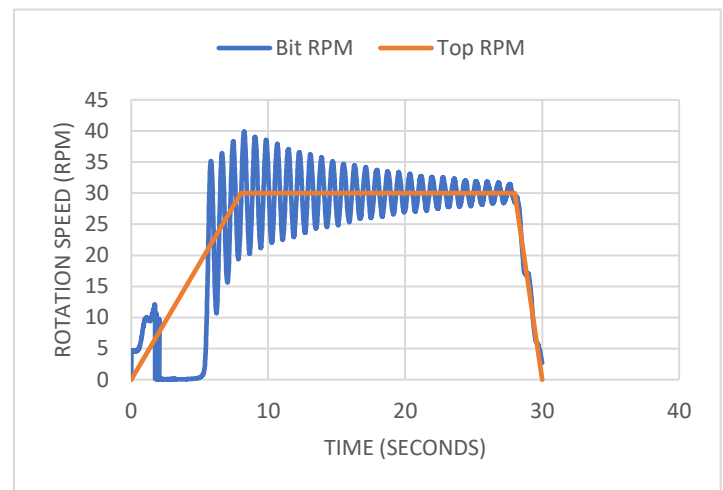


Figure 4: TOB at 500 kgf.m

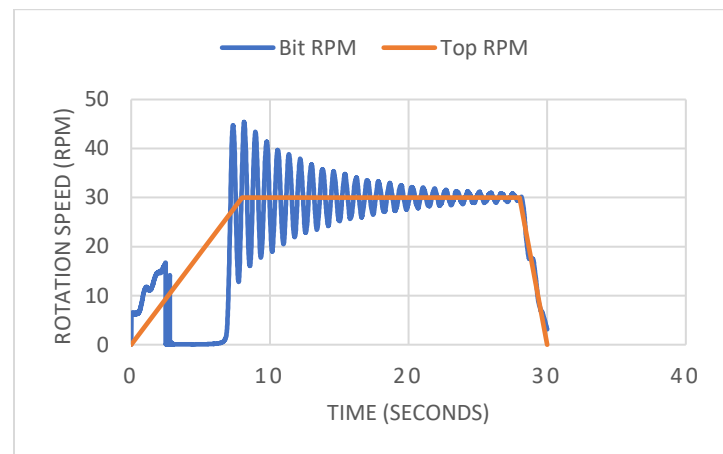


Figure 5: TOB at 800 kgf.m

In the simulation's next phase, the TOB value was kept at 800 kgf.m, resulting in notable changes in the drilling dynamics. Figure 5 vividly illustrates the impact of this TOB modification on the system's behavior. Specifically, the sticking phase exhibits a substantial extension in duration, lengthening from 3 seconds to an extended period of 6.8 seconds. This lengthened sticking phase suggests that the higher TOB introduces increased resistance or challenges in the drilling process, causing the drillstring to adhere to the wellbore for an extended duration. The graph serves as a visual representation of this phenomenon, providing a clear understanding of how alterations in TOB can influence the temporal aspects of sticking. Furthermore, a distinct slip event is observed in the data, occurring at 7.3 seconds into the run. During this slip phase, the rotation speed at the bit increased to 45 RPM, highlighting the system's dynamic response to the elevated TOB. The slip at this specific moment adds an additional layer of complexity to the drilling dynamics, signifying a momentary

release of rotational constraints before potentially readjusting to a new equilibrium state.

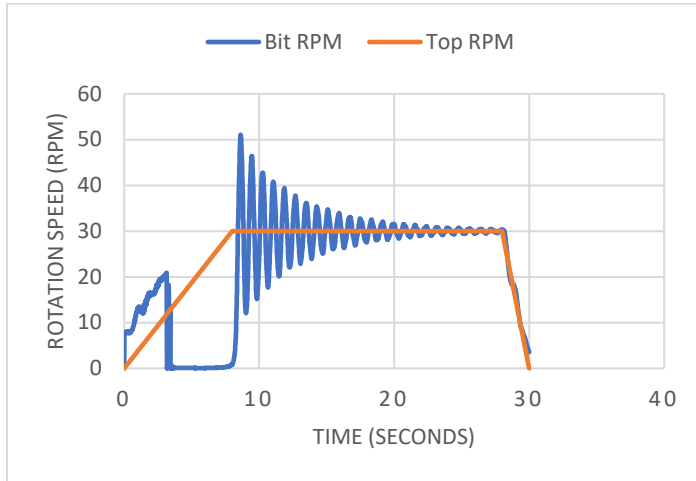


Figure 6: TOB at 1100 kgf.m

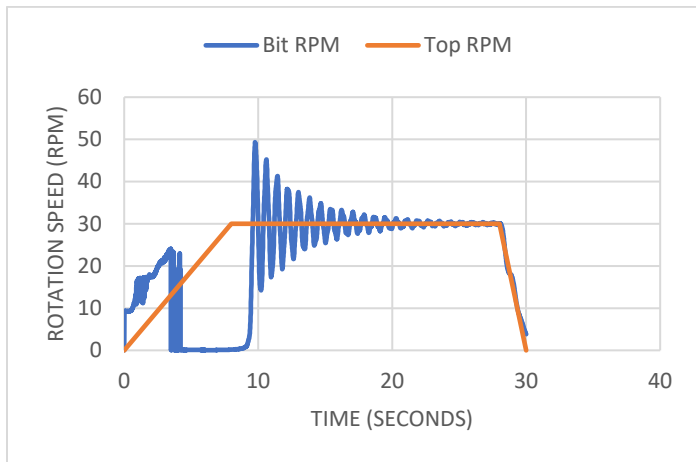


Figure 7: TOB at 1400 kgf.m

With a further escalation in TOB to 1100 kgf.m, the drilling dynamics underwent significant changes, as depicted in Figure 6. The graph highlights a distinct stick-slip occurrence, which spans from 3.1 seconds to 3.4 seconds. This interval indicates a brief period of rotational slip followed by adherence, reflecting the complex interaction between the drillstring and the wellbore under the influence of the heightened TOB. Following the stick-slip phase, the system transitions into an extended sticking period, persisting from 3.3 seconds to 8 seconds. It can be observed that the maximum slip recorded at the bit during this interval reached 52 RPM. This peak slip value provides quantitative insight into the momentary release of rotational constraints and the subsequent acceleration of the bit's rotation. Such detailed information is crucial for understanding the transient behavior of the system during stick-slip events. Interestingly, at 27 seconds into the run, a convergence is observed in the rotation speeds between the bit and the top of the drillstring. This synchronization indicates the leveling of rotation speeds between different sections of the drilling

assembly. The system seems to have adapted to the applied TOB, leading to this alignment in rotation speeds.

Upon increasing the TOB to 1400 kgf.m, Figure 7 shows a distinct shift in drilling dynamics. Right from the onset, there is a visible divergence in rotation speeds, with the bit consistently rotating at twice the speed of the top, as evidenced by the blue spikes and orange line, respectively. The graph captures a notable stick-slip event that initiates at 3.5 seconds and persists until 4.2 seconds. This interval showcases a cyclic pattern of slips followed by sticking, illustrating the intricate rotational dynamics induced by the elevated TOB. The bit remains static during the sticking phase from 4.2 seconds to 9.1 seconds and do not rotate. This extended period of bit immobility explains the challenges caused by the high value of TOB. At 9.2 seconds, a sudden release from the sticking phase is observed as the bit experiences a slip of 50 RPM. Notably, this slip occurs while the rotation speed at the top remains constant at 30 RPM, underscoring the system's sensitivity to changes in TOB. The temporal progression of these events provides an understanding of how the drilling system dynamically responds to variations in applied torque, highlighting the phases of sticking and slipping and their intricate influence on the overall drilling process.

Discussions

The exploration of various components and casing strategies within an upscaled drillstring aimed to bridge the gap between theoretical understanding and practical application. The study successfully builds upon prior research by employing a detailed model and advanced simulation techniques. The choice of the drillstring components adds credibility to the study, ensuring that the materials used in the simulation align with industry standards. The results presented a dynamic response of the system to variations in TOB, revealing intricate stick-slip events and their influence on rotational behavior. The impact of TOB on drilling dynamics was evident in the simulations. For instance, under a TOB of 500 kgf.m, an initial slip at the bit led to a prolonged sticking phase, emphasizing the sensitivity of the system to torque variations. As TOB increased, the sticking phases extended, posing challenges to drilling efficiency. The system, however, exhibited adaptability, with rotation speeds converging after stick-slip events, indicating a dynamic equilibrium.

Conclusions

- The study explains the significance of time domain modeling in understanding torsional stick slip in vertical wells
- A detailed numerical simulation of an upscaled drillstring is presented using a WellScan simulator.
- Carefully chosen input parameters, including TOB adjustments and casing friction factors, drive a focused drilling dynamics analysis.

- The study provides valuable insights into the rotational behavior of the drillstring under different TOB conditions, explaining the transient nature of the drilling process.
- The results reveal the dynamic response of the system to variations in TOB. At 500 kgf.m, distinct stick-slip events occur, with subsequent adjustments leading to rotational synchronization..
- The escalating TOB to 1,400 kgf.m results in pronounced divergence in rotation speeds, extended sticking phases, and dynamic responses, including static periods of the bit.
- Increasing TOB intensifies sticking phases, prolonging drilling challenges.
- The ability to create multiple scenarios for a given situation is a great help to understand and mitigate dangerous stick slip situations.

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Nomenclature

PDC = Polycrystalline diamond compact

NBS = Near bit stabilizer

SDC = Short drill collar

VGS = Variable gauge stabilizer

DC = Drill collar

MWD = Measurement while drilling

HWDP = Heavy weight drill pipe

DP = Drill pipe

MD = Measured depth

TVD = True vertical depth

OD = Outer diameter

TOB = Torque on bit

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